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*in Mechanical Engineering*

# **ENGINEERING THERMODYNAMICS**

# ENGINEERING

# *Thermodynamics*

## Theory and Applications

SECOND EDITION

5425

*By*

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## PREFACE TO THE SECOND EDITION

Careful re-examination of the First Edition of **ENGINEERING THERMODYNAMICS** has resulted in a revision in which many passages, and in some instances whole sections, have been reworked. This has been done without changing the original purpose of the book: to present thermodynamics in a simple, straightforward manner so that the average student can grasp this important phase of engineering without undue difficulty.

Also, a chapter on heat transfer has been added, in recognition of the growing importance of that subject for the several fields of engineering.

The problems which in the previous edition had been included at the end of each chapter have now been grouped together at the end of the book, and to these have been added many new problems, so that the total number now exceeds eight hundred. These problems have not been devised as routine exercises to provide work for the student, but rather to give him an opportunity to apply fundamental thermodynamic principles. Students should be encouraged to compare with data from their past experience the answers they obtain from the problems, in order to discover the purpose of the problems and in order to grasp the significance of contrasting conditions.

The authors wish to thank those who used the First Edition, for their kind suggestions, all of which have received careful consideration and many of which have been incorporated in this edition.

J. S. D.

A. H. Z.

### THE FIRST EDITION



Thermodynamics has been treated in many  
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ideas. This consideration has, to a large extent, determined the order of presentation.

The text has been planned to cover a year's work in thermodynamics. Often, a briefer treatment is desired for non-mechanical engineering students, for this purpose, a number of articles can be by-passed without the thread of the text being lost. The articles which may thus be omitted in a shorter course are designated by a star for convenience. In each chapter, the drill problems are sufficiently varied so that both short and long problems are available.

In the derivation of expressions, many alternate forms of equations have purposely been omitted, as it is felt that all too often the student is overwhelmed by a large array of equations. The basis of this text has been to present as few equations as possible and then to show how problems may be analyzed and solved by the application of a few fundamental principles.

J S D  
A H Z

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## CHAPTER 1

### INTRODUCTION

**1-1. Need for Mechanical Power.**—From the dawn of history until about two hundred years ago, man has depended entirely on his own muscular energy and upon that of animals for doing work. This dependency on muscular energy has so limited his physical progress that his mode of living changed very little for thousands of years. To be sure, man learned to refine metals and to fashion them into crude tools, gunpowder was invented, and printing and the allied arts came into being. However, the restricted amount of available power prevented sufficient utilization of these developments to materially alter the way of life of the average individual from one century to another.

With the development of water and steam power in the last two centuries, there has been an extremely rapid growth of the physical world. Basic and far reaching discoveries have been made in the fields of physics, chemistry, and other sciences. It is true that our whole present-day civilization rests on these basic inventions. However, mechanical power is necessary for the utilization of these basic inventions. Transportation in all its forms, mining, manufacturing, and, in fact, our food, our clothing, and our shelter are made possible because we have learned to utilize mechanical energy. Not only has the utilization of energy other than muscular made possible our civilization, but the degree of our development can be determined by how extensively we use mechanical energy. There is an excellent correlation between the material wealth of a nation and its installed horsepower.

**1-2. Sources of Mechanical Energy.**—Various sources of energy have been utilized to produce mechanical power. For centuries, the Dutch have utilized *wind power* to pump water from their lowlands behind the dikes. The intermittent nature of the wind, together with the necessity of large structures, has prevented further development of wind-power plants of any appreciable size until quite recently. In 1941 a 1000-kw wind-turbine power plant was installed on Grandpa's Knob in Vermont. Much operating data were obtained before it threw one of its blades in 1945. However, because this was an experimental machine, the installation costs of large commercially made machines are as yet unknown; and, although some engineers feel quite confident of the ability of wind turbines to compete in the future with other types of prime movers, final decision on the economic value of wind power must wait for further information.

The difference between high and low tides varies greatly in various parts of the world. In some localities this difference is large and may exceed 30 ft. By virtue of this difference between high and low tides, power may be generated in small quantities intermittently. To generate a large amount of power continuously, such large storage basins are required that the initial costs of *tidal power* plants are excessive. This high initial cost plus the lack of a ready local market for electrical power led the United States Government to abandon the large tidal power project started in the Passamaquoddy Bay on the northeast coast of Maine in the early nineteen thirties.

It has been estimated that the energy received by the earth from the sun in an hour is equivalent to the heating value of 21 billion tons of coal. Thus, in a period of 3 hr the earth receives from the sun an amount of energy which is equivalent to the heating value of all the coal consumed annually in America. Attempts have been made to utilize this *solar energy*. Model power plants have been constructed, in which mirrors focus sunlight on water so as to produce sufficient steam to actually run model steam engines. Although the energy received by the entire earth is tremendous, the energy received per acre of the earth's surface is very small, particularly as the hours of sunshine are a rather small per cent of the total time. The original costs of devices for collecting and transforming solar energy are so large in comparison to the final mechanical energy delivered that solar power seems out of the question at present. It is possible that other means of collecting and storing solar energy may be found. Research has been initiated along this line in the field of photo-chemistry and photo-electricity, but there is not yet sufficient evidence to permit any deductions in regard to the economical feasibility of direct use of solar energy.

In the last few years, attention has been focused on *hydraulic power*. Enormous projects have been developed by the Federal Government on the Tennessee, Colorado, Columbia, Missouri, and Sacramento Rivers. Parts of the costs of many of these projects were chargeable to flood control measures or to irrigation works. Under these conditions hydraulic power costs are low. However, the sites available for hydraulic power plants are limited in number and in many cases, because of the local topography of the region surrounding a river, the installation costs of hydraulic power plants are so great that such plants cannot compete with steam power plants.

It should be evident that, although there are many natural sources from which we may derive mechanical energy, it is not feasible economically to derive the major portion of needed mechanical energy from these sources. Hence, our civilization must depend on *heat power* developed in engines receiving energy from burning a fuel, such as coal, oil, or gas,

or—for the future—from energy released by atomic fission. Although it has been shown that power can be produced by atomic fission, much development work must be done before commercial power will be produced by this means. Some major problems are (1) proper shielding to prevent the escape of radioactive particles, (2) operating and maintaining equipment from outside the shielded furnace, (3) developing metals and other materials to stand the continuous bombardment of the radioactive particles, (4) developing a suitable working substance, and (5) disposing of the highly radioactive wastes. The solution of all these problems will not assure widespread commercial use of atomic energy, as there is much uncertainty about the amounts of fissionable materials available, about the cost of preparation of the usable fissionable “fuels,” and about the original cost of an atomic power plant.

To develop mechanical energy in a “heat” engine, it is necessary to have (1) a source of heat, or a *hot body*, (2) an engine, (3) a “heat” sink, or a *cold body*, and (4) a working substance. The working substance picks up heat from the hot body and, by suitable changes of state in the engine, transforms part of the heat it receives into work. The rest of the heat received from the hot body is rejected to the cold body. The hot body is maintained at the desired temperature by combustion of a fuel. For an internal combustion engine, the fuel is burned within the engine and the products of combustion are both the hot body and the working substance. If atomic power is developed, atomic fissionable material will replace the fuel. It appears that the rest of the heat-engine setup will remain as it is at present.

*Thermodynamics* includes all heat-work transformations. It deals with working substances (vapors, gases, and liquids) under all conditions. Of particular importance is work produced by means of heat engines. Before the heat-work transformations are considered, it is desirable to review certain concepts, such as forms of energy and the various terms commonly used in thermodynamics.

**1-3. Definitions of Commonly Used Terms**—Several terms already used in physics will be defined here for convenient reference.

*Properties* of a substance are those quantities that define the substance and its state.

*State* of a substance is the condition of existence of the substance.

*Phase* of a substance is the nature of the condition of the substance. A substance may be in the gaseous phase, the liquid phase, or the solid phase.

*Force*,  $F$ , is a push or a pull which tends to change the motion of a body, in either magnitude or direction or in both.



*Mass, M*, denotes the quantity or amount of matter in a body. The larger the mass of the body, the greater resistance it offers to a change in motion when acted upon by a given force. Thus, one may see the relation between force and mass. The unit of force is that force which produces a certain acceleration of a unit mass. Also, the total force is the product of mass *M* and acceleration *a*, or

$$F = Ma = \frac{M \times d(\text{Vel})}{d(\text{time})} \quad (1-1)$$

In the English system, the usual unit of force is the pound, and the unit of acceleration is feet per second per second or feet per second squared.

The force of gravity acting on a freely falling body in a vacuum produces an acceleration of 32.17\* ft per sec<sup>2</sup>. From equation 1-1 it may be seen that the force of gravity, in pounds, is 32.17 times the mass of the body.

The *weight, W*, of a body is equal to the force existing between the earth and the body. As the force of gravity is taken as 32.17 times the mass of the body, the weight of the body, in pounds, is also 32.17 times the mass of the body. Hence,

$$W = 32.17 M$$

or

$$F = \frac{W}{32.17} a \quad (1-1a)$$

*Work, Wk*, is the energy transmitted to a body when a force applied to the body causes a displacement against a resistance or causes an acceleration of the body. The work done equals the product of the force and the distance, *L*, through which it acts when in motion, or

$$Wk = FL \quad (1-2)$$

The common unit of work is the foot-pound, abbreviated ft-lb.

*Power* is the time rate of doing work. The work required to raise an elevator to the top of a building is a definite quantity, regardless of the length of time required. If the time available is long a small motor will suffice, but, as the time available is reduced the size of the motor must be increased so that it will do more work in a unit of time. In a mechanical sense, the unit of power is known as the horsepower (abbreviated hp), which is equivalent to 33,000 ft lb of work per min.

\* This figure varies with the locality but as the variation rarely exceeds  $\frac{1}{2}$  per cent, it may be neglected.

The electrical unit of power is the watt or the kilowatt (abbreviated kw), which is equal to 1,000 watts. The relation between the horsepower and the kilowatt is

$$1 \text{ hp} = 0.746 \text{ kw} \quad (1-3)$$

Pressure,  $P$ , is defined as the force per unit area, or

$$P = \frac{F}{A} \quad (1-4)$$

Pressure may be expressed in either pounds per square inch (abbreviated psi) or pounds per square foot (psf), the choice depending on the units used for the area.

Large pressures normally are measured by means of a pressure gage. The common type of pressure gage, called the Bourdon tube, consists of an elliptical tube bent in the form of a semicircle and closed at one end. The closed end is free to move. Attached to the free end is a linkage which in turn is connected to the needle on the gage dial. The movement of the needle is proportional to the movement of the closed end of the tube. The open end of the tube is connected to the source of pressure. As the outside of the tube is surrounded by the atmosphere, the movement of the closed end depends on how much the measured pressure is above or below atmospheric pressure. Thus, a pressure gage reads pressures above or below atmospheric. The pressure read on a pressure gage is termed *gage pressure*. Pounds per square inch of gage pressure is abbreviated psig.

As air has mass, and hence weight, each particle of air exerts a pressure on the air beneath it. Under standard atmospheric conditions the air exerts a pressure of 14.7\* psi. The pressure of the atmosphere varies not only with the locality but also with the atmospheric conditions; it is determined by means of a barometer and is often called barometric pressure. In order to obtain the total or absolute pressure (pounds per square inch absolute is abbreviated psia), the atmospheric pressure must be added to the gage pressure. Thus,

$$P_{\text{abs}} = P_{\text{gage}} + P_{\text{atmos}} \quad (1-5)$$

If the pressure is less than atmospheric,  $P_{\text{gage}}$  is negative.

When pressures are small they are conveniently measured by balancing them against columns of various fluids. A common method is to use a U-tube manometer, one end of which is connected to the source of pressure and the other end being open to the atmosphere. The difference between

\* The exact figure is 14.696, but 14.7 is sufficiently close for most engineering calculations.

the heights of the columns of liquid in the two legs of the U-tube is proportional to the pressure above or below atmospheric. The force exerted on a given area by a column of a liquid equals the weight of the fluid above that area. The pressure, in pounds per square inch, exerted by a column of a fluid 1 inch high equals the weight of the fluid in pounds divided by the cross-sectional area in square inches. This quantity equals the weight per cubic inch, which is the weight density.\*

The gage pressure indicated by a manometer equals the product of the density of the liquid and the difference in height of the two columns, or

$$P = \text{density} \times \text{height} \quad (1-6)$$

As mercury weighs 0.491 lb per cu in,† a column of mercury 1 inch high exerts a pressure of 0.491 psi. Water weighs 62.4 lb per cu ft ‡ Hence, a column of water 1 foot high exerts a pressure of 62.4 psf or 0.433 psi.

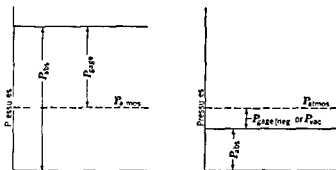


Fig 1-1 Relations of Pressures

The relation of gage pressure and absolute pressure is represented in Fig 1-1. In case the pressure is atmospheric, there is no vacuum. If the pressure is a little less than atmospheric, a slight vacuum is said to exist. If the pressure is close to absolute zero, there is a high vacuum. Thus, vacuum is used to determine how much a pressure is below atmospheric. Vacuums are generally expressed in inches of water or inches of mercury, whether they are determined by means of a manometer or a Bourdon-tube gage.

**Volume,  $V$ ,** is the space occupied by a substance. When gases or vapors are considered, volume is normally expressed in cubic feet. **Specific volume,  $v$ ,** is the volume of a unit weight of the substance, common units

\* Weight density is the weight of a unit volume. It is recognized that some authorities restrict the term density to mass density, i.e. the mass of a unit volume. However, in accordance with common engineering practice, the term density used in this text will refer to weight density.

† At 32 F. This figure varies slightly with the temperature.

being cubic feet per pound. Density is the reciprocal of specific volume and has as units pounds per cubic foot.

The concept of temperature will be more readily understood after the definitions of heat and molecular kinetic energy are established. Hence, temperature will not be discussed until the end of this chapter.

**1-4. Forms of Energy.**—If a substance has the ability to produce physical or chemical changes, this substance is said to possess energy. In a restricted sense, energy is often defined as the ability to do work. In some forms energy is stored in a substance; in other forms energy may be transferred from one substance to another substance. Thus, energy may be classified as stored energy or energy in transit.

(a) *Stored Energy*: The energy of elevation is known as *potential energy* (P.E.). When an object is elevated, work is required to overcome the gravitational force existing between the object and the earth. In its new position it possesses the energy required to elevate it. Its potential energy has been increased by the work done on it.

The amount of potential energy stored in an object in a certain position depends on the position in which its potential energy would be considered zero. Since the position in which the potential energy would reach absolute zero cannot be readily established, the engineer does not attempt to determine the absolute value of the potential energy of an object. A relative value is satisfactory. The same is true for many other forms of stored energy. Furthermore, when the elevation of an object is changed, the engineer is interested only in the changes of the energy stored in the object (*i.e.*, the energy which the object will give up or will receive as it changes elevation). For these reasons, the potential energy of a body is evaluated above some arbitrary datum plane, quite often the lowest elevation that will be encountered in a given problem. Potential energy equals the product of the weight of the body (a force) and the distance above the datum plane. Thus,

$$\text{P.E.} = W \times \text{height} \quad (1-7)$$

In the English system of units, the weight is normally taken in pounds, and the height is taken in feet. The product, which equals the potential energy, is in foot-pounds.

A body in motion is said to possess *kinetic energy* (K.E.). This is the energy required to accelerate the body from rest (in relation to the earth) to its given velocity. The kinetic energy equals one-half the product of the mass of the body and the square of its velocity. As the engineer generally deals with weights rather than masses, kinetic energy is expressed as

$$\text{K.E.} = \frac{W(\text{Vel})^2}{2g} \quad (1-8)$$

In the English system,  $W$  is in pounds,  $V_{el}$  is in feet per second, and  $g$  is 32.17 feet per second per second. Kinetic energy is thus in foot pounds.

Many demonstrations may be made to show that substances are composed of extremely small particles called molecules and that these are in continuous motion. Molecules possess kinetic energy by virtue of their motions. If molecules are close together, there is intermolecular attraction. When molecules are separated they acquire the energy required to separate them. The energy thus acquired is of the potential form. The energy possessed by molecules is often called *internal\** or *intrinsic energy* and is designated by the symbol  $U$ . The subscript  $k$  is used to designate the kinetic portion and the subscript  $p$  is used for the potential portion. Thus,

$$U = U_k + U_p \quad (1-9)$$

When there is a chemical reaction, atoms of one substance combine with atoms of another substance, energy being released or absorbed. This energy is known as *chemical energy* and is designated as  $Ch E$ . The mechanical engineer is particularly interested in the chemical energy of fuels.

The electrical engineer has been concerned with *sub-atomic energy* in various forms such as static electricity and ionized substances. There are many indications that sub-atomic energy, released by atomic fission, may play a very important role in power generation in the future.

(b) *Energy in Transit* Work is accomplished whenever a force acts through a distance. If the force applied to an object is exactly equal to a restraining force acting on the body, so that there is no motion, work cannot be done. But, if there is an unbalance of forces, work is done by virtue of energy being transferred to the object. Work is, then, the energy transferred because the unbalance of forces causes motion or change in motion.

*Heat*, up to a little over one hundred years ago, was thought to be an imponderable fluid—a substance without weight or mass—which, when added to an object, increased its temperature. Many of the definitions concerning heat and temperature which are in common use today arose at that time. For instance, if some of this imponderable fluid (heat) was added to an object, the object was said to contain more heat. Today, although it is realized that substances are composed of molecules and that one result of addition of heat may be the increasing of molecular energy, many people use the term "heat" interchangeably for that which has

\* Although the term "internal" means in the strict sense all forms of energy stored within a body, its use in this text will be restricted to the energy of molecules as a whole rather than to include sub-molecular energy.

been added and for the energy increase of the molecules. This use is very unfortunate and confusing, as the energy of the molecules may be increased without adding any heat. For example, as air is compressed in a bicycle pump, its mean molecular energy is increased by virtue of work done on it and not because of addition of heat. To distinguish between heat and molecular energy, most modern authors of texts on Thermodynamics and on Physics have agreed on the following conception of heat: Heat is the *energy transferred* from one place to another place because of a *temperature difference* existing between the two places.

The unit of heat called the British thermal unit, Btu, was originally defined as the amount of heat required to increase the temperature of 1 pound of water 1 degree Fahrenheit. Because this quantity varies with the water temperature, it is necessary to specify the water temperature to be used. Lack of agreement on which temperature to use led to the use of the mean Btu. This is defined as  $\frac{1}{180}$  times the amount of heat required to increase the temperature of 1 pound of water from 32 F to 212 F at atmospheric pressure. Realizing that it is undesirable to express the unit of heat (which is an energy quantity) in terms of the properties of water, the International Steam Table Conference, held in London in 1929, defined the calorie in terms of the International watt-hour. This is equivalent to defining the Btu as 778.16 ft-lb.

The concept of temperature must be introduced here. If two bodies having the same mean molecular kinetic energies are brought in contact with one another, their mean molecular kinetic energies are unchanged.\* But, if the mean molecular kinetic energy is higher in one body, there will be a flow of energy (heat) between the two bodies until there is equalization of the mean molecular kinetic energy. In the first case we say that the bodies are at the same temperature. In the second case we say that the temperature of the body with the high mean molecular kinetic energy is higher.

*Temperature* may be defined as a measure of the mean molecular kinetic energy of a body. It is, however, more than that. It is the driving force which causes that form of energy called heat to flow from one place to another place.

Two common scales of temperature are in use today, the centigrade and the Fahrenheit. In each case, the unit is called the degree. The centigrade degree of temperature is  $\frac{1}{180}$  times the difference between the temperatures of freezing water and boiling water at atmospheric pressure. The zero on the centigrade scale is taken as the temperature of freezing water at atmospheric pressure. The Fahrenheit degree of temperature is

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\* Provided, of course, no chemical change occurs.

1.80 times the difference between the temperatures of freezing water and boiling water at atmospheric pressure. The zero on the Fahrenheit scale is 32 degrees below the freezing temperature of water, being the lowest temperature that Fahrenheit was able to achieve with the means at hand. The following equations express the relations between centigrade and Fahrenheit temperatures

$$F = 1.8C + 32 \quad (1-10)$$

or 
$$C = \frac{5}{9}(F - 32) \quad (1-10a)$$

in which  $F$  is the number of Fahrenheit degrees and  $C$  is the number of centigrade degrees

The symbol  $t$  will be used for temperature. The subject of absolute temperature will be discussed in Chapter 2. Absolute temperature will be symbolized by  $T$ .

TABLE 1-1  
FORMS OF ENERGY

Stored	In Transit
I In bodies	I Work
(a) Potential	II Heat
(b) Kinetic	III Electrical Energy
II In molecules	
(a) Potential	
(b) Kinetic	
III In atoms (chemical)	
IV Sub-atomic	

The exact nature of *radiant energy* is subject to debate. Some authorities have felt that radiant energy is energy shot out of a body in a series of small amounts known as *quanta*. These quanta travel through space, through air, and even through glass without materially increasing the temperature of those media. The energy received by the earth from the sun is radiant energy. Much heat may be transferred by radiation.

Another form of energy in transit is *electrical energy*. It is that energy transferred through a conductor because of a difference in electrical potential existing at the two ends of the conductor.

**1-5 Energy Transformation**—The Law of Conservation of Energy states that energy cannot be created or destroyed but may be transformed from one form into another\*. The changes from one form to another are going on continuously in nature. A pendulum at the start of its swing

\* It is realized that when the atom is split vast quantities of energy are released as part of the mass of the atom is transformed into energy or vice versa. This statement however, simply broadens the concept of the Law of Conservation of Energy which after all is simply the method of accounting for energy being transformed.

possesses potential energy. As it falls, its potential energy decreases, being converted into kinetic energy. This kinetic energy becomes a maximum at the bottom of the swing and is then transformed back into potential energy by the time the end of the swing is reached. A more complicated example is the case of a steam-power plant. Radiant energy, received a long time ago from the sun, was stored as chemical energy in coal. This energy is released when the coal is burned, and is given to the gaseous products of combustion, greatly increasing their molecular kinetic energy. By virtue of the high temperature of the gases, heat is transferred to the water in the boiler, turning it into steam. The steam carries with it its energy (molecular) as it passes into a steam engine or turbine. Here some of the energy is transformed into work. If a generator is driven by the engine or turbine, this work is transformed into electrical energy. As such, it is transmitted by means of wires and may be reconverted to mechanical energy (work) before being used. In each of these steps, the conversion of energy is incomplete, the energy not transformed passing off as heat and increasing the temperature of the air or the earth or other surroundings.

1-6. **The First Law of Thermodynamics.**—The First Law of Thermodynamics recognizes that heat is a form of energy and, as such, may be converted into other forms. A common form of this law is: Heat and work are mutually convertible in fixed quantitative relationship.

The First Law can be demonstrated experimentally in many ways. Joule demonstrated this law indirectly when he demonstrated that heat was a form of energy. He arranged paddles in a tank of water and connected the paddles to an overhead weight by means of a rope and pulley. As the weight descended, it turned the paddles, agitating the water and increasing its temperature. The falling weight gave up its potential energy and produced the same effect on the water as if a definite quantity of heat had been added. In this and in other experiments of a similar nature, Joule measured the work done and the increase in temperature of a known quantity of water. By this means he formulated his mechanical equivalent of heat, which was remarkably close to the true value of 778.16\* ft-lb per Btu, particularly when his crude apparatus is taken into account.

The First Law of Thermodynamics may be demonstrated by making an energy balance on any "heat"-engine plant. If, in the case of the steam-power plant mentioned in Art. 1-5, the heat passing into the steam was measured and an accurate accounting was made of all heat leaving the steam or water at any point, it would be found that the portion of the heat added which was otherwise unaccounted for would exactly equal the heat equivalent of the work produced in the engine.

\* For most work, the value 778 may be used.



## CHAPTER 2

### EQUATIONS OF STATE FOR PERFECT GASES

**2-1. Boyle's Law and Charles' Law.**—The early chemists did considerable experimental work concerning the compression and expansion of gases, together with the production of changes in pressure, volume, and temperature accompanying these compressions and expansions.

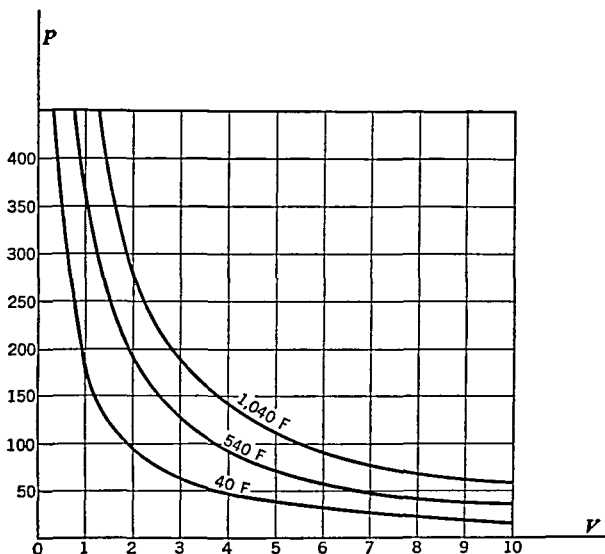


Fig. 2-1. Relation of Pressure to Volume of Gas at Constant Temperature

Boyle worked with gases, such as air and nitrogen, and found that, if he compressed them while keeping the temperature constant, the pressure (absolute) was inversely proportional to the volume. Curves showing the relation between  $P$  and  $V$  for three different temperatures are shown in Fig. 2-1. Thus, for 40 F,

$$P_1V_1 = P_2V_2 = C^*$$

Similarly, for 540 F and 1040 F,

$$\begin{aligned} P_3V_3 &= P_4V_4 = C' \\ P_5V_5 &= P_6V_6 = C'' \end{aligned}$$

\* Values of the constants  $C$ ,  $C'$ , and  $C''$  depend on the kind of gas, the amount of the gas, and the gas temperature.

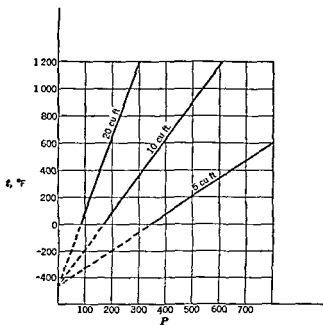


Fig 2-2 Relation of Pressure to Temperature of Gas at Constant Volume

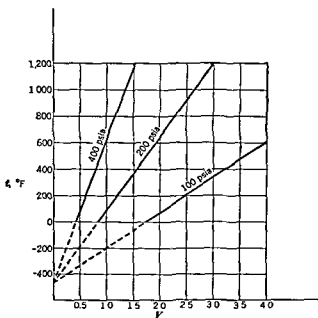


Fig 2-3 Relation of Volume to Temperature of Gas at Constant Pressure

Charles, working with similar gases, found that when he heated or cooled a gas at constant volume the change in pressure was proportional to the change in temperature. This means that a plot of observed temperatures versus observed absolute pressures at constant volume will be a straight line, as shown in Fig. 2-2.

When Charles heated or cooled these gases at constant pressure, he found that the change in volume was proportional to the change in temperature. At constant pressure, the plot of volumes against temperatures will be a straight line, as indicated in Fig. 2-3.

It is to be noted that when the volume lines in Fig. 2-2 are extended, they come together at zero pressure (absolute). The temperature at which this occurs is approximately 460 degrees below Fahrenheit zero. In a like manner, the extended pressure lines in Fig. 2-3 come together at zero volume, which is also approximately 460 degrees below Fahrenheit zero. These graphs were plotted for air, but other gases, such as oxygen, nitrogen, hydrogen, and helium, will produce similar results. As it is impossible to conceive of a gas having less than zero volume, it is impossible to conceive of a temperature less than 460 degrees below the Fahrenheit zero. Temperatures measured above this point are known as absolute temperatures. If the unit of temperature is the Fahrenheit degree, the absolute temperature is given as degrees Rankine ( $^{\circ}\text{R}$ ). If centigrade degrees are used, the absolute temperature is spoken of as degrees Kelvin ( $^{\circ}\text{K}$ ). According to the latest evaluation of absolute zero, observed temperatures may be converted to absolute temperatures as follows:

$$R = F + 459.69^* \quad (2-1)$$

$$K = C + 273.16^* \quad (2-2)$$

in which  $R$ ,  $F$ ,  $K$ , and  $C$  represent numbers of degrees in the respective systems.

If scales of absolute temperature  $T$  are used for ordinates in Fig. 2-2 and Fig. 2-3, it may be seen that at constant volume Charles' law may be expressed as follows:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = C \text{ and } \frac{P_3}{T_3} = \frac{P_4}{T_4} = C'$$

At constant pressure, Charles' law may be expressed as

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = C \text{ and } \frac{V_3}{T_3} = \frac{V_4}{T_4} = C'$$

**2-2. Characteristic Equation for a Perfect Gas.**—During most actual changes of state, there are changes in pressure, temperature, and

\* For most work, the value 460 or 273 may be used.

volume of the gas. Under these conditions, Boyle's and Charles' laws cannot be used. Nevertheless, a relationship between the pressure, temperature, and volume is desired. This may be obtained as follows.

Let the graph in Fig. 2-4 be drawn for 1 pound of a gas. Select any point on this graph and call it point 1. A relation between the pressure, temperature and volume at this point and those at any other point, as point 2, is desired. Point 2 is established by the pressure, temperature, and volume selected but any process or series of processes may be used

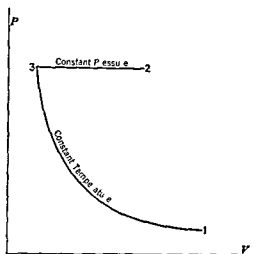


Fig. 2-4 Relation of Pressure, Temperature, and Volume of a Gas

in going from point 1 to point 2. For example, point 2 may be reached by compressing the gas to point 3 at constant temperature until its pressure equals that at point 2 and then heating the gas at this pressure until point 2 is reached. Because process 1-3 is at constant temperature, it follows from application of Boyle's Law that

$$P_1 v_1 = P_3 v_3$$

Also, by application of Charles' Law between points 3 and 2,

$$\frac{v_3}{T_3} = \frac{v_2}{T_2}$$

If each of these equations is solved for  $v_3$ , it is found that

$$v_3 = \frac{P_1 v_1}{P_3}$$

and

$$v_3 = \frac{v_2 T_3}{T_2}$$

Thus, 
$$\frac{P_1 v_1}{P_3} = \frac{v_2 T_3}{T_2}$$

But  $P_3 = P_2$  and  $T_3 = T_1$ . Therefore,

$$\frac{P_1 v_1}{P_2} = \frac{v_2 T_1}{T_2} \text{ or } \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

Although point 1 is any random point, the pressure, temperature, and volume at that point are definitely chosen quantities. Hence, the quantity  $\frac{P_1 v_1}{T_1}$  is a fixed quantity and is a constant. Thus,

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} = \text{Constant}$$

To distinguish this constant from other constants, the symbol  $R$  is used to designate it. The foregoing equation may now be written as:

$$P_1 v_1 = R T_1 \text{ and } P_2 v_2 = R T_2$$

To obtain an equation that is applicable to any weight of the gas, each side of either of the foregoing equations may be multiplied by the weight  $W$ . The result is

$$P_1 v_1 W = W R T_1$$

But the product of the specific volume  $v$  and the weight  $W$  equals the total volume  $V$ . Hence,

$$PV = WRT \quad (2-3)$$

As gases are not normally very dense, their volumes are generally measured in cubic feet. To be consistent, pressures are *expressed* in pounds per square foot. This introduces some difficulty, as the pressures are generally *measured* in pounds per square inch. Hence, observed pressures must be converted to pounds per square foot before being used. It is to be noted that in the mathematical expressions of Boyle's and Charles' laws, the pressures and temperatures were both absolute values. Hence, when equation 2-3 is applied, care must be taken to use both *absolute temperature* and *absolute pressure*.

The units of  $R$  may be determined by solving equation 2-3 for  $R$ . In the English system,  $R$  has the units ft-lb per lb per deg Rankine. It is customary, however, not to express the units of  $R$ . This is unfortunate in a way, as the units and hence the value of  $R$  will depend on the units used for other quantities in the equation.

At given conditions of temperature and pressure, different gases have different volumes. Hence, each gas has its own value of  $R$ . For this reason,  $R$  is known as a *particular gas constant*. Values of  $R$  are given in Table 2-1. In Chapter 13 the universal gas constant is discussed.

TABLE 2-1  
PROPERTIES\* OF VARIOUS GASES

Gas	Chemical Formula	Molecular Weight	$R$	$c_p$	$c_v$	$k = \frac{c_p}{c_v}$
Air		28.97	53.35	0.240	0.1715	1.40
Oxygen	$O_2$	32.00	48.3	0.219	0.157	1.40
Nitrogen	$N_2$	28.016	55.2	0.248	0.177	1.40
Carbon monoxide	$CO$	28.005	55.2	0.248	0.177	1.40
Hydrogen	$H_2$	2.016	767	3.390	2.404	1.41
Helium	$He$	4.000	386	1.250	0.754	1.66

*Example 2-1*—Calculate the tank volume required to store 4 lb of air at 140 F and at 200 psi, gage. Atmospheric pressure is 14.2 psia.

*Solution*—The value of  $R$  for air as given in Table 2-1, is 53.35. Rearranging equation 2-3, we obtain

$$\begin{aligned}
 V &= \frac{WRT}{P} \\
 &= \frac{(4)(53.35)(140+460)}{(144)(200+14.2)} \\
 &= 4.151 \text{ cu ft} \quad \text{Ans}
 \end{aligned}$$

**2-3 Perfect and Permanent Gases**—Although Boyle and Charles based their laws on experiments made with actual gases, later experiments with more refined apparatus showed that no gases obeyed either of these laws exactly. Some gases showed marked deviations from these laws under normal conditions, and all gases showed deviations from these laws at low temperatures and high pressures. However, many gases show little deviations from these laws under normal conditions of temperature and pressure. Any gas obeying these laws exactly is said to be a perfect gas, provided also that it has constant specific heat (see Chapter 3). Thus, equation 2-3 holds only for perfect gases.

The early chemists and physicists found that some gases, such as carbon dioxide, could be liquefied quite readily. Other gases resisted liquefaction to a high degree and many could not be liquefied at all with the means that they had. This led them to the conclusion that these gases were gases under all conditions and hence they termed them *permanent gases*. Today, all known gases have been liquefied, hence, the term permanent gas is a misnomer. However, the term permanent gas has been retained because, for gases thus classified, the characteristic equation for perfect gases (equation 2-3) may be used for most engineering purposes under normal conditions of temperatures and pressures. Some of these

\*The gas constants are determined at 14.7 psia and room temperature. For perfect gases these constants cannot vary. The values of  $R$  have units of ft-lb per lb per degree Rankine, being based on pressures in pounds per square foot, volumes in cubic feet, weights in pounds, and temperatures in degrees Rankine. The values of specific heats  $c_p$  and  $c_v$  are in Btu per lb per degree Fahrenheit.

so-called permanent gases are air, oxygen, nitrogen, hydrogen, helium, and carbon monoxide.

In the discussion of gases in the first four chapters of this text, the equations derived will be those for perfect gases. However, some equations, by their very nature, will hold for all gases and also for liquids. No separate equations will be used for imperfect gases until Chapter 14. It is to be assumed in solving problems in these first four chapters that solutions are to be based on perfect-gas laws.

**2-4. Polytropic Changes of State.**—In the discussion of Boyle's and Charles' laws, three common processes\* have been discussed: the *constant pressure*, sometimes called the *isobaric* or *isopiestic*; the *constant volume*, sometimes called the *isovolumic* or *isometric*; and the *constant temperature*, often called by the misnomer *isothermal*. In addition to these three changes of state, many processes in practice approach an *adiabatic* change of state. Such a change of state was defined by Clausius as "any change occurring within an envelope impermeable to heat." In other words, an adiabatic process is one in which no heat is added or removed during the process.

In actual cases, some heat is generally added or removed and there are also changes in pressure, temperature, and volume. It has been found that actual compressions and expansions obey the equation  $PV^n = C$  fairly well, provided that the proper value is selected for the exponent  $n$ . Because  $n$  may have so many values, any process obeying this law is called a *polytropic process* (many forms).

This general  $P$ - $V$  relation holds for the special processes just mentioned. It may be written as follows:

$$P_1 V_1^n = P_2 V_2^n = C \quad (2-4)$$

When  $n=0$ ,  $P_1 = P_2 = C$ ; or the pressure is constant when  $n=0$ .

When  $n=1$ ,  $P_1 V_1 = P_2 V_2 = C$ , which is the  $P$ - $V$  relation set forth by Boyle for a constant temperature; or the temperature is constant when  $n=1$ .

Taking the  $n$ th root of equation 2-4, we get

$$P_1^{\frac{1}{n}} V_1 = P_2^{\frac{1}{n}} V_2 = C'$$

When  $n = \infty$  in this equation,  $V_1 = V_2 = C$ ; or the volume is constant when  $n = \infty$ .

The value of  $n$  in an adiabatic process will be determined in Chapter 3.

In Fig. 2-5 are shown curves representing many polytropic changes of state, with the proper value of  $n$  placed on each curve. It should be noted that the values of  $n$  on the various curves increase in an orderly manner,

\* A systematic change of state.

from negative infinity to positive infinity. As the value of  $n$  increases from zero, the slope  $\frac{dP}{dV}$  has a larger negative value. Once the value of  $n$  is known, the curve may be spotted in roughly on Fig 2-5 without the necessity of calculations. This will give a qualitative idea of how the pressure and the volume vary with respect to each other.

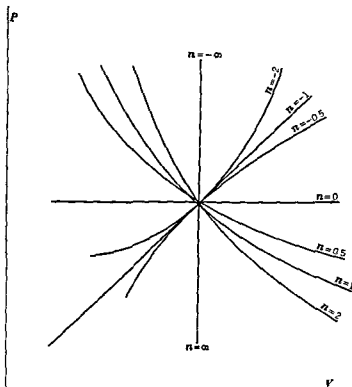


Fig 2-5 Polytropic Changes of State

The pressure temperature-volume relation for polytropic changes of state between states 1 and 2 may be evolved by writing the characteristic equation of state at each of these points. Thus,

$$P_1 V_1 = W R T_1 \text{ and } P_2 V_2 = W R T_2$$

or 
$$\frac{P_1 V_1}{T_1} = W R \text{ and } \frac{P_2 V_2}{T_2} = W R$$

Therefore, as shown in the derivation of equation 2-3,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad (2-3a)$$

Although the exponent  $n$  does not appear in equation 2-3a, this equation is none the less valid for all polytropic changes of state, the



temperature  $T$  taking the place of  $n$  in expressing the pressure-volume relation.

Equation 2-4 should be rearranged as follows before it is used to solve problems:

$$\frac{P_1}{P_2} = \left( \frac{V_2}{V_1} \right)^n \quad (2-4a)$$

It is suggested that, in case  $V_1$  is greater than  $V_2$ , the fractions on both sides of the equation be inverted because the average student can handle whole numbers to powers more readily than fractional numbers. In case the fractions on both sides are inverted, the exponent remains unchanged. If only one fraction is inverted, the sign of the exponent must be changed. In case both pressures are known and a volume ratio is desired, then the first step in the solution of the problem is to take the  $n$ th root of each side of equation 2-4a. This procedure may be illustrated in the following example.

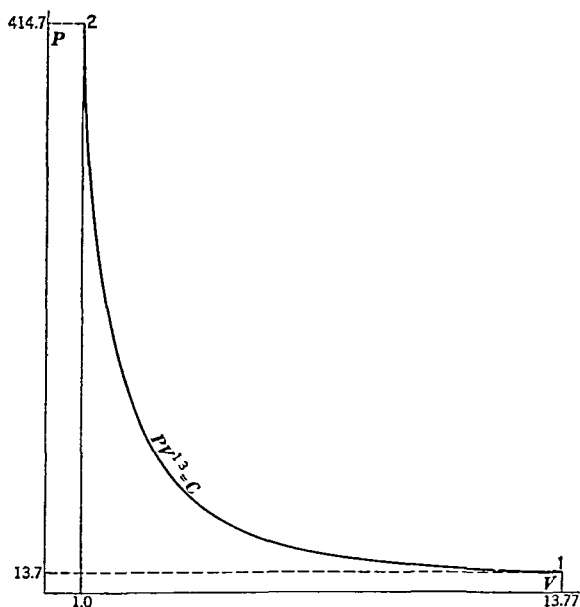


Fig. 2-6. Graph for Example 2-2

**Example 2-2.**—Calculate the compression ratio (ratio of the volume before compression to the volume after compression) necessary to raise the pressure in a diesel engine to 400 psi, gage. As indicated by the graph in Fig. 2-6, compression obeys the law  $PV^{1.3} = C$ . There is a vacuum of 1 psi at the start of compression.

**Solution.**—As the barometric pressure is not given, standard barometric pressure will be assumed. Thus,  $P_1 = 14.7 - 1.0 = 13.7$  psia and  $P_2 = 400 + 14.7 = 414.7$  psia. Rearranging equation 2-4a, we obtain

$$\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = \frac{V_1}{V_2}$$

$$\text{or } \frac{V_1}{V_2} = \left(\frac{414.7}{13.7}\right)^{\frac{1}{1.3}} = (30.27)^{0.769} = 13.77 \quad \text{Ans.}$$

Relationships of pressures and temperatures and relationships of temperatures and volumes are also desired. These may be found as follows:

$$P_1 V_1^n = P_2 V_2^n; P_1 V_1 = W R T_1 \text{ or } P_1 = \frac{W R T_1}{V_1}; \text{ and}$$

$$P_2 V_2 = W R T_2 \text{ or } P_2 = \frac{W R T_2}{V_2}$$

Therefore,

$$\frac{W R T_1}{V_1} V_1^n = \frac{W R T_2}{V_2} V_2^n$$

$$T_1 V_1^{n-1} = T_2 V_2^{n-1}$$

$$T_1 V_1^{n-1} = T_2 V_2^{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} \quad (2-5)$$

Starting again with  $P_1 V_1^n = P_2 V_2^n$  and with  $V_1 = \frac{W R T_1}{P_1}$  and  $V_2 = \frac{W R T_2}{P_2}$ , we obtain

$$P_1 \left(\frac{W R T_1}{P_1}\right)^n = P_2 \left(\frac{W R T_2}{P_2}\right)^n$$

$$P_1 P_1^{-n} T_1^n = P_2 P_2^{-n} T_2^n$$

$$\left(\frac{P_1}{P_2}\right)^{\frac{1-n}{n}} = \frac{T_2}{T_1}$$

$$\text{or } \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} \quad (2-6)$$

**Example 2-3**—The compression in a gasoline engine obeys the law  $P V^{1.3} = C$ . Calculate the maximum allowable compression ratio if the temperature at the end of compression is not to exceed 500 F. The initial temperature is 120 F.

**Solution.**—Rearranging equation 2-5, we get

$$\frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{n-1}}$$

or

$$\frac{V_1}{V_2} = \left( \frac{500 + 460}{120 + 460} \right)^{\frac{1}{1.3-1}} = \left( \frac{960}{580} \right)^{\frac{1}{0.3}}$$

$$= (1.655)^{3.333} = 5.36$$

Ans.

*Example 2-4.*—The expansion curve in a diesel engine obeys the law  $PV^{1.28} = C$ . At the start of expansion, the temperature is 2940 F and the pressure is 600 psia. The exhaust valve starts to open at a point where the pressure is 70 psia. What is the temperature at this point?

*Solution.*—Rearranging equation 2-6, we get

$$\frac{T_1}{T_2} = \left( \frac{P_1}{P_2} \right)^{\frac{n-1}{n}} = \left( \frac{600}{70} \right)^{\frac{1.28-1}{1.28}}$$

from which

$$T_2 = 3400 \div 8.571^{0.2188} = 3400 \div 1.600 = 2125 \text{ R}$$

Hence,

$$t_2 = 1665 \text{ F}$$

Ans

The value of  $n$  to be used in predicting conditions at the end of a compression or an expansion may be based on a knowledge of values for similar machines that have been made and tested. The actual value of  $n$  to be used in calculating work, heat added or removed, etc. can be determined from a knowledge of properties at the beginning and the end of a process. Perhaps the easiest properties to obtain are the pressures and volumes, which are determined from an indicator card.\*

*Example 2-5.*—The suction at the point where the cylinder volume is 1.12 cu ft is 1.0 psi. The pressure reaches a value of 446 psi, gage, when the cylinder volume is reduced to 0.08 cu ft. Atmospheric pressure is 14.0 psia. Calculate the value of  $n$ .

*Solution.*—Rearranging equation 2-4, we may write

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^n$$

or

$$\frac{460}{13} = \left( \frac{1.12}{0.08} \right)^n$$

Hence,

$$35.38 = 14^n$$

and

$$n = 1.351$$

Ans.

**2-5. The Kinetic Theory of Gases.**—The equations used so far have been based on experimental observable facts. These equations are independent of the conception of molecular existence. If it could be shown that molecules are a pure invention of the mind, the science of thermodynamics would not be affected in the least. However, many demonstrations may be made to indicate the existence of molecules. To the average student molecules have come to be a reality. Because the acceptance of

\* This is really a pressure-volume diagram for the substance within the cylinder. The ordinates are proportional to cylinder pressures, and the abscissas are proportional to the piston positions, which in turn are proportional to cylinder volumes.

the existence of molecules and a study of their actions will explain many of the fundamental laws of thermodynamics and will aid in the establishment of more involved ones, the kinetic theory will now be considered

To understand molecular actions it is helpful to picture the molecule as a spherical entity, even though it has been shown that the actual structure of the molecule is quite complex, particularly in the case of a molecule containing many atoms. From experimental studies, the following five hypotheses have been set up regarding molecules and their actions when the substance is in the gaseous state

- (1) Under normal conditions, molecules are moving with high velocities
- (2) They are moving in all directions and with various velocities. Some stray molecules move with extremely high velocities, and other stray molecules move at relatively very low velocities. Both the magnitudes and the directions of the velocities are being changed by innumerable collisions of molecules with one another
- (3) In these collisions, the time of which is extremely short compared to the time of uninterrupted travel, the laws of elastic impact are followed perfectly. (Behaving as spheres, the molecules can have no frictional effects on one another during impact.)
- (4) Under normal conditions, the length of the path traveled between encounters (the mean free path) is, on the average, very large compared with the size of the molecule
- (5) In spite of the random motion of the molecules, the molecules of a body as a whole are in a state of dynamic equilibrium, there being an attracting force—which is small to be sure under some conditions—between the molecules

The subject of the kinetic theory of gases in its entirety is quite complex, complete texts having been written on the subject. However, the bare fundamental concepts will suffice for the purpose of this text.

The kinetic theory may also be applied quite readily to liquids. For the purpose of comparing the actions of molecules of liquids with those of gases, the following facts may be noted about liquids

- (1) Molecular size is appreciable in comparison with the space moved through during the time of uninterrupted travel of the molecules
- (2) The time of impact is appreciable in comparison with the time of uninterrupted travel
- (3) The intermolecular attractive force is so large that all molecules are deflected from their paths by other molecules

The kinetic theory cannot be applied so readily to solids, particularly to crystalline ones. This fact is not much of a drawback here, as solids are infrequently encountered in a study of thermodynamics

To formulate postulates concerning the behavior of gases, by means of the kinetic theory, it is desirable to simplify the problem by picturing a perfect gas. In a perfect gas, according to the kinetic theory, the following conditions exist:

- (1) Intermolecular attractive forces are negligible. Hence, the molecules travel in straight lines.
- (2) The time of impact is negligible in comparison with the time of uninterrupted travel.
- (3) The size of the molecule is negligible in comparison with the length of the mean free path.

★2-6. Pressure Exerted by Molecules of a Perfect Gas.—Because the walls of a containing chamber restrict the motion of gas molecules, these molecules exert pressures on the walls. The pressure on any selected unit area may be evaluated as follows:

$$P = \frac{F}{A}$$

$$F = Ma$$

in which  $a$  = acceleration = change in velocity per unit time = (change in velocity per impact)  $\times$  (number of impacts per unit time).

Thus, it becomes necessary to evaluate the change in velocity per impact as well as the number of impacts per unit time. As the molecules are traveling with random velocities in random directions, it is desirable to consider first one molecule and the pressure it exerts on a certain plane, as  $Xyz$  in Fig. 2-7. This molecule is traveling with a velocity  $Vel$ , but only the component of its velocity parallel to  $OX$ , or the component  $Vel_x$ , can exert a force on plane  $Xyz$ .

When the molecule strikes the plane  $Xyz$ , the velocity parallel to  $OX$  will change abruptly from  $+Vel_x$  to  $-Vel_x$ , or the change will be  $2 Vel_x$ .

The molecule must travel a distance equal to  $2 OX$  each time it strikes plane  $Xyz$ . Thus, the number of impacts per unit of time will be  $\frac{Vel_x}{2 OX}$ .

If  $m$  = the mass of the molecule, the force is

$$F_x = m \cdot 2 Vel_x \times \frac{Vel_x}{2 OX} = m \frac{(Vel_x)^2}{OX}$$

and, for one molecule,

$$P_x = \frac{m(Vel_x)^2}{OX} \div (Xy \times Xz) = \frac{m(Vel_x)^2}{Vol}$$

---

★ This section, like others similarly marked (★), may be omitted in a short course.

From this equation it may be seen that pressure is proportional to the square of the velocity of a molecule. Since the molecules of a gas travel with random velocities, the pressure exerted by all the molecules must be

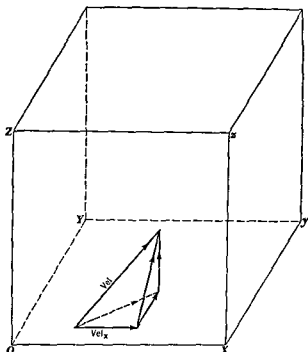


Fig 2-7 Pressure Produced by Gas

determined by using the average of the squares of the velocities of the molecules (mean root square velocity, denoted as  $\overline{Vel}^2$ ). If  $N$  = the number of molecules, the pressure exerted by all the molecules becomes

$$P_x = \frac{Nm \overline{Vel}_x^2}{Vol}$$

The relation between a velocity and its three components may be expressed as follows

$$\overline{Vel}^2 = \overline{Vel}_x^2 + \overline{Vel}_y^2 + \overline{Vel}_z^2$$

As the pressure must be the same in all directions,

$$\overline{Vel}_x^2 = \overline{Vel}_y^2 = \overline{Vel}_z^2$$

and

$$\overline{Vel}^2 = 3 \overline{Vel}_x^2$$

Then

$$P = P_x = \frac{1}{3} \frac{Nm \overline{Vel}^2}{Vol}$$

★2-7. **Temperature as Defined by Means of the Kinetic Theory.** The total weight of a gas is proportional to the number of molecules  $N$  and the weight  $w$  of a molecule. The value of  $w$  is different for each gas but is constant for any given gas. Then,

$$N = \frac{W}{w} = C \times W$$

Hence, pressure may be expressed as follows:

$$P \times \text{Vol} = \frac{1}{3} C \times W \times m \overline{\text{Vel}}^2$$

If this equation is compared with equation 2-3, which is  $PV = WRT$ , it may be seen that the two equations are quite similar; but one contains the term  $m \overline{\text{Vel}}^2$  and the other contains the term  $T$ . This being the case, the absolute temperature must be some function of  $m \overline{\text{Vel}}^2$  of the molecules and also of the mean kinetic energy of the molecules, or  $\frac{m \overline{\text{Vel}}^2}{2}$ . Thus, by

comparing the expression for pressure obtained by combining Boyle's and Charles' laws with that obtained by means of the kinetic theory of perfect gases, the following concise definition of temperature may now be given: *Temperature is a measure of the mean molecular kinetic energy of a substance.*

This definition of temperature leads to a new concept of absolute zero of temperature. As long as molecular motion persists, there is some temperature. The temperature of a perfect gas (as defined by the kinetic theory) becomes zero when molecular motion ceases. As it is impossible to conceive of gas molecules having less than zero velocity, it is also impossible to conceive of a temperature lower than that at which molecular motion ceases. This temperature is known as the absolute zero of temperature.

## CHAPTER 3

### ENERGY EQUATIONS FOR NON-FLOW PROCESSES

**3-1. The General Energy Equation.**—In practice there are many cases in which heat is added to a fluid and causes it to expand and do work. In other cases, a fluid is compressed by virtue of work done on it. As it is being compressed, some heat may be taken from the fluid. In still other cases, a fluid may be heated or cooled at constant volume. Examples of some of the foregoing processes are the cooling of air in a tank, the heating of water in a boiler before it is put on the line, the compression of air in an air compressor, the expansion of steam in a steam engine, and the compression of the air-fuel mixture in a gasoline engine. In each of these cases the fluid may expand or contract, but it stays in its container. Because the fluid neither enters nor leaves its container, these processes are classified as *non-flow processes*.

The law of conservation of energy may be applied to any non-flow process. In the application of this law it is customary to omit those forms of energy that remain constant in the normal non-flow process. In many non-flow processes encountered by the engineer, there is no change in the chemical energy of the fluid in the container.\* The fluid as a whole does not experience a change in either potential energy or kinetic energy. Neither radiant energy nor electrical energy is brought into the fluid or removed from it. Thus, the only energies normally involved in non-flow processes are molecular energies, heat, and work. If the law of conservation of energy is applied to these quantities, the following equation may be set up: The original molecular or intrinsic energy,  $U_1$ , plus the net amount of heat added between the original and final states,  ${}_1Q_2$ , equals the final molecular or intrinsic energy,  $U_2$ , plus the energy removed in the form of work,  $Wk$ . Thus,

$$U_1 + {}_1Q_2 = U_2 + \frac{{}_1Wk_2}{J}$$

As the work is normally determined in foot-pounds, it must be divided by  $J$ , or 778 ft-lb per Btu, to make the units of the equation consistent. Sometimes, instead of being divided by  $J$ , the work is multiplied by  $A$ , which is the reciprocal of  $J$ . It should be kept in mind that the absolute value of the molecular energy is not readily determinable and what is

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\* An exception to this is a non-flow combustion. In this case chemical energy must be taken into account.



really desired is the change in molecular energy. The foregoing equation should therefore be rearranged as follows

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1Wk_2}{J} \quad (3-1)$$

Note that in this equation  ${}_1Q_2$  represents the net amount of heat that has been added between the original and final states. If heat is removed,  ${}_1Q_2$  becomes a negative quantity. Also,  ${}_1Wk_2$  represents work done by the fluid as it expands. In case work is done on the fluid,  ${}_1Wk_2$  becomes a negative quantity.

**3-2. Specific Heat**—In accordance with common engineering practice, specific heat is a definite quantity of heat added to a definite quantity of a substance and producing a definite result. In the English system of units it is the number of Btu which must be added to increase the temperature of 1 pound of the substance 1 degree Fahrenheit. If the symbol  $c_x$  is used for specific heat, the total amount of heat added to a weight  $W$  of a substance is

$${}_1Q_2 = Wc_x(T_2 - T_1) \quad (3-2)$$

Tables of specific heats of solids and liquids give one value of specific heat for each substance in a given phase.\* When heat is added to a solid or a liquid under normal conditions, there is little change in volume and practically no work is done. Thus, all the heat which is added is converted into intrinsic energy ( $i.e.$ , internal or molecular energy), and there is only one value of specific heat.

In Table 2-1 on page 18, two specific heats are given,  $c_p$  and  $c_v$ . These designate the specific heat of the substance during a constant-pressure process and a constant volume process, respectively. It is to be noted that the value of  $c_p$  is larger than  $c_v$ . In a constant-volume process all of the heat added goes to increase the molecular energy, and hence to increase the temperature, as no work can be done in the constant-volume non-flow process. In a constant-pressure addition of heat, part of the heat added goes to do work. Hence more heat must be added in the constant pressure process, per pound of substance, to produce a temperature rise of 1 degree.

When heat is added to a gas, various proportions of it may go to do work, the amount depending on the process, and hence various quantities of heat must be added, to increase the temperature of 1 pound of the gas 1 degree Fahrenheit. As there are an infinite number of kinds of processes, so are there an infinite number of specific heats. The subscript  $x$  has been used in connection with the specific heat, because the specific heat—and hence the heat added—is unknown until the nature of the process is known.

\* This statement disregards the effect of temperature on specific heat.

When the process—and hence the value of  $n$ —is known, the value of the specific heat  $c_x$  may be calculated (see Art. 3-6). The values of the specific heats for an adiabatic and an isothermal (constant temperature) process may be determined as follows.

In an adiabatic compression, the work done on a gas goes to increase its molecular energy. Hence its temperature is increased, even though no heat has been added. Solving equation 3-2 for  $c_x$  shows that the value of  $c_x$  in an adiabatic is zero.

If a large quantity of heat is added to a definite weight of a gas with a small temperature rise, the specific heat, by equation 3-2, is large. As the temperature rise is decreased, the value of the specific heat increases, approaching a value of infinity as the process approaches a constant-temperature one.

**3-3. Reversible Processes.**—In Fig. 3-1 is shown a vertical insulated cylinder containing a gas under pressure applied by a closely fitting piston carrying a weight  $W$ . Because of its pressure, the gas has ability to do work. If the entire weight could be removed instantaneously, the piston would immediately shoot upward. However, if the piston is considered to be weightless and the space above it is assumed to be a perfect vacuum, the expanding gas can do no work. Nevertheless, to restore the gas to its original state, work must be done on it from the outside. Free expansion, such as that resulting from the removal of the weight, is therefore undesirable because it uses up some of the ability of the gas to do work. This fact may be stated in another way by saying that the gas has less ability to do work after the expansion.

If the weight shown in Fig. 3-1 is divided into two equal parts and only one half is removed instantaneously, the gas will expand to some extent and will do some work. If the weight is divided into four equal parts, instead of two, and the parts are removed one at a time, the expansion will be more gradual and more work will be obtained. As the original weight is divided into more and more smaller parts, the amount of work obtainable increases. The maximum amount of work will be performed by the gas when each part of the weight becomes infinitesimally small.

When the entire large weight is removed at once, a non-uniform condition exists within the gas. The pressure on the gas in contact with the piston tends to be lower in value than that at the bottom of the cylinder. Because of non-uniformity of pressures, there will also be a non-uniformity

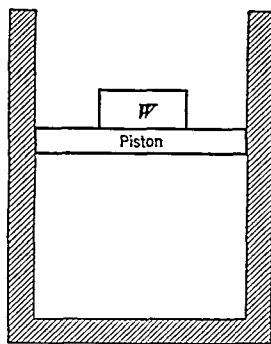


Fig. 3-1. Insulated Cylinder Containing Gas

of temperatures. Therefore, it is not possible to specify the state of the gas within the cylinder during such an expansion. Furthermore, to bring the gas back to its original condition, much energy, in the form of work, must be brought in from the outside. On the other hand, when the infinitesimally small weights are removed one at a time, equilibrium will exist throughout the gas and its state may be specified at all times during the expansion. At any time during this expansion the gas may be brought back to its original state by introducing only an infinitesimal amount of work from the outside. Such a process, that is, one in which the gas will substantially of its own accord return to its original state, is known as a reversible process.

A reversible process is a limiting process. As friction, turbulence, and pressure differences are minimized, a process approaches reversibility. It will be shown in Chapter V that whenever there is heat flow, there will be non-uniform conditions at any instant and the process will be irreversible if the temperature differences causing heat flow are finite ones. As these temperature differences become smaller and smaller, the heat-transfer process approaches reversibility.

In practice no process is a reversible one. However, it is desirable to study reversible processes because they are criteria of perfection. In an expansion a reversible process will indicate the maximum possible amount of work obtainable. In a compression such a process will show the minimum amount of work required. Some processes in practice approach the reversible one so nearly that no serious error will be made by assuming them to be reversible.

Because the state of a working substance cannot be specified during an irreversible change of state, an irreversible process will be shown by broken lines when plotted. The broken line does not indicate the actual path taken by the substance between two states, and this fact must be kept in mind continually.

**3-4 Work in Terms of Pressure and Volume**—In the fundamental equation for work,  $W_k = F \times L$ , it is assumed that the force  $F$ , acting through the distance  $L$ , is a constant one. In most cases, as gases or vapors expand or are compressed in a cylinder, there is a variation in their pressure. Hence, the force acting on the face of the piston varies, and the equation for work must be modified to allow for this varying force.

Let the piston in Fig. 3-2 move an infinitesimal distance,  $dL$ . During this extremely small movement of the piston, there will be no appreciable change in pressure, and the force  $F$  will act on the piston during this motion. The infinitesimal amount of work done is

$$d(W_k) = F dL$$

Since  $F = P A$ , where  $A$  is the piston area,

$$d(\text{Wk}) = P A dL$$

But,  $A \times dL = dV$ , where  $dV$  is the volume swept through during the motion  $dL$ . Hence,

$$d(\text{Wk}) = P dV$$

The work done on the face of the piston as it moves from the initial state to the final state equals the summation of infinitesimal amounts of work during the infinitesimal piston motions. Thus,

$$\int_1^2 d(\text{Wk}) = \int_1^2 P dV$$

and

$${}_1\text{Wk}_2 = \int_1^2 P dV \quad (3-3)$$

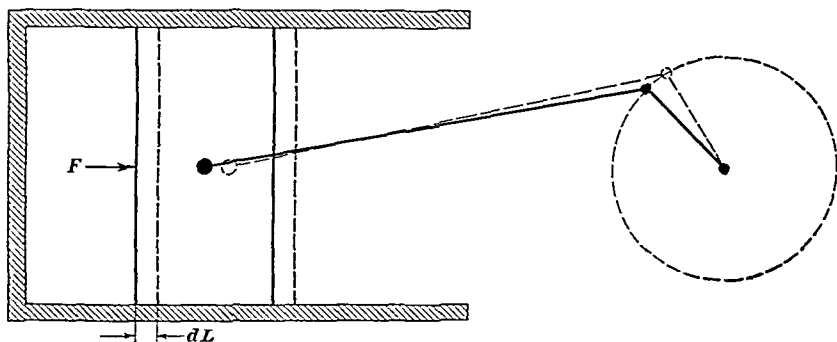


Fig. 3-2. Work Done in Cylinder

In equation 3-3, the pressure  $P$  is the pressure which acts on the face of the piston. This equation is based on the assumption that the process is a reversible one, so that the pressures can be specified for the entire process. If the process is so close to reversibility that the pressure on the face of the piston is known at all times, the work can be evaluated by use of equation 3-3. It should be noted, particularly for a high-speed engine, that the pressure exerted on the piston may be less than the pressure exerted on the cylinder walls. Frequently, an internal-combustion engine contains one or more auxiliary chambers. As there may be a restriction, and hence a pressure differential, between the auxiliary and main chambers, care must be taken not to use the pressure in the auxiliary chamber as that in the main chamber.

In Fig 3-3 is shown an isothermal expansion. As the piston sweeps through the infinitesimal volume  $dV$ , the work done, or  $d(Wk)$ , which equals  $P dV$ , is shown by the cross hatched area. As the total area underneath the curve of the process 1-2 equals  $\int_1^2 P dV$ , and as  ${}_1Wk_2$  equals this integral, the area on a  $P$ - $V$  plane underneath the curve of a non flow process represents the work done during the process.

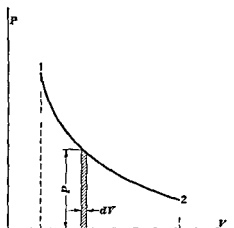


Fig 3-3 Polytropic Expansion

The equation  $PV^n = C$  permits the evaluation of  $\int_1^2 P dV$  and hence the work. From  $PV^n = C$ ,  $P = CV^{-n}$ . Therefore,

$${}_1Wk_2 = C \int_1^2 V^{-n} dV = C \left[ \frac{V^{-n+1}}{-n+1} \right]_1^2$$

Since  $C = PV^n$ ,

$${}_1Wk_2 = \left[ \frac{PV^n V^{-n+1}}{1-n} \right]_1^2 = \left[ \frac{PV}{1-n} \right]_1^2$$

or

$${}_1Wk_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad (3-3a)$$

This equation holds for any process obeying the law  $PV^n = C$ . However, in the case of an isothermal,  $P_1 V_1 = P_2 V_2$ , and  $n = 1$ . As 0 divided by 0 is indeterminate, equation 3-3a is useless in this case, and a special work equation must be derived for the isothermal.

As  $PV = C$  in an isothermal,  $P = \frac{C}{V}$ . Hence,

$${}_1Wk_2 = C \int_1^2 \frac{dV}{V} = C \log_e V \Big|_1^2 = C \log_e \frac{V_2}{V_1}$$

Substituting the value  $PV$  for  $C$  gives

$${}_1Wk_2 = PV \log_e \frac{V_2}{V_1} \text{ for an isothermal only} \quad (3-3b)$$

*Example 3-1.*—Heat is added to 8 cu ft of air at a constant pressure of 20 psia until the volume is increased to 12 cu ft. Calculate the work done.

*Solution.*—As the pressure is constant,  $n=0$  and  $P_1=P_2$  in equation 3-3a. Therefore,

$${}_1Wk_2 = P(V_2 - V_1) = 144 \times 20 \times (12 - 8) = 11,520 \text{ ft-lb} \quad \text{Ans.}$$

*Example 3-2.*—How much work is done when 4 lb of air expands at a constant temperature of 240 F until the pressure is reduced to one-half of its original value?

*Solution.*—In equation 3-3b both the pressure and the volume are unknown. However, as only the product  $PV$  is desired, it may be found from the relation  $PV = WRT$ .

As  $P_1V_1 = P_2V_2$  in an isothermal process,  $\frac{V_2}{V_1} = \frac{P_1}{P_2}$ . Making these substitutions in equation 3-3b gives

$$\begin{aligned} {}_1Wk_2 &= WRT \log_e \frac{P_1}{P_2} \\ &= 4 \times 53.35 \times (240 + 460) \log_e 2 \end{aligned}$$

Since  $\log_e 2 = 0.6931$ ,

$${}_1Wk_2 = 4 \times 53.35 \times 700 \times 0.6931 = 103,500 \text{ ft-lb} \quad \text{Ans.}$$

*Example 3-3.*—Calculate the work of compression in a diesel engine having a 16 to 1 compression ratio. At the start of compression the volume is 0.96 cu ft and the pressure is 14 psia; and  $n=1.3$ .

*Solution.*—To solve for  $P_2$ , rearrange equation 2-4 as follows:

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^n = 16^{1.3} = 36.76$$

and

$$P_2 = 14 \times 36.76 = 514.6 \text{ psia}$$

Also,  $V_2 = V_1 \div 16 = 0.96 \div 16 = 0.06 \text{ cu ft}$ . Therefore, by equation 3-3a,

$$\begin{aligned} {}_1Wk_2 &= \frac{P_2V_2 - P_1V_1}{1-n} = \frac{144}{1-1.3} (514.6 \times 0.06 - 14 \times 0.96) \\ &= 480(30.88 - 13.44) \\ &= 480(17.44) = 8,370 \text{ ft-lb} \quad \text{Ans.} \end{aligned}$$

**3-5. Changes in Intrinsic or Molecular Energy for a Perfect Gas.**—One of the hypotheses concerning a perfect gas, according to the kinetic theory, is that the molecules are so far apart that there is no appreciable intermolecular attraction. Therefore, no energy is required to move them further apart and the molecules give up no energy as they come closer

together. There can be then, no change in the molecular potential energy of a perfect gas. Temperature has been defined as a measure of the mean molecular kinetic energy of a substance. Thus, for a perfect gas,

$$U=f(T)$$

If the kinetic theory is disregarded, it is possible to show that  $U=f(T)$  for any gas obeying the law  $PV=WR T$  (See Chapter 14). This should be satisfactory evidence that the molecular energy of a perfect gas depends on temperature, and only on temperature. However, as it is customary to do so, the classical experiments of Joule will be introduced here.

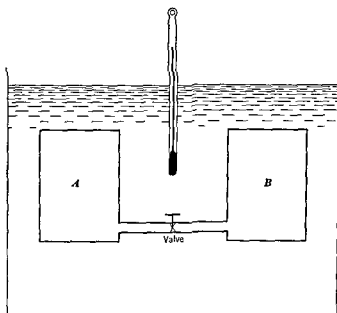


Fig 3-4 Joule's Experiment

Joule took two interconnected cylinders, as *A* and *B* in Fig 3-4, and immersed them in water. In cylinder *A* he had air at a pressure of about 22 atmospheres. In cylinder *B* he had as high a vacuum as possible. Immersed in the water was a specially constructed thermometer which would detect slight variations in water temperatures.

After equilibrium conditions were reached, Joule opened the valve and allowed the air to rush from *A* into *B*. He noted his thermometer very closely and found no change in water temperature. To draw conclusions from this experiment, equation 3-1 should be rewritten, it being realized that molecular energy is dual in its nature. Thus,

$${}_1Q_2 = U_{K_2} - U_{K_1} + U_{P_2} - U_{P_1} + \frac{{}_1Wk_2}{J} \quad (3-1a)$$

When this equation was applied to the air,  ${}_1Q_2$  was zero, as no heat passed into or out of the gas; otherwise, there would have been a temperature change in the water. There was no change in  $U_K$ , nor was any work done. Hence, the change in molecular potential energy must have been zero. Stated in another way, there had been a change in both the pressure and the volume (specific) of the air and there had been no change in molecular energy. Therefore, the molecular energy of a perfect gas is a function of temperature and temperature alone.

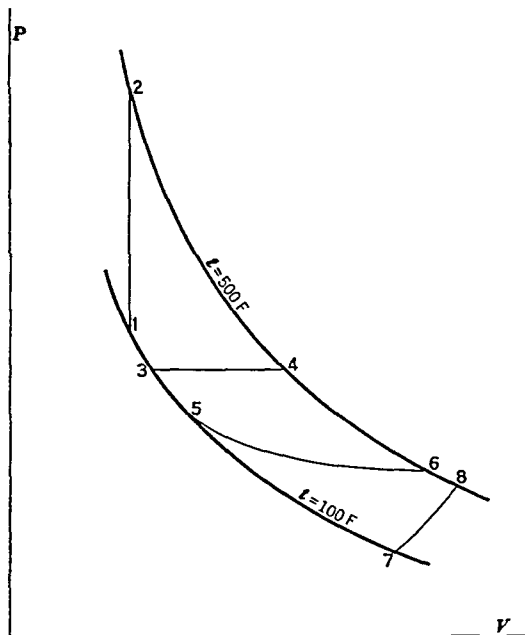


Fig. 3-5. Equal Changes of Molecular Energy Between Two Temperatures, Regardless of the Nature of Processes

Joule later found by refinements in his apparatus that there was a slight change in water temperature. However, this simply demonstrated, as did refinements in apparatus for repeating Boyle's and Charles' experiments, that air is not exactly a perfect gas.

The nature of molecular energy does not permit its direct measurement. However, as it is a direct function of temperature, the evaluation of *change* in molecular energy in terms of the *change* in temperature for *any* process must hold for *all* processes.

In Fig. 3-5 are shown two constant-temperature lines, one for 100 F and the other for 500 F. As the molecular energy depends only on temperature and is independent of volume and pressure,



$$U_2 - U_1 = U_4 - U_3 = U_6 - U_5 = U_8 - U_7 = U_{500\text{ F}} - U_{100\text{ F}}$$

As these changes are all equal, the evaluation of the change in molecular energy for *one* of these processes will give the change in molecular energy for *all* processes

Some of the heat added goes to do work in all of these processes except the constant volume one, or 1-2. For this reason the constant-volume case is used to aid in evaluating the change in molecular energy. In a constant-volume case, all the heat added goes to increase the molecular energy. Thus,

$${}_1Q_2 = U_2 - U_1$$

But, for a constant-volume addition of heat,

$${}_1Q_2 = Wc_v(T_2 - T_1)$$

Therefore,  $U_2 - U_1 = Wc_v(T_2 - T_1)$  at constant volume. Since the change in molecular energy is the same in all the other cases,

$$U_2 - U_1 = Wc_v(T_2 - T_1) \text{ for all processes}^* \quad (3-4)$$

Stated in another way, the change in molecular energy in *any process* equals the *heat added in a constant-volume process*

**3-6 Specific-Heat Relations**—The terms in the general energy equation for a constant pressure process may be evaluated in terms of temperature as follows

$${}_1Q_2 = Wc_p(T_2 - T_1)$$

$$U_2 - U_1 = Wc_v(T_2 - T_1)$$

$${}_1Wk_2 = P(V_2 - V_1) = WR(T_2 - T_1)$$

Substituting these values in the general energy equation 3-1 gives

$$Wc_p(T_2 - T_1) = Wc_v(T_2 - T_1) + \frac{WR(T_2 - T_1)}{J}$$

or

$$c_p = c_v + \frac{R}{J} \quad (3-5)$$

A constant-pressure process was chosen to aid in establishing a relation between  $c_p$  and  $c_v$ , because both of these quantities can be introduced into the general energy equation in the constant-pressure process. However, as the specific heats  $c_p$  and  $c_v$  are fixed quantities for any given perfect gas, equation 3-5 must hold for all processes

\* This applies only to perfect gases

The value of the specific heat of a substance applying to a polytropic process may be determined in terms of the exponent  $n$  or by evaluating the terms of the general energy equation in terms of temperature. Thus,

$${}_1Q_2 = Wc_z(T_2 - T_1)$$

$$U_2 - U_1 = Wc_v(T_2 - T_1)$$

$${}_1Wk_2 = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{WR(T_2 - T_1)}{1-n}$$

Therefore,

$$Wc_z(T_2 - T_1) = Wc_v(T_2 - T_1) + \frac{WR(T_2 - T_1)}{J(1-n)}$$

$$\text{or} \quad c_z = c_v + \frac{R}{J(1-n)} \quad (3-6)$$

Although equation 3-6 may be put into various forms, there is little to be gained by doing this.

**3-7. The Reversible Adiabatic.**—The special value which the exponent  $n$  assumes in a reversible adiabatic may be determined by evaluating the quantities in the general energy equation in terms of temperature.\* Thus,

$${}_1Q_2 = 0$$

$$U_2 - U_1 = Wc_v(T_2 - T_1)$$

$${}_1Wk_2 = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{WR(T_2 - T_1)}{1-n}$$

Hence,

$$0 = Wc_v(T_2 - T_1) + \frac{WR(T_2 - T_1)}{J(1-n)}$$

from which

$$-Wc_v(T_2 - T_1) = \frac{WR(T_2 - T_1)}{J(1-n)}$$

$$\text{and} \quad -c_v = \frac{R}{J(1-n)}$$

$$\text{or} \quad -c_v(1-n) = \frac{R}{J}$$

Since  $\frac{R}{J} = c_p - c_v$  (see equation 3-5),

$$-c_v + nc_v = c_p - c_v$$

$$\text{or} \quad n = \frac{c_p}{c_v}$$

---

\* A rigorous proof validating the reversibility of this is contained in Problems 13, 14, and 15 of Chapter 5.

For convenience  $\frac{c_p}{c_v}$  is symbolized by  $k$ . Thus, in a reversible process in which no heat is added,

$$n = \frac{c_p}{c_v} = k \text{ for a reversible adiabatic} \quad (3-7)$$

**3-8 Path Versus Point Functions**—A property of a substance may be defined as a characteristic quality of the substance. Some thermodynamic properties are pressure, volume, temperature, and intrinsic or molecular energy. As with other properties the values of these thermodynamic properties are fixed once the state of the substance is fixed. This being the case, if the substance changes from an initial state to a final state, the changes in the values of the properties will depend on their values in the initial and final states and will be independent of *how* the change proceeded from the initial state to the final state. For this reason properties are known as *point functions*. This principle was tacitly recognized in Chapters 2 and 3 in evaluating the four thermodynamic properties just mentioned.

Although it is not capable of mathematical proof, experimental evidence has led to the following conclusion. If any *two independent* properties or point functions are known for a substance at rest and existing in a given state, then all other properties or point functions are fixed for that state. Their determination may, in some cases, be quite involved and a trial and error solution may be necessary, but these functions can be evaluated. It should be noted that the two properties given must be independent of each other. For a perfect gas, molecular energy is a direct function of temperature. The specification of the molecular energy in addition to the temperature adds nothing to our knowledge of the state of a perfect gas. Another property that is independent of temperature must be determined in order to fix the state of the gas. The temperature at which a liquid boils is fixed by the pressure, and the specification of the boiling temperature in addition to the pressure aids in no way whatever in determining the condition of a mixture of boiling water and steam.

If two or more quantities are point functions, their values being fixed by the state of the substance, the result obtained by multiplying them together, adding them together, dividing one by another, or subtracting one from another will be a fixed quantity and hence will indicate a true property or point function. Such a property is known as a compound property. A very useful compound property called enthalpy will be discussed in Chapter 4. Compound properties are of as much value in determining the state of a substance and its other properties as are simple properties.

If two states of a substance are known, the work done cannot be evaluated until it is known how the change of state took place between the initial and final conditions. If the change was a reversible one and the substance passed directly from the initial state to the final state, then the work may be evaluated. If, however, the process is irreversible to any degree, or if the change proceeds to some other point first and then to the final state in another manner, the work cannot be evaluated until a knowledge of the manner in which the change took place is gained. This is shown in Fig. 3-6. The work done in passing from state 1 to state 2 is much less if state 2 is reached by first going to state 4 than if the path is from 1 to 3 and then to 2. An intermediate amount of work is obtained

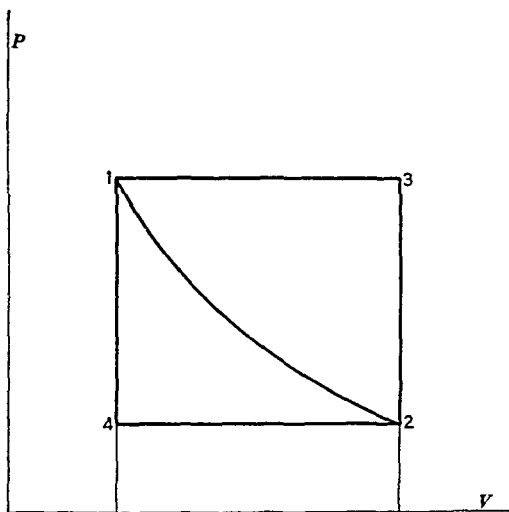


Fig. 3-6. Significance of Work as a Path Function

by going to state 2 along the curved path. If the final state is reached merely by virtue of a free expansion, of course, no work has been accomplished. As points 1 and 2 are fixed points, the change in molecular energy is also fixed. If in this case  $t_1 = t_2$ , the change is zero. Thus, the amount of heat added will equal the work done between state 1 and state 2 and cannot be evaluated until the nature of the path taken between 1 and 2 is known.

Because it is essential to know the path taken in order to evaluate both the heat added and the work done, these two quantities are known as *path functions*. No attempt should be made to evaluate path functions if only the initial and final states are known.

**3-9. Sample Problems.**—The principles discussed in the foregoing articles are applied in the following typical problems.

**Example 3-4**—The combustion process occurring after top dead center in a certain gasoline engine obeys the law  $PV^{-1.2} = C$ . At the start of the process, the volume is 0.02 cu ft, the pressure is 200 psia, and the temperature is 1600 F. Combustion proceeds until the volume is 0.028 cu ft. Calculate the work done and the heat added, treating the cylinder's contents as air and treating air as a perfect gas. (Strictly speaking, no "heat" is added. Chemical energy is released, producing the same effect as if heat were added. What is desired is this equivalent amount of heat added.)

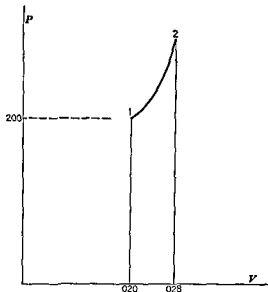


Fig 3-7 Graph for Example 3-4

**Solution**—In Fig 3-7 is shown the path of the process on the  $P$ - $V$  plane, a simple sketch such as this should always be drawn when a problem of this nature is to be solved. Mere numerical handling of numbers without taking cognizance of the variation of the physical properties involved is to be avoided.

By equation 3-3a,

$${}_1W_{k2} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

In this case,

$$\begin{aligned} P_2 &= P_1 \left( \frac{V_1}{V_2} \right)^{-1.2} = 200 \left( \frac{0.020}{0.028} \right)^{-1.2} \\ &= 200 \left( \frac{0.028}{0.020} \right)^{1.2} = 200(1.4)^{1.2} \\ &= 200 \times 1.498 = 299.6 \text{ psia} \end{aligned}$$

Hence,

$$\begin{aligned} {}_1W_{k2} &= \frac{144(299.6 \times 0.028 - 200 \times 0.02)}{1 - (-1.2)} \\ &= \frac{144(8.389 - 4)}{2.2} = 287.3 \text{ ft-lb} \end{aligned}$$

Also

$${}_1Q_2 = W_{k2}(T_2 - T_1)$$

in which

$$W = \frac{PV}{RT} = \frac{144 \times 200 \times 0.02}{53.35 \times 2060} = 0.00524 \text{ lb}$$

$$c_x = c_v + \frac{R}{J(1-n)}$$

$$= 0.1715 + \frac{53.35}{778[1 - (-1.2)]} = 0.2027$$

and

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{n-1}$$

$$= 2060 \left( \frac{0.020}{0.028} \right)^{-1.2-1} = 2060 \left( \frac{0.028}{0.020} \right)^{2.2}$$

$$= 2060(1.4)^{2.2} = 2060 \times 2.096$$

$$= 4318 \text{ R}$$

Then,

$${}_1Q_2 = 0.00524 \times 0.2027 \times (4318 - 2060)$$

$$= 2.398 \text{ Btu}$$

Ans.

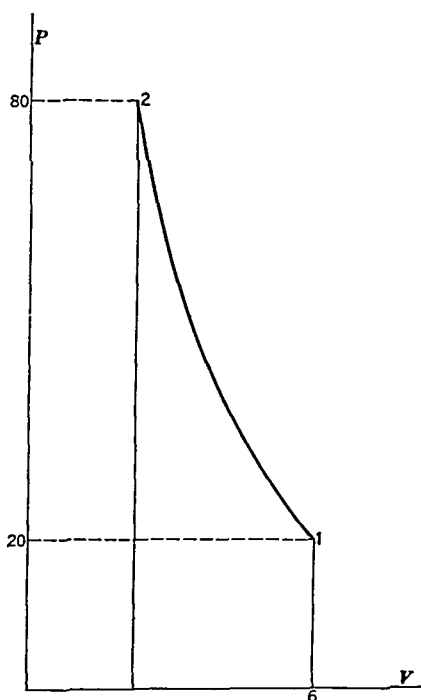


Fig. 3-8. Graph for Example 3-5

An alternative solution for the heat added is as follows:

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1Wk_2}{J}$$

$$U_2 - U_1 = Wc_v(T_2 - T_1)$$

$$= 0.00524 \times 0.1715 \times (4318 - 2060)$$

$$= 2.029 \text{ Btu}$$

$${}_1Q_2 = 2.029 + \frac{287.3}{778} = 2.398 \text{ Btu}$$

Ans.

*Example 3-5* — Air is compressed adiabatically from 20 psia to 80 psia. The air originally occupies 6 cu ft at 240 F. Assuming that the compression is a reversible one, calculate the change in intrinsic energy.

*Solution* — As this change is adiabatic,  ${}_1Q_2 = 0$  and

$$U_2 - U_1 = -\frac{{}_1Wk_2}{J}$$

In this case,

$${}_1Wk_2 = \frac{P_2V_2 - P_1V_1}{1-n}$$

By referring to Fig. 3-8, one can see that the volume at the end of the process is unknown, then

$$P_1V_1^n = P_2V_2^n$$

Transferring the exponent to the known (pressure) ratio, we get

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

from which

$$V_2 = \frac{6}{\left(\frac{80}{20}\right)^{\frac{1}{1.4}}} = 2.23 \text{ cu ft}$$

Hence,

$${}_1Wk_2 = \frac{144(80 \times 2.23 - 20 \times 6)}{1 - 1.4} \\ = -21,020 \text{ ft lb}$$

and

$$U_2 - U_1 = \frac{21,020}{778} = 27.0 \text{ Btu} \quad \text{Ans}$$

An alternative solution is as follows

$$U_2 - U_1 = Wc_v(T_2 - T_1)$$

$$W = \frac{PV}{RT} = \frac{144 \times 20 \times 6}{53.35 \times 700} = 0.4628 \text{ lb}$$

$$T_2 - T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 700 \left(\frac{80}{20}\right)^{\frac{1.4-1}{1.4}} = 1040 \text{ R}$$

$$U_2 - U_1 = 0.4628 \times 0.1715(1040 - 700) = 26.98 \text{ Btu} \quad \text{Ans}$$

*Example 3-6* — Air in a tank is heated until the pressure increases from 80 psia to 120 psia. If the original temperature is 80 F and the tank volume is 20 cu ft, calculate the heat added.

*Solution* — As the volume is constant, there is no area under the curve on the  $P$ - $V$  plane and no work is performed. Hence,

$${}_1Q_2 = Wc_v(T_2 - T_1)$$

in which

$$W = \frac{PV}{RT} = \frac{144 \times 80 \times 20}{53.35 \times 540} = 8.0 \text{ lb}$$

Since the volume is constant,

$$T_2 = T_1 \frac{P_2}{P_1} = \frac{540 \times 120}{80} = 810 \text{ R}$$

and

$${}_1Q_2 = 8.0 \times 0.1715(810 - 540) = 370 \text{ Btu} \quad \text{Ans}$$

*Example 3-7.*—A quantity of air at 180 F originally occupies 20 cu ft at a pressure of 30 psia. How much heat must be removed from the air to keep its temperature constant at 180 F as it is compressed until the pressure reaches 180 psia?

*Solution*—The  $P$ - $V$  curve, which is shown in Fig 3-9, is a rectangular hyperbola since the temperature is constant and  $n$  equals unity. In this case,

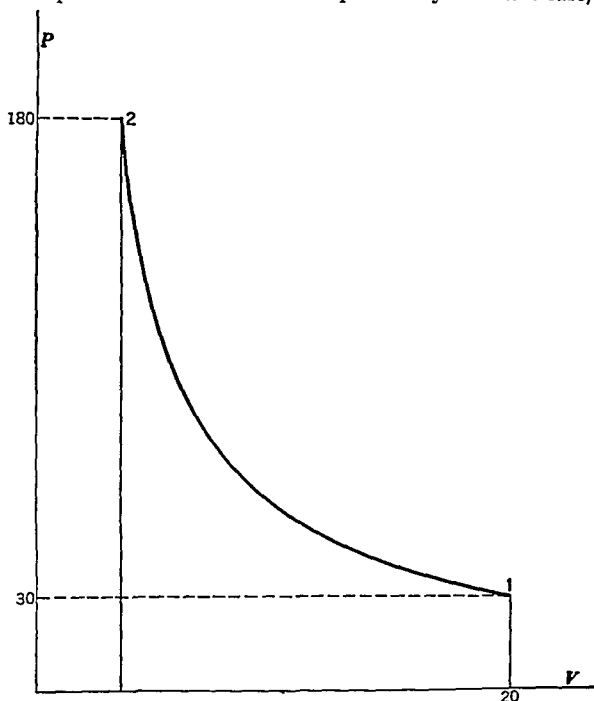


Fig. 3-9. Graph for Example 3-7

$${}_1Q_2 = \frac{{}_1Wk_2}{J} = \frac{PV}{J} \log_e \frac{V_2}{V_1}$$

Since  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$  reduces to  $\frac{V_2}{V_1} = \frac{P_1}{P_2}$ ,

$${}_1Q_2 = \frac{PV}{J} \log_e \frac{P_1}{P_2}$$

But

$$\log_e \frac{P_1}{P_2} = \log_e P_1 - \log_e P_2 = -(\log_e P_2 - \log_e P_1) = -\log_e \frac{P_2}{P_1}$$

Hence,

$${}_1Q_2 = \frac{-144 \times 30 \times 20}{778} \log_e \frac{180}{30} = -199.0 \text{ Btu added or } 199.0 \text{ Btu removed} \quad \text{Ans.}$$

*Example 3-8*—A quantity of air is originally at 200 F and occupies 60 cu ft. If 400 Btu are added to the air at a constant pressure of 50 psia, calculate the work done.

*Solution*—This is a constant-pressure process. The non-flow general energy equation is



$${}_1Q_2 = U_2 - U_1 + \frac{{}_1Wk_2}{J}$$

from which

$$\frac{{}_1Wk_2}{J} = {}_1Q_2 - (U_2 - U_1) = {}_1Q_2 - Wc_p(T_2 - T_1)$$

Since  ${}_1Q_2 = Wc_p(T_2 - T_1)$ ,

$$T_2 - T_1 = \frac{{}_1Q_2}{Wc_p}$$

Also,

$$W = \frac{PV}{RT} = \frac{144 \times 50 \times 60}{53.35 \times 660} = 12.27 \text{ lb}$$

Hence,

$$T_2 - T_1 = \frac{400}{12.27 \times 0.24} = 135.8$$

and

$$Wc_p(T_2 - T_1) = 12.27 \times 0.1715 \times 135.8 = 285.8 \text{ Btu}$$

The work done is

$${}_1Wk_2 = (400 - 285.8)778 = 88,900 \text{ ft-lb}$$

Ans

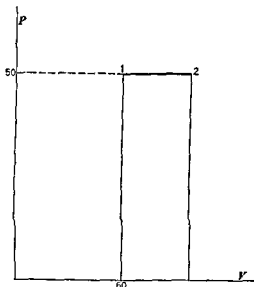


Fig 3-10 Graph for Constant Pressure Process in Example 3-8

An alternative solution is as follows. As can be seen in Fig 3-10, the work area on  $P$ - $V$  coordinates is rectangular, and hence

$${}_1Wk_2 = P(V_2 - V_1)$$

In this case,

$$T_2 = 135.8 + 660 = 795.8 \text{ R}$$

Also, since  $P_1 = P_2$ , the expression  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$  reduces to

$$V_2 = V_1 \frac{T_2}{T_1} = 60 \left( \frac{795.8}{660} \right) = 72.35 \text{ cu ft}$$

Hence,

$${}_1Wk_2 = 144 \times 50(72.35 - 60) = 88,900 \text{ ft-lb}$$

Ans

## CHAPTER 4

### STEADY FLOW

4-1. **The Field of Steady Flow.**—With the exception of certain isolated cases, such as the heating or cooling of a tank of air, most problems encountered in dealing with liquids, vapors, and gases are flow problems. Flow is encountered in a boiler, its furnace, the turbine, and all auxiliaries in a steam power plant, such as pumps, condensers, fans, economizers, and air preheaters. Flow is met with in a diesel engine and its auxiliaries, such as water pumps, oil pumps, and superchargers. A study of a refrigerating compressor and the other equipment needed to produce refrigeration, such as the evaporator, condenser, and brine pump, involves a flowing fluid. In air conditioning, flow is involved in a study of the dehumidifier, the heater, and the fans. In the case of each of these individual devices, as well as innumerable others, a fluid flows into and out of the device.

In the centrifugal and rotary types of pumps, engines, and compressors, there is continuous flow of a fluid into and out of a device. In the case of reciprocating machinery, the flow is pulsating, because of the opening and closing of valves. With the valves closed, a truly non-flow process takes place within the cylinder. The over-all picture of such a device is, however, one of flow. If conditions are observed in a pipe line at some distance from a reciprocating machine, it will be found that the fluctuations are dampened out to a large extent. Furthermore, if a reciprocating piece of equipment is carrying a steady load, it will be found that the weight of fluid flowing into the device per unit of time equals that flowing out of the device for the same unit of time. Thus, although the flow in the case of a reciprocating piece of machinery is pulsating, the fluid flows into and out of the device at a steady rate.

4-2. **The Steady Flow Energy Equation.**—The equation of steady flow is a restatement of the law of conservation of energy. Provided that the flow into and out of a device is at a steady and equal rate, and provided that no energy is stored up in or given up by the device, the energy in all forms entering the device in a given time interval must equal the energy in all forms leaving that device in the same time interval.

In case the temperature within the device differs from the outside temperature, *heat* may flow into or out of the device through the walls.

It is necessary to know the nature of the device in order to know whether or not the internal temperature is higher or lower than that of the surroundings and thus know whether heat is flowing in or out of the device. If the device is a complex one, heat may flow out of part of it and into another part of it. Both quantities of heat may be introduced into the equation, or the difference between the two may be found and used as a single quantity.

Work may be transmitted into or out of the device, the direction depending on the nature of the device. This energy is transmitted through moving parts of the device.

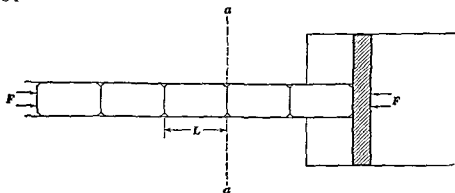


Fig 4-1 Transmission of Flow Energy

The fluid passing into and out of the device will carry energy with it. The fluid may possess *molecular energy*, *chemical energy* (Ch E), *potential energy* (P E), and *kinetic energy* (K E). In addition, the fluid will carry with it energy exerted on it to cause it to flow. *Electrical energy* also may, conceivably, pass into or out of the device. As this is not a normal case in mechanical devices it will be neglected in this discussion.

The energy transmitted by virtue of flow is illustrated diagrammatically in Fig 4-1. In this figure a resisting force  $F'$  exists in a cylinder to the right of a piston. A pipe leading into the cylinder is filled with cylindrical blocks. If a sufficiently large force  $F$  is exerted on the left hand block, the blocks will move into the cylinder, doing work on the piston. The energy required to do the work is transmitted through the blocks from the source of energy acting on the left hand block to the piston. Each block passing section  $a-a$  carries with it the energy necessary to force it past  $a-a$ . This energy equals the product of the following quantities: the pressure  $P$  exerted on the block by the preceding block, the pipe area  $A$  and the length  $L$  of the block. Since the product  $AL$  equals the volume  $V$  of the block, the energy carried by the block as it passes a section equals the product of the pressure  $P$  on the block and the block volume  $V$ , or  $PV$ .

There is no well-established term to designate this energy transmitted by virtue of flow. To avoid a lengthy phrase, the term *flow energy* will be used here.

The concept applied to the solid block may be applied to a flowing fluid—a gas, vapor, or liquid. Instead of dealing with blocks, the fluid stream may be conceived of as being broken up into a series of volumes of fluid, each volume having unit weight. In this case the flow energy per unit weight is  $Pv$ , where  $v$  is the specific volume.

Care should be taken to avoid confusion of flow energy with kinetic energy of the flowing fluid. The flow energy is a *transmitted energy* and, per unit of material, it is independent of the length of time required to move the fluid but depends solely on the work necessary to move the fluid. Kinetic energy, on the other hand, is a function of the velocity of the fluid.

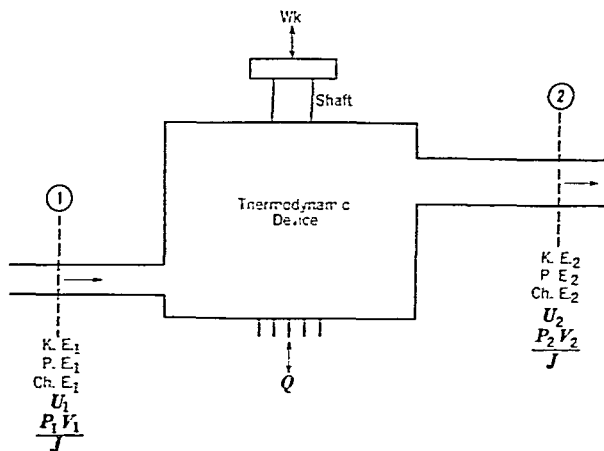


Fig. 4-2. General Application of Steady Flow Energy Quantities

Although this chapter deals with flow, because the term  $PV$  is introduced here in the case of flow, it is also desirable to consider the term  $PV$  in case no flow exists. Referring to Fig. 4-1 again, suppose that the same force is exerted on the left-hand block as before but the resisting force  $F'$  is so large that no movement takes place. If there is no movement, no work is done on the blocks and no energy is transmitted through them. Nevertheless, the same pressure exists on the blocks and the blocks have the same volume. Hence, there is a term  $PV$  having the same value as in the case of flow, but this term  $PV$  cannot now represent energy being transmitted by flow, because none is being transmitted. Thus, the term  $PV$  represents energy only in case of flow. In the case of non-flow, the term  $PV$  does not enter into the energy equation.

Refer to Fig 4-2 The equation of steady flow is based on the fact that *energy in* per unit of time is equal to *energy out* per unit of time If the various forms of energy in are given the subscript 1 and the forms of energy out are given the subscript 2,

$$\frac{K E_1}{J} + \frac{P E_1}{J} + Ch E_1 + U_1 + \frac{P_1 V_1}{J} = Q + \frac{Wk}{J} = \frac{K E_2}{J} + \frac{P E_2}{J} + Ch E_2 + U_2 + \frac{P_2 V_2}{J} \quad (4-1)$$

Note that each of these energy quantities must be evaluated for the same length of time This may well be called the most important equation in engineering thermodynamics

A further knowledge of the individual device is required before it can be determined whether the work and the heat are going into or out of the device In case they are going into the device, they should be placed on the left-hand side of the equation In case they are energies leaving the device, they should be put on the right hand side of the equation If these quantities are so placed, they will have positive values

Equation 4-1 appears complex and involved However, in specific applications several terms drop out of the equation as the changes in the energy quantities are so small that they may be neglected The condition that determines whether or not terms may be omitted from equation 4-1 is not the amount of the change in energy under consideration on an absolute basis but rather its amount in comparison with the amounts of the other energy changes involved

The work term must be zero unless work from mechanical power transmitters, such as shafts, goes into or out of the device Even when such a method of transmitting mechanical power is used, the work going out of the device will still equal zero if the machine is idling

The heat transmitted through the walls of the device approaches a value of zero as the temperature within the device approaches that of the surroundings and also as the amount of insulation on the walls is increased

If point 2 is 10 ft above point 1, then the potential energy of 1 lb of the fluid at point 2 exceeds that at point 1 by 10 ft-lb, or 0.0128 Btu This energy change, while large in comparison with changes in other forms of energy in the case of liquids, is normally an exceedingly small part of the other energy changes for gases and vapors and hence may be neglected for such fluids

In well designed pipe lines, the fluid velocities must be kept low, otherwise, excessive friction losses will be encountered Thus, if points 1 and 2 are taken in the pipe lines entering and leaving the machine, the

kinetic energies at entrance and exit are very small and may be neglected for most cases of flowing gases and vapors.

Unless combustion or some other chemical reaction is taking place within the device, there will be no change in chemical energy.

In most cases there will be a change in the molecular energy  $U$  and in the flow energy  $PV$ .

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4-3. **Enthalpy.**—In general, there is no need for evaluating either the change in the molecular energy or the change in the flow energy. What is desired is the change in the *summation* of these two energy quantities. As molecular energy, pressure, and volume are true properties and may be definitely evaluated once the state of the substance is known, the summation of the molecular energy and the pressure-volume term, or  $U + \frac{PV}{J}$ , must be fixed. If the states of the fluid entering and leaving a device are

known, the quantity  $\left(U_2 + \frac{P_2 V_2}{J}\right) - \left(U_1 + \frac{P_1 V_1}{J}\right)$  is fixed and is readily determinable without the necessity of determining the individual terms involved.

At least 75 years ago Prof. Willard Gibbs saw the desirability of using the summation of the molecular energy and the flow energy. Realizing that no one term could picture the summation of these dissimilar quantities, he used the symbol  $\chi$  (*chi*) to designate this summation, or  $\chi = U + \frac{PV}{J}$ .

Shortly after this, someone, not appreciating the full significance of this summation, designated it as total heat. More recently it has been termed heat content. Unfortunately, both of these terms were widely used and for years students of thermodynamics have been confused by them, as they were misnomers. In the first place,  $U$  does not represent heat; it is molecular energy. In the second place, the term  $\frac{PV}{J}$  does not represent heat; it represents flow energy in case of flow and has no physical significance unless there is flow. Therefore, the quantity  $U + \frac{PV}{J}$  is neither the total heat nor the heat content of a body.

Modern writers on thermodynamics have recognized this difficulty. However, instead of reverting to Gibbs' original term, they coined the term *enthalpy*. This term has the advantage of not raising an incorrect physical concept; but, unless correctly pronounced,\* it has the disadvantage of sounding similar to *entropy* (see Chapter 5). The accepted symbol for

\* Accent is on the second syllable.

enthalpy, which is  $H$ , is a poor choice as it is often incorrectly associated with heat. However, following common practice,  $H$  will be used in this text to designate enthalpy. The defining equation is

$$H = U + \frac{PV}{J} \quad (4-2)$$

This relation should be read as follows. The enthalpy equals the summation of the molecular energy and the product of pressure and volume expressed in Btu (or simply the term  $\frac{PV}{J}$ ). Little is to be gained by any further attempt to picturize this summation which is termed enthalpy.

In dealing with a perfect gas, enthalpy changes may be evaluated in terms of temperature as follows

$$H = U + \frac{PV}{J}$$

$$H_2 - H_1 = U_2 - U_1 + \frac{P_2 V_2 - P_1 V_1}{J}$$

But  $U_2 - U_1 = W c_v (T_2 - T_1)$

and  $P_2 V_2 - P_1 V_1 = W R (T_2 - T_1)$

Hence,  $H_2 - H_1 = W c_v (T_2 - T_1) + \frac{W R}{J} (T_2 - T_1) = W \left( c_v + \frac{R}{J} \right) (T_2 - T_1)$

Since  $c_p = c_v + \frac{R}{J}$  (see equation 3-5),

$$H_2 - H_1 = W c_p (T_2 - T_1) \quad (4-3)$$

Equation 4-3 recognizes that enthalpy is a true property and that, for a perfect gas, the change in enthalpy is a direct function of the change in temperature. *Change in enthalpy is independent of the nature of the change of state taking place, and equation 4-3 holds equally well for flow and non flow processes.*

As mentioned earlier, the term  $\frac{PV}{J}$  does not represent energy in a non-flow process. Because of this enthalpy has little use in a non flow process except in the case of a constant-pressure process. This is shown as follows

$$H = U + \frac{PV}{J}$$

$$dH = dU + \frac{P dV}{J} + \frac{V dP}{J}$$

From the general energy equation for a non-flow process,

$$dQ = dU + \frac{d(Wk)}{J}$$

or

$$dQ = dU + \frac{p dV}{J}$$

By comparing  $dH$  and  $dQ$  it may be seen that, under *non-flow conditions*, the change in enthalpy in a constant-pressure process equals the heat added. This fact is valuable when dealing with non-flow constant-pressure processes for liquids, vapors, and imperfect gases for which tables of enthalpy are available. (See Chapter 14.)

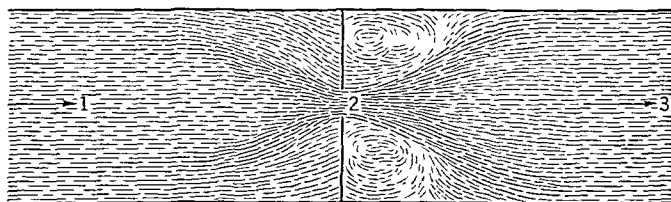


Fig. 4-3. Throttling

**4-4. Throttling.**—One of the simplest applications of steady flow is that of throttling. The fundamental concept of throttling is the choking off of, or the offering of an obstruction to, a flowing fluid. Although it is true that any pipe has friction, and hence restricts flow, the idea of throttling generally implies the introduction into the line of flow of some object, such as an orifice plate or a partly opened valve, which offers materially more resistance to flow than does the rest of the system.

Throttling is illustrated in Fig. 4-3. A plate containing a small orifice (hole) has been inserted in a pipe line. Because the area of the hole is small in comparison to the area of the pipe, the velocity of the fluid passing through the orifice is large in comparison to that in the pipe. More must be known about the system, however, before a statement can be made about the absolute values of the various velocities. If the pipe discharges into the atmosphere at point 3, and the pressure at point 1 is only slightly above atmospheric, then the absolute velocity of the fluid will not be large at any point. On the other hand, if a high pressure is maintained at point 1 and at the same time discharge into the atmosphere is retained, the difference in pressure between points 1 and 3 will cause a higher rate of flow and all velocities will be greater. Under most conditions, however, very large pressure differences do not exist between points 1 and 3 unless the area of the opening at point 2 is very small. This small opening so restricts flow that the velocity at point 1 is small. If the location of point



3 is chosen at such a distance from point 1 that the flowing stream fills the entire pipe, then the velocity at point 3 is also very small. Thus, under most conditions, the change in kinetic energy between points 1 and 3 can be neglected.

If the temperature of a flowing fluid is substantially atmospheric, it may be assumed that there is no appreciable loss of heat between points 1 and 2. Under this condition, the equation of steady flow becomes (kinetic energy at point 1 being neglected)

$$H_1 = H_2 + \frac{K E_2}{J}$$

The greater the kinetic energy at point 2, the larger is the drop in enthalpy between points 1 and 2. For a perfect gas, the change in enthalpy is a direct function of the temperature change (see equation 4-3). With a large pressure differential existing between points 1 and 2, the temperature at point 2 is much lower than that at point 1. In case the temperature at point 1 is about atmospheric, there will be radiation into the pipe between points 2 and 3 unless precautions are taken to prevent it.

If the radiation into or out of the pipe may be neglected and if the difference in kinetic energies can also be neglected, the equation of steady flow for a throttling process becomes

$$H_1 = H_2$$

This equation does not state that enthalpy remains constant during a throttling process, but it does state that, when the kinetic energy set up by throttling is dissipated (i.e., turned back into enthalpy), the final enthalpy equals the enthalpy before the start of the throttling.

As the enthalpy of a perfect gas is a direct function of temperature, it follows that there is no net change in temperature when a perfect gas is throttled, provided there is no appreciable net change in kinetic energy and no appreciable radiation.

When an imperfect gas or a vapor is throttled, there will be a net change in temperature, the amount of which is sometimes used to determine the imperfection of the gas.\*

The loss of ability to do work occasioned by the throttling of a fluid is discussed in Chapter 5.

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\* This is called the Joule-Thomson effect and is expressed mathematically as  $\left(\frac{dT}{dP}\right)_h$ .

For a perfect gas this coefficient is zero and for most of the so-called "permanent" gases it is very small, i.e., for atmospheric air, the temperature drop is approximately 0.03 F degree per psi drop. For vapors near condensing conditions, the Joule-Thomson effect is naturally large.

4-5. Other Applications of Steady Flow.—Other applications of the steady-flow principle are most readily shown by the use of selected examples. However, the student is strongly urged not to memorize the particular forms of the steady-flow equation set up for the various devices. Rather he should understand the method used so that he may reduce the steady-flow equation to apply to these and other steady-flow devices.

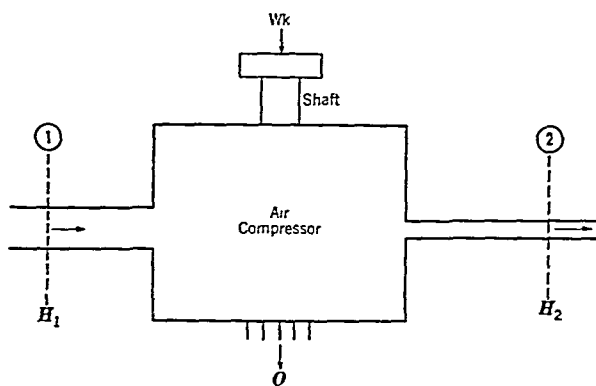


Fig. 4-4. Energy Quantities for Air Compressor

(a) *The Air Compressor:* The temperature of air entering and leaving an air compressor may be determined in the pipe lines. The pipe lines are normally of sufficient size to make the difference in kinetic energies entering and leaving the compressor insignificant in comparison to the changes in the forms of energy. As shown at the bottom of page 52, the change in potential energy can be neglected. Furthermore, there is no change in chemical energy. Thus the only energy quantities to be considered are those indicated in Fig. 4-4.

*Example 4-1.*—Air at the rate of 60 cu ft per min enters an air-cooled air compressor at 14.0 psia and 75 F, and leaves at 275 F. If the horsepower input to the machine is 6, calculate the radiation from the compressor per minute.

*Solution.*—Equating the energies in and out, we may write the following relationship:

$$H_1 + \frac{W_k}{J} = H_2 + Q$$

or

$$Q = H_1 - H_2 + \frac{W_k}{J}$$

To evaluate  $H_1 - H_2$ , the weight of air must be known. It is

$$W = \frac{PV}{RT} = \frac{144 \times 14.0 \times 60}{53.35 \times 535} = 4.238 \text{ lb per min}$$

Then,

$$\begin{aligned} H_1 - H_2 &= W c_p (T_1 - T_2) \\ &= 4.238 \times 0.24 (75 - 275) = -203.4 \text{ Btu per min} \end{aligned}$$

Also, 
$$\frac{Wk}{J} = 6 \times \frac{33,000}{778} = 254.5 \text{ Btu per min}$$

Hence, 
$$Q = -203.4 + 254.5 = 51.1 \text{ Btu per min} \quad \text{Ans}$$

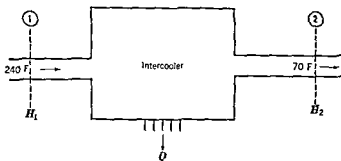


Fig 4-5 Intercooler of Example 4-2

(b) *Intercooler* As far as the air in an intercooler of a compound compressor is concerned, the only energy change it experiences is a change in enthalpy. As there are no moving parts, no work is done.

*Example 4-2*—Air enters the intercooler of a compound compressor at 40 psi, gage, and 240 F, and leaves at 39 psi, gage, and 70 F. The barometer is 26.48 in and the air flow in is 200 cu ft per min. Calculate the heat removed from the air per min by the cooling water.

*Solution*—The equation of steady flow becomes (see Fig 4-5)

$$H_1 = H_2 + Q$$

or 
$$Q = H_1 - H_2 = Wc_p(T_1 - T_2)$$

In this case,

$$P = 40 + 0.491 \times 26.48 = 40 + 13 = 53 \text{ psia}$$

and

$$W = \frac{PV}{RT} = \frac{144 \times 53 \times 200}{53.35 \times 700} = 40.87 \text{ lb per min}$$

Hence,

$$H_1 - H_2 = 40.87 \times 0.24(240 - 70) = 1667 \text{ Btu per min}$$

and

$$Q = 1667 \text{ Btu per min}$$

Ans

Note that  $Q$  can be evaluated here by the change in  $H$  (even though the pressure drops) because the flow is steady, contrast this with the non-flow statements at the end of Art. 4-3.

(c) *Steam Engine*—A steam engine does work, but a small quantity of heat may leave the engine by radiation and convection.

*Example 4-3*—Steam enters a steam engine with an enthalpy of 1250 Btu per lb and leaves with an enthalpy of 1075 Btu per lb\*. Steam flow is 4290 lb per hr. The engine delivers 290 hp. Calculate the radiation loss per hour from the engine.

\* These values of enthalpies are determined from the condition of the steam entering and leaving the engine by use of a steam table. These are not absolute values but are values above a datum plane (see Art. 6-3).

*Solution.*—The energies involved are shown in Fig. 4-6. The steady-flow energy equation is

$$H_1 = H_2 + Q + \frac{Wk}{J}$$

or

$$Q = H_1 - H_2 - \frac{Wk}{J}$$

In this case,

$$H_1 - H_2 = 4290(1250 - 1075) = 750,800 \text{ Btu per hr}$$

and

$$\frac{Wk}{J} = \frac{290 \times 33,000 \times 60}{778} = 738,000 \text{ Btu per hr}$$

Hence,

$$Q = 750,800 - 738,000 = 12,800 \text{ Btu per hr}$$

Ans.

(d) *Turbine Nozzle:* The common form of a turbine nozzle for steam or gas is shown in Fig. 4-7. The fluid is supplied to the turbine with a

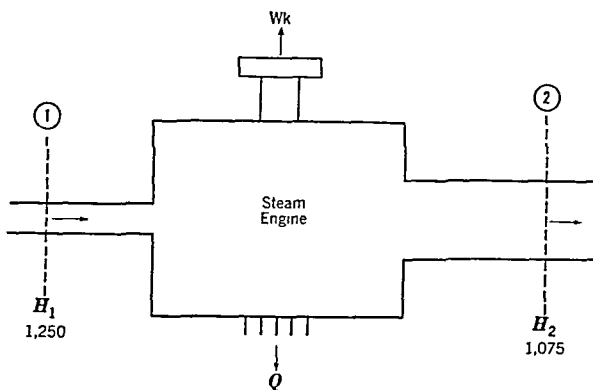


Fig. 4-6. Application of Steady Flow Energy Analysis of Example 4-3

large value of enthalpy and with a pressure higher than that at exhaust. The purpose of the nozzle is to convert as large a percentage as possible of the enthalpy into kinetic energy and to direct the high-velocity fluid onto blades where work is done. The fluid normally moves through the nozzle in such a short length of time and in such quantities that the heat lost per unit weight of fluid is small enough to be neglected. Under this condition, the equation of steady flow becomes

$$\frac{\text{K.E.}_1}{J} + H_1 = \frac{\text{K.E.}_2}{J} + H_2$$

In most installations, the area of the pipe leading to the nozzle is large and hence the original kinetic energy is so small that it can be neglected.

*Example 4-4.*—The gases leave the combustion chamber of a gas turbine and enter the nozzle at 25 psi, gage, and 1200 F. There is atmospheric exhaust. The area of the approach to the nozzle is large in comparison to the nozzle area. Barometric pressure is standard. Calculate the velocity at the nozzle exit, assuming perfect nozzle action.

*Solution*—As the temperature leaving the nozzle is as low as 1200 F, it is assumed that so much air has been used that the gases leaving the combustion chamber may be treated as air without serious error. A small error will be introduced by neglecting the change in specific heat of the air with a change in temperature. The steady-flow equation reduces to

$$\frac{KE_2}{J} = H_1 - H_2$$

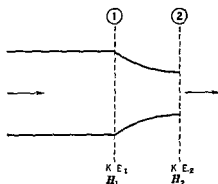


Fig 4-7 Turbine Nozzle

If 1 lb of air is considered,

$$\frac{V_2^2}{2gJ} = c_p(T_1 - T_2)$$

In this case,

$$T_2 = \frac{T_1}{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}} = \frac{1660}{\left(\frac{39.7}{14.7}\right)^{\frac{0.4}{1.4}}} = \frac{1660}{1.328} = 1250 \text{ R}$$

and

$$\frac{V_2^2}{2gJ} = 0.24(1660 - 1250) = 98.4 \text{ Btu}$$

Hence,

$$V_2^2 = 4,930,000$$

and

$$V_2 = 2220 \text{ ft per sec}$$

Ans

## CHAPTER 5

### CARNOT CYCLE, AVAILABLE ENERGY AND ENTROPY

5-1. **The Carnot Principle.**—The first law of thermodynamics states that heat may be turned into work and vice versa. It seems evident that the earlier developers of the steam engine did not realize that a limitation existed. Perhaps the first to appreciate this limitation was a French engineer, Sadi Carnot. In 1824\* he published his deductions in a paper entitled “Reflections on the Motive Power of Heat and the Proper Machines to Develop this Power.”

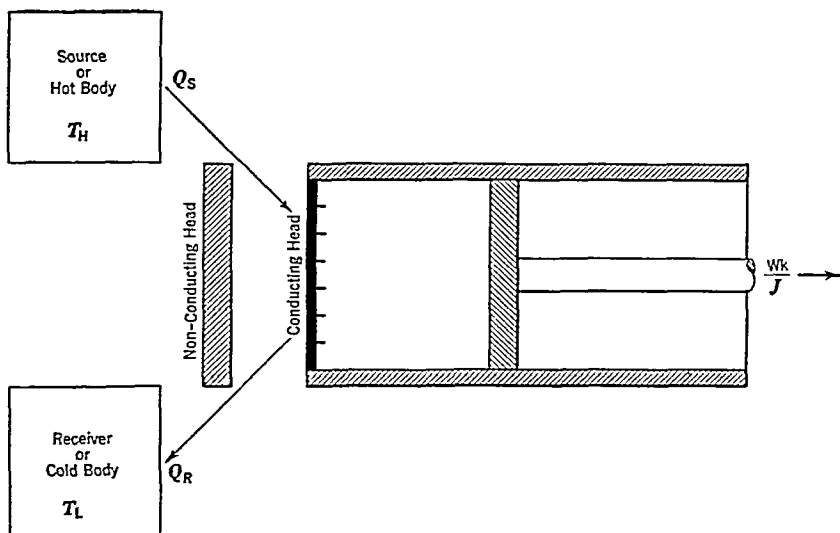


Fig. 5-1. Engine Using Carnot Cycle

Carnot's deduction was that "heat cannot perform work except when it passes from a higher to a lower temperature level." He pointed out that the amount of heat thus transformed into work was proportional to the difference existing between the temperatures at which heat was supplied and rejected.

Carnot presented no details of an engine to obtain work. He avoided a discussion of the nature of heat. This work was presented some 20 years before Joule's brilliant demonstrations led to the discard of the caloric theory of heat and the recognition that heat is a form of energy.

\* Carnot was 28 years of age at this time. It is believed that he wrote this paper three or four years earlier.

Whether or not Carnot realized the true nature of heat, he so framed his deductions that they could be accepted, regardless of the nature of heat.

In 1849, Lord Kelvin, realizing the fundamental and far reaching significance of Carnot's concepts, presented Carnot's Principle to the Royal Society of Edinburgh. He improved on Carnot's original paper

by suggesting means whereby work could be obtained from heat supplied to an engine. This engine operated in a manner implied but not specified by Carnot.

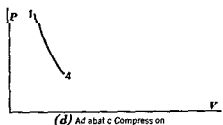
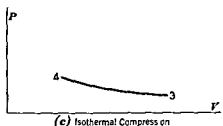
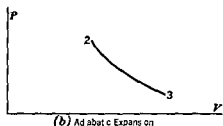
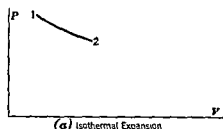


Fig 5-2 Steps in Carnot Cycle

**5-2 The Carnot Cycle.**—The Carnot Cycle, as proposed by Kelvin, was carried out by having the following parts, as indicated in Fig 5-1.

(1) A cylinder with a frictionless piston, non-conducting walls and piston, a perfectly conducting head, and a detachable non-conducting head,

(2) A source of heat, or a so-called *hot body*,

(3) A receiver of heat, or a so-called *cold body* or *heat sink*,

(4) A working substance

It is to be noted that these four items are not peculiar to the Carnot Cycle, but are required for the operations of other theoretical cycles. The manner in which heat is abstracted from the hot body, the method of producing work in the engine, and the way in which heat is rejected to the cold body, as is to be described, particularize the Carnot Cycle.

In the case of the Carnot Cycle, both the hot body and the cold body are said to be infinite hot or cold bodies. This does not imply that these

bodies are of unlimited size but rather implies that an addition of heat to them or a removal of heat from them does not change their temperatures. If energy is supplied to a body by combustion of a fuel at the same

rate as heat is removed from the body, then its temperature will remain constant. Condensing steam gives up heat at constant temperature as long as its pressure is constant. There is little material change in atmospheric temperature caused by man's addition of heat to the atmosphere or removal of heat from it. All these objects are, then, considered infinite hot or cold bodies.

Carnot did not specify the nature of the working substance. Indeed he stated that the work obtained for a given set of temperatures was independent of the working substance. In accordance with common practice, the working substance will be taken as air. The air will be treated as a perfect gas, and all processes will be assumed to be reversible.

The first process of the Carnot Cycle starts with the temperature of the working substance equal to that of the hot body which is in contact with the conducting cylinder head. When the restricting force on the piston rod is reduced by a suitable mechanism, the piston moves outward and does work, as indicated in Fig. 5-2(a). The work done comes, immediately, from the molecular energy of the working substance, the temperature of which would tend to drop. As the working substance is in perfect contact with the hot body, heat will flow into the working substance, replacing the molecular energy transformed into work and thus maintaining constant temperature.

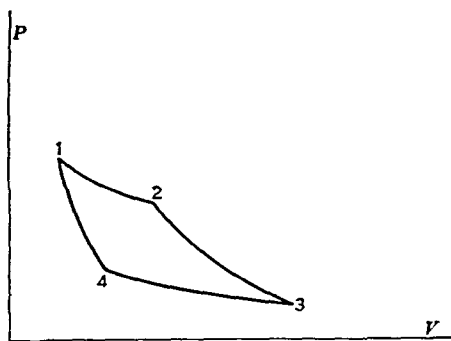


Fig. 5-3. P-V Diagram for Carnot Cycle

When the desired amount of heat has been transferred to the working substance, the non-conducting head is attached to the cylinder and the working substance is allowed to expand adiabatically, as indicated in Fig. 5-2(b), until its temperature reaches that of the cold body.

It is assumed that the engine is so designed that the end of the stroke is reached at the termination of the reversible adiabatic expansion. In order to continue to have work delivered by the engine, it is necessary to return the working substance to its original state. Unless the pressure



of the working substance acting on the piston during the return stroke is less than that on the forward stroke, as indicated in Fig 5-2 (c) and Fig 5-3, no net work can be done. Without discarding the working substance, the only way to obtain a lower pressure on the return stroke is to remove heat from the working substance at the beginning of the return stroke. In the Carnot Cycle this is accomplished by removing the non-conducting head and bringing the cold body in contact with the conducting cylinder head. The working substance is now so compressed, as indicated in Fig 5-2 (c), that heat is rejected or discarded to the cold body without any change in temperature of the working substance.

The isothermal compression is stopped at such a time that a further adiabatic compression, represented in Fig 5-2 (d), with the non-conducting cylinder head in place, will return the working substance to its initial condition. The cycle may now be repeated indefinitely.

**5-3 The Carnot-Cycle Efficiency**—The purpose of any heat engine is to transform into work as much as possible of the heat supplied to the working substance. The efficiency (thermal) of such an engine may be defined as the net work done by an engine divided by the heat supplied to its working substance by the hot body. Thus,

$$\text{Eff} = \frac{\frac{\text{Net Wk}}{J}}{Q_s} \quad (5-1)$$

From the law of conservation of energy, that portion of the heat supplied which is not rejected must be delivered as work. Hence,

$$\frac{\text{Net Wk}}{J} = Q_s - Q_r$$

An alternate form of the efficiency equation becomes

$$\text{Eff} = \frac{Q_s - Q_r}{Q_s} \quad (5-2)$$

All the heat supplied to the working substance in the Carnot Cycle is supplied at constant temperature. For this constant temperature process,

$$Q_s = {}_1Q_2 = \frac{{}_1W_2}{J} = \frac{P_1 V_1 \log_e \frac{V_2}{V_1}}{J} = \frac{W R T_H}{J} \log_e \frac{V_2}{V_1}$$

where  $T_H$  is the temperature of the hot body.

The heat rejected by the working substance is rejected at constant temperature. Hence,

$$Q_R = -{}_3Q_4 = -\frac{{}_3Wk_4}{J} = \frac{-P_3V_3 \log_e \frac{V_4}{V_3}}{J} = \frac{WRT_L}{J} \log_e \frac{V_3}{V_4}$$

where  $T_L$  is the temperature of the cold body.

Substituting in equation 5-2, we obtain

$$\text{Carnot Eff.} = \frac{WRT_H \log_e \frac{V_2}{V_1} - WRT_L \log_e \frac{V_3}{V_4}}{WRT_H \log_e \frac{V_2}{V_1}}$$

But

$$\frac{V_3}{V_2} = \left( \frac{T_2}{T_3} \right)^{\frac{1}{k-1}} = \left( \frac{T_H}{T_L} \right)^{\frac{1}{k-1}}$$

and

$$\frac{V_4}{V_1} = \left( \frac{T_1}{T_4} \right)^{\frac{1}{k-1}} = \left( \frac{T_H}{T_L} \right)^{\frac{1}{k-1}}$$

Therefore,

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ or } \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

and

$$\text{Carnot Eff.} = \frac{T_H - T_L}{T_H} = 1 - \left( \frac{T_L}{T_H} \right) \quad (5-3)$$

The net work of the cycle could have been found by evaluating the work of each of the four processes of the cycle and adding these values algebraically. The work thus calculated must equal the difference between the heat supplied and the heat rejected.

Since the area under the graph of any non-flow reversible process on a  $P$ - $V$  plane represents the work done during that process, it should be evident that the algebraic summation of the work of the four processes of the Carnot Cycle must equal the enclosed area, as area 1234 in Fig. 5-3, on the  $P$ - $V$  plane.

An analysis of equation 5-3 shows that the efficiency of the Carnot Cycle increases as the cold-body temperature,  $T_L$ , is decreased, and the efficiency approaches 100 per cent as  $T_L$  approaches absolute zero. However, work must be done to artificially keep the temperature of the cold body below that of its surroundings. It will be shown later in this chapter that the work done to keep the cold body at an artificially depressed temperature is, theoretically, equal to the added work obtained by artificially depressing the temperature.

An analysis of equation 5-3 shows also that the Carnot-Cycle efficiency increases as the source temperature  $T_H$  increases, and the efficiency approaches 100 per cent as the source temperature approaches infinity.

Although this statement is derived from an analysis of the Carnot-Cycle efficiency, the fundamental concept holds for all heat engines. This concept, that the efficiency is increased as the temperature at which heat is added is increased, explains the concerted effort of heat engine designers in the last decade or two to increase the temperature at which heat is added. This is true whether the heat engine is a gas turbine plant, a steam-turbine plant, or an internal combustion-engine plant.

**5-4 The Second Law of Thermodynamics**—In Art 5-2 it was pointed out that, when the end of the stroke was reached in the Carnot Cycle engine, it was necessary to throw away to the cold body part of the heat which was supplied by the hot body, in order to return the working substance to its original state and repeat the cycle. One form of the Second Law of Thermodynamics expresses this condition as follows: No engine actual or ideal, can completely and continuously transform into work all the heat added to its working substance.

The Second Law is incapable of direct proof, either mathematical or experimental. However, whenever heat is being transformed into work, the limitation on the amount of the transformation exists in all cases. This incomplete transformation may appear to a beginner as an artificial limitation introduced by man's lack of imagination, rather than by any natural phenomena. The student should keep in mind, however, that experience has verified the Carnot Principle, which may be stated as follows: As heat is added at a higher level of temperature, a greater portion of it is turned into work. Extrapolation of existing data indicates that heat must be added at an infinitely high temperature if all of it is to be turned into work.

There are many forms of the Second Law. Clausius' form is "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at higher temperature."

Kelvin's statement is "It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of surrounding objects."

Reeve's statement is "Energy tends ever, so long as it undergoes no transformation, to gravitate to a lower degree of intensity."

It is difficult to combine these various statements into a single form of the Second Law because the Second Law covers so much territory. In general, the Second Law states that heat is a low-grade form of energy, that is, it cannot be completely converted into some other forms. The temperature at which heat is added is a measure of the possible degree of its transformation into work. Heat tends to gravitate to a lower temperature plane where there is less possibility of transforming it into work.

This is an irreversible process. Heat can never, of its own accord, flow to a higher level of temperature. The maximum proportion of heat may be turned into work when the unused heat is thrown away at the lowest naturally available temperature. Nothing can be gained by rejecting it at an artificially lower temperature.

★5-5. The Reversed Carnot-Cycle Engine.—As each process of the Carnot Cycle is a reversible one, it is possible to reverse the direction of operation of the Carnot Cycle. If the working substance expands at constant temperature when in contact with the cold body, it will abstract heat from the cold body, as indicated by process 1-2 in Fig. 5-4. If the

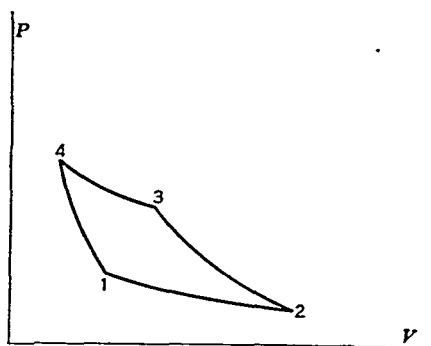


Fig. 5-4. Reversed Carnot Cycle

working substance is compressed in a reversible adiabatical manner, as indicated by process 2-3, until it reaches the temperature of the hot body, compression at the hot-body temperature, as indicated by process 3-4, will result in rejection of heat to the hot body. The working substance may now be expanded reversibly and adiabatically, as in process 4-1; and the process may then be repeated.

The heat put into the hot body is equal to the heat picked up from the cold body plus the net work supplied from the outside to run the engine. If, for given temperatures of the hot and cold bodies, a quantity of heat is supplied to a straight heat engine, a definite amount of work will be produced. If the same amount of work is put into a reversed heat engine working between the same temperature limits, the reversed heat engine will return the same quantity of heat to the hot body as was originally abstracted from it. This must be true, or the cycle would not be reversible.

A reversed Carnot-Cycle engine—as are all reversed heat engines—is a refrigerating machine, because it is capable of maintaining a cold body at a low temperature by abstracting heat from it.

The concept of a reversed Carnot-Cycle engine may be utilized to demonstrate several axioms related to the Carnot Principle. In Fig 5-5, engine *A* is a normal heat engine and engine *B* is a reversed heat engine. The two engines are coupled together.

In the first case, let both engines be Carnot-Cycle engines. Then the amount of heat abstracted from the hot body by engine *A* is returned to

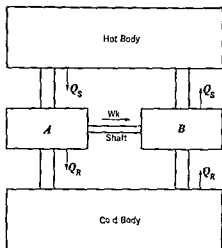


Fig 5-5 Actions of Coupled Normal and Reversed Heat Engines

the hot body by engine *B*. Likewise, engine *B* removes from the cold body a quantity of heat equal to that rejected to the cold body by engine *A*. As should be expected when two engines are operated so that one exactly reverses the performance of the other, no net change is produced by the operations of these engines. Such engines are reversible engines.

The foregoing statement must mean that the efficiency of a Carnot-Cycle engine is independent of the nature of the working substance. To demonstrate this, assume that the temperature of the hot body is 1000 R and the cold body is at 600 R. Let engine *A* take 100 Btu from the hot

body. If the working substance in engine *A* is a perfect gas, then, according to equation 5-3, the efficiency is 40%, and 40 Btu of heat is converted into work which is used to run engine *B*. If the working substance in engine *B* is also a perfect gas, then this work would deliver 100 Btu of heat to the hot body. If it is felt that the use of some other working substance would increase the efficiency of the Carnot Cycle engine, let that working substance be used in engine *A*. Then, by virtue of its higher efficiency, engine *A* would transform into work more than 40 Btu out of the 100 Btu it received. By virtue of receiving a larger quantity of work, engine *B* will pump more than 100 Btu into the hot body. This would result in a gradual transfer of energy from the cold body to the hot body. As experience has demonstrated that heat will not, unaided by an outside agency, flow to a higher-temperature object, it is impossible to find a working substance giving higher efficiency than the perfect gas.

If it is believed that the use of a working substance other than a perfect gas will result in a lower efficiency in the Carnot-Cycle engine, let that fluid be used in engine *B*. If engine *B* were run under the foregoing conditions as a heat engine, an amount of work equivalent to less than 40

Btu would be required by it to deliver 100 Btu to the hot body. If engine *A* delivered work equivalent to 40 Btu, then engine *B* would pump more than 100 Btu into the hot body. This, as before stated, is impossible. Therefore, the efficiency of the Carnot Cycle is independent of the nature of the working substance and equals that determined by equation 5-3 for the engine using a perfect gas.

In a like manner, it is possible to show that no engine can have a higher efficiency than that of the Carnot-Cycle engine operating between the same temperature limits. It may also be shown that the efficiencies of all reversible engines are the same when operating between the same temperature limits.

**5-6. Available Energy and Unavailable Energy.**—According to the Second Law, only a portion of the heat transferred at a given temperature can be turned into work. This portion which is capable of being transformed into work is called *available energy*, that is, energy available for doing work. The remaining portion of the heat transferred, which under no circumstances can be turned into work, is *unavailable energy*.

When trying to produce work from heat, it is desirable to know at all times how much heat is available energy and how much is unavailable. It is also desirable to know what happens to both the available and unavailable energies possessed by a body when the body experiences a change in state, to make certain that there is a minimum loss in the ability to do work.

The concept of available energy will be presented here from the temperature viewpoint, to concentrate the student's thoughts on heat flow for thermodynamic reasons. The subject of available energy can be extended to a system that is at the same temperature as its surroundings but has a higher pressure, velocity, or elevation, as in a hydraulic power plant.

The yardstick used for measuring the available and unavailable portions of heat transferred is a theoretical engine, receiving heat at the transfer temperature and rejecting heat at the lowest available temperature. For instance, suppose 50,000 Btu is added to boiling water at a temperature of 400 F and it is desired to determine the portion of the 50,000 Btu which is available energy. The lowest available temperature is taken to be 60 F. This available energy can be determined by supposing the 50,000 Btu to be added to a theoretical engine at a temperature of 400 F with the theoretical engine rejecting at 60 F. The calculated work of the theoretical engine is the available energy in the 50,000 Btu which is actually added to the water. The rest of the 50,000 Btu is unavailable energy. The theoretical engine to be used depends on the particular heat-power field under consideration. For example, when dealing with

steam it is customary to use the Rankine engine as the yardstick (See Chapter 7) As the efficiencies, and hence the power outputs, of all reversible cycles are the same for the same conditions, any reversible cycle may be used for the theoretical engine In accordance with normal procedure, the Carnot engine will, in general, be taken as the standard to measure the available and unavailable energy

As the efficiency of the Carnot-Cycle engine equals  $1 - \frac{T_L}{T_H}$ , its output and hence the available portion of the heat transferred is

$$\text{Available Energy Portion} = Q_s \left( 1 - \frac{T_L}{T_H} \right) \quad (5-4)$$

The remainder of the heat transferred is unavailable energy Thus,

$$\text{Unavailable Energy Portion} = Q_s - Q_s \left( 1 - \frac{T_L}{T_H} \right) = Q_s \frac{T_L}{T_H} \quad (5-5)$$

*Example 5-1* — Assume that 50 000 Btu is added to boiling water at a temperature of 400 F and the lowest available temperature is 60 F Determine the available and unavailable portions of the 50 000 Btu added

*Solution* — By use of equation 5-4 the available energy portion equals

$$50\,000 \left( 1 - \frac{520}{860} \right) = 1000(1 - 0.6047) = 19\,765 \text{ Btu} \quad \text{Ans}$$

The unavailable energy portion equals

$$50\,000 - 19\,765 = 30\,235 \text{ Btu} \quad \text{Ans}$$

$$\text{or} \quad 50\,000 \times \frac{520}{860} = 30\,235 \text{ Btu} \quad \text{Ans}$$

Equations 5-4 and 5-5 are based on the assumption that heat is transferred at a *constant temperature* In the majority of cases when heat is transferred to or from a body, the body undergoes a temperature change Thus, each unit of heat is transferred at its own particular temperature In order to determine the available and unavailable portions of heat transferred at a varying temperature, it is necessary to imagine a series of Carnot-Cycle engines, each engine receiving the unit of heat at the particular temperature at which it is transferred Each engine will reject heat at the lowest available temperature For each unit of heat,  $dQ_s$ , the available portion equals

$$dQ_s \left( 1 - \frac{T_L}{T_H} \right) = dQ_s - dQ_s \frac{T_L}{T_H}$$

where  $T_H$  is the particular temperature at which  $dQ_s$  is transferred

The unavailable portion of  $dQ_s$  equals

$$dQ_s \frac{T_L}{T_H}$$

The total available portion of the heat transferred equals the summation of all the available portions. Thus,

$$\text{Available Energy Portion} = \int_1^2 dQ_s - \int_1^2 dQ_s \frac{T_L}{T_H}$$

As the lowest available temperature,  $T_L$ , is considered to be constant,

$$\text{Available Energy Portion} = Q_s - T_L \int_1^2 \frac{dQ_s}{T_H} \quad (5-6)$$

Likewise, the total unavailable energy portion of the heat transferred is

$$\text{Unavailable Energy Portion} = T_L \int_1^2 \frac{dQ_s}{T_H} \quad (5-7)$$

The limits on the above integrals are the values of  $T_H$  between which heat is supplied. Equations 5-6 and 5-7 are general equations. If the temperature at which heat is transferred is constant, these equations will reduce to equations 5-4 and 5-5, respectively.

*Example 5-2.*—Heat amounting to 200 Btu is added to 4 lb of water having an initial temperature of 100 F. The lowest available temperature is 60 F. Calculate the available energy and the unavailable energy added. Assume that the specific heat of water is unity.

*Solution.*—The addition of heat to the water will produce a temperature rise of the water. The final water temperature is needed. In this case,

$${}_1Q_2 = Wc_x(T_2 - T_1)$$

$$\text{or} \quad 200 = 4 \times 1 \times (T_2 - T_1)$$

$$\text{Hence} \quad T_2 - T_1 = 50$$

$$\text{and} \quad T_2 = 610 \text{ R}$$

The unavailable energy added may be found by use of equation 5-7. This equals

$$\begin{aligned} T_L \int_1^2 \frac{dQ_s}{T_H} &= 520 \int_1^2 \frac{Wc_x dT}{T_H} \\ &= 520 \times 4 \times 1 \int_1^2 \frac{dT}{T_H} \\ &= 520 \times 4 \times 1 \log_e \frac{T_2}{T_1} \\ &= 520 \times 4 \times 1 \log_e \frac{610}{560} \\ &= 177.8 \text{ Btu} \end{aligned} \quad \text{Ans.}$$

The rest of the 200 Btu supplied is available energy. This equals

$$200 - 177.8 = 22.2 \text{ Btu} \quad \text{Ans.}$$

In a particular state, a given body possesses a definite amount of energy. Some of this energy can be used to do work and hence is available energy. The remaining portion of the energy of a body is unavail-



able energy The available energy of a body can be increased by adding energy to the body in the form of heat or work Available energy can be removed from a body by making it do work or by taking heat from it The unavailable energy of a body can be increased by adding heat to it The unavailable energy of a body can be decreased only by removing heat from it If there is any irreversible process within the body, there is a decrease of available energy and a corresponding increase in unavailable energy (it being assumed that there is no energy exchange with the outside)

In Art 3-3 it was pointed out that there is a loss of ability to do work in every irreversible process This is another way of saying that in an irreversible process some of the available energy of a substance becomes unavailable When the working substance undergoes an irreversible change of state, its final molecular energy content is higher than it would have been had the change been a reversible one If the process were a reversible expansion, the substance would give up some of its molecular energy to do work Less work is done by the substance in an irreversible expansion, and hence the final energy left will be higher in this case More work is required in an irreversible compression Again, the final molecular energy is higher If friction reduces the velocity of a flowing fluid, its molecular energy is increased by a quantity equal to the decrease in kinetic energy

This excess of molecular energy resulting from irreversible action causes an increase in unavailable energy of the same amount as would be caused by the addition of a quantity of heat equal to this excess molecular energy The increase of unavailable energy resulting from irreversible action is

$$\text{Increase in Unavailable Energy} = T_L \int_1^2 \frac{dQ_f}{T_H} \quad (5-8)$$

where  $dQ_f$  is the equivalent amount of heat which would have to be added to produce the same effect as that of the irreversible action

If heat is added to a substance, increasing its temperature from  $T_1$  to  $T_2$ , and then an irreversible process takes place within the substance, increasing its temperature from  $T_2$  to  $T_3$ , the total change in unavailable energy of the substance is

$$\text{Total Increase in Unavailable Energy} = T_L \int_1^2 \frac{dQ_f}{T_H} + T_L \int_2^3 \frac{dQ_f}{T_H} \quad (5-9)$$

**5-7 Entropy Changes**—When a body has a temperature above the lowest available temperature, it possesses ability to do work by virtue of its temperature This amount of available energy possessed by the

body is a function of its state and the lowest available temperature. The rest of the energy possessed by the body is unavailable energy. This unavailable energy is also a function of the lowest available temperature, as well as state. When unavailable energy is divided by the lowest available temperature, a quantity will be obtained which is a function of state only. This quantity is called *entropy*. In other words, the entropy of a body is the unavailable energy of the body divided by the lowest available temperature. Because it is a function of state, entropy is a property.

The evaluation of the total energy possessed by a body is difficult. Furthermore, a knowledge of the total energy of the body is of little value to an engineer. What he is concerned with is an evaluation of energy changes of a body when it changes state. Likewise, the engineer has little interest in the absolute value of the entropy of a substance, but he is vitally concerned with the change in entropy of a body as it changes its state. Because entropy is a property, the change in entropy between two states is independent of the path and depends solely on the two states. If the entropy change can be evaluated for one path, this will be the entropy change for any process between the two states. To determine the entropy change, rearrange equation 5-9. Thus,

$$\frac{\text{Increase in Unavailable Energy}}{T_L} = \int_1^2 \frac{dQ_s}{T_H} + \int_2^3 \frac{dQ_f}{T_H} \quad (5-10)$$

$$\text{Also,} \quad S_2 - S_1 = \int_1^2 \frac{dQ_s}{T_H} + \int_2^3 \frac{dQ_f}{T_H} \quad (5-11)$$

where  $S$  represents entropy.

Equation 5-11 is valuable as it shows that the entropy of a body may be increased by adding heat to the body or by allowing any irreversible process to take place within the body. Because of the difficulty of evaluating the second term in equation 5-11, this equation is not used in calculating the increase in entropy of a body. The entropy increase between two states, being independent of paths, may be determined by calculating the entropy change for any reversible processes or series of reversible processes between the two states. For reversible processes only,

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} \quad (5-11a)$$

where  $dQ$  is the infinitesimal amount of heat added in the assumed reversible process and  $T$  is the absolute temperature at which that quantity of heat is added. As  $dQ = Wc_x dT$ , equation 5-11a will become

$$S_2 - S_1 = \int_1^2 \frac{W c_z dT}{T} \quad (5-11b)$$

If the specific heat is constant for the given process,

$$S_2 - S_1 = W c_z \log_e \frac{T_2}{T_1} \quad (5-11c)$$

*Example 5-3* — If 2 lb of air at 30 psia and 200 F changes its state to 15 psia and 90 F what is the change in entropy?

*Solution* — As nothing is known about the nature of the process between the two states a reversible polytropic expansion will be assumed. To determine the value of  $n$ , use equation 2-6 inverting both sides. Then

$$\frac{30^{\frac{n-1}{n}}}{15} = \frac{660}{550} \quad \text{or} \quad 2^{\frac{n-1}{n}} = 1.2$$

Hence,

$$\frac{n-1}{n} = 0.263 \quad \text{and} \quad n = 1.36$$

By use of equation 3-6 the specific heat is

$$c_z = 0.1715 + \frac{53.35}{778(1-1.36)} = 0.1715 - 0.1905 = -0.019$$

Using equation 5-11c for the change in entropy we get

$$\begin{aligned} S_2 - S_1 &= 2(-0.019) \log_e \left( \frac{550}{660} \right) \\ &= 2(0.019) \log_e \left( \frac{660}{550} \right) \\ &= 2 \times 0.019 \times 0.1826 = 0.00693 \text{ unit} \end{aligned} \quad \text{Ans}$$

Strictly, the units of entropy in the English system are Btu per degree Rankine, but it is customary simply to say so many units of entropy.

The choice of other reversible processes may lead to equations which are easier to use than the single polytropic. Such equations are not introduced here as it is felt desirable to keep the number of equations to a minimum.

The evaluation of the change in entropy for any reversible process simply calls for the substitution of the value of  $c_z$  for the process into equation 5-11b. If the process is an isothermal one equation 5-11b becomes indeterminate. It is then necessary to use equation 5-11a in which  $T$ , being constant, may be taken outside the integral sign.

Entropy change is merely a mathematical term or a ratio, the student should learn to use it as he does other ratios. However even though no satisfactory physical significance can be given to an entropy change, it is very desirable to see the uses of entropy as a property as well as its application in connection with both unavailable energy and heat.

Entropy, being a property, is useful in fixing the state of a substance. Equation 5-10 shows that in a reversible adiabatic there is no entropy change, since each  $Q$ -term equals zero. If the entropy\* can be evaluated at the initial condition, then the final entropy is also known. Knowledge of only one other independent property at the final condition will provide sufficient information for the evaluation of all other properties at the second state. This fact is frequently used in reversible adiabatic processes of vapors, as will be shown in Chapter 6. Because there is no change in entropy in a reversible adiabatic, such a process is often called an *isentropic* (i.e., same entropy).

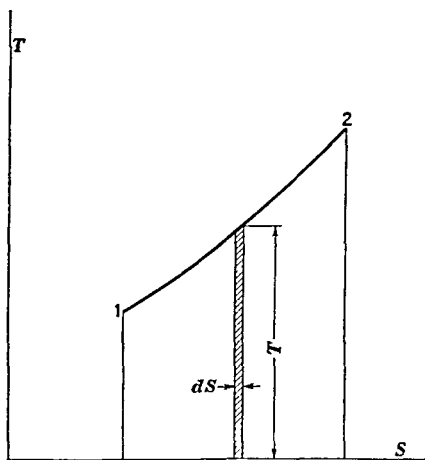


Fig. 5-6. Temperature-Entropy Diagram

Some properties are extensive (such as volume); others are intensive (such as pressure). The product of an intensive property and an extensive property may represent an energy quantity. Thus the product of pressure and volume change during a reversible non-flow process represents work. Hence the pressure-volume diagram is widely used, because the area under the curve for any reversible non-flow process on this diagram equals the work done. For the same reason a temperature-entropy diagram has much value, because the area under the curve for any reversible process on this diagram represents the heat transferred. Here temperature is the intensive property, and entropy is the extensive one. In Fig. 5-6 is shown a reversible addition of heat to a body. The cross-hatched strip has an area equal to  $T dS$ . The total area under the curve between points 1 and 2

\* Strictly speaking, this is the entropy above a datum plane or the change in entropy from the datum plane to the initial conditions.

equals  $\int_1^2 T dS$  From equation 5-11a,  $dQ = T dS$  and  ${}_1Q_2 = \int_1^2 T dS$

Hence the area under the curve for any *reversible* process on a body must equal the heat added to or removed from the body

The temperature-entropy diagram also has value in showing changes in unavailable energy of a body The product of the intensive property, lowest available temperature, and the extensive property, entropy change, equals the change of unavailable energy of a body for *any* process

The slope of a curve on the  $T$ - $S$  plane equals  $\frac{dT}{dS}$  In a reversible process,  $dS = \frac{dQ}{T}$  Then the slope equals  $dT - \frac{dQ}{T}$  If  $W c_x dT$  is substituted for  $dQ$ , the slope equals  $\frac{T dT}{W c_x dT}$  or

$$\text{Slope} = \frac{T}{W c_x}$$

This equation shows that the greater the specific heat, the smaller is the slope In a constant-temperature process, the specific heat is infinity and the slope is zero For an *isentropic\** process, the specific heat is zero and the slope is infinity

The four common processes are shown in Fig 5-7, together with their values of  $n$  It is to be noted that values of  $n$  for these curves progress in an orderly fashion in a clockwise direction This is helpful because the curve of a process may be spotted in roughly if its value of  $n$  is known (Compare Fig 2-5 for  $P$ - $V$  curves) Equation 5-10a shows that the entropy can increase in a reversible process only when heat is added A removal of heat is accompanied by a decrease in entropy A study of a curve on the  $T$ - $S$  plane should reveal whether heat is being added or removed by noting whether or not the entropy is increasing This is illustrated in Fig 5-7 by the dotted curve for  $n = 1.25$  In case it is known that the temperature increases during the process, the curve must proceed upward and to the left This means that the entropy is decreasing and heat is being removed

**Example 5-4**—A quantity of air weighing 20 lb originally at 200 F, is heated at constant pressure by the addition of 400 Btu of heat If the lowest available temperature is 80 F, calculate the change in entropy of the air and the available energy portion of the heat added to the air

\* Isentropic is here used in the sense of reversible adiabatic All reversible adiabatic processes are isentropic but the converse is not necessarily true If in equation 5-10 the two integrals were to add up to zero by having heat removed to counter balance the irreversibility effect the operation would not be adiabatic

*Solution.*—Refer to Fig. 5-6. In this case the final temperature is needed. From the conditions,

$$T_2 - T_1 = \frac{Q}{W c_p} = \frac{400}{20 \times 0.24} = 83.3$$

Hence,

$$T_2 = 743 \text{ R}$$

From equation 5-11c,

$$S_2 - S_1 = 20 \times 0.24 \log_e \left( \frac{743}{660} \right) = 0.0567 \text{ unit}$$

Ans.

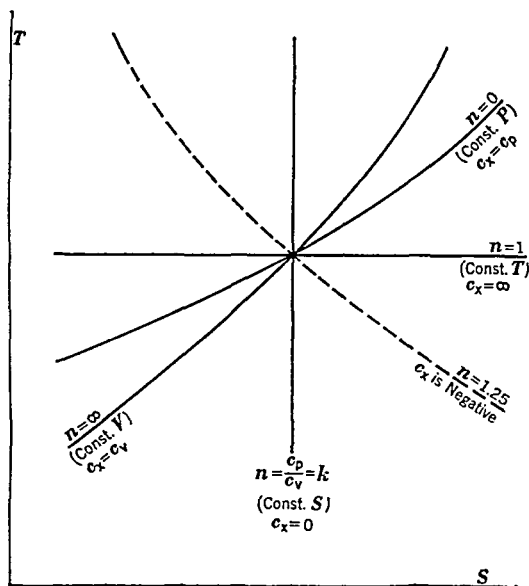


Fig. 5-7. T-S Diagrams for Common Processes

Because entropy change equals the change in unavailable energy divided by the lowest available temperature, the increase in unavailable energy equals  $0.0567(460 + 80) = 306$  Btu.

This 306 Btu is the increase in unavailable energy of the air. It is also the unavailable portion of the heat added. The available portion of the heat added is the remainder of the 400 Btu. Thus,

$$\text{Available Energy Portion} = 400 - 306 = 94 \text{ Btu}$$

Ans

*Example 5-5.*—A quantity of air weighing 4 lb is compressed in a centrifugal compressor from 14 psia and 70 F to 42 psia and 310 F. If the lowest available temperature is 40 F, calculate the loss of available energy during the compression process.

*Solution.*—Because the air flows through a centrifugal compressor in such a short period of time, it can be assumed that no heat is lost during the compression. To find

the change in entropy during the compression assume a reversible polytropic process between the two state points. Then

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{770}{530} = \left(\frac{42}{14}\right)^{\frac{n-1}{n}}$$

from which  $1.453 = 3^{\frac{n-1}{n}}$

Hence,  $\frac{n-1}{n} = 0.34$  and  $n = 1.515$

The specific heat is

$$c_v = 0.1715 + \frac{53.35}{778(1.515)} = 0.1715 - 0.133 = 0.0385$$

and the change in entropy is

$$S_2 - S_1 = 4 \times 0.0385 \times \log_e \left(\frac{770}{530}\right) = 0.00575 \text{ unit}$$

The increase in unavailable energy equals  $0.00575 \times 500 = 28.8 \text{ Btu}$ . As no heat was added or removed, this increase in unavailable energy must come from the available energy. Thus the loss in available energy equals  $28.8 \text{ Btu}$ . Ans

**NOTE** The difference between the actual work and the theoretical work of the compressor exceeds  $28.8 \text{ Btu}$  as part of the difference in work is energy possessed by the air which is available for doing work.

**★5-8 Reversible and Irreversible Heat Transfers**—In Art. 5-7 reversible heat transfers were described as  $T$ - $S$  processes. Their importance justifies a more detailed analysis. In Art. 5-6 it was shown that the amount of the available portion of heat added to or taken from a body depends on the temperature at which the heat transfer takes place. If  $1000 \text{ Btu}$  is removed from a body at a constant temperature of  $400^\circ \text{F}$ , and  $60^\circ \text{F}$  is the lowest available temperature, then the available energy removed is  $1000 \left( \frac{400-60}{400+460} \right) = 395.3 \text{ Btu}$ . In case the same amount of heat is added to a body at a temperature of  $300^\circ \text{F}$ , with the same lowest available temperature, the available energy added is  $1000 \left( \frac{300-60}{300+460} \right) = 315.8 \text{ Btu}$ .

Thus, because the heat is added to the second body at a lower temperature than that at which it was removed from the first body, less available energy is added to the second body than is removed from the first body. Ability to do work is lost as a result of such a heat transfer. As the total energy added to the second body is the same as that removed from the first, this means that some of the available energy has become unavailable.

This loss of available energy and gain of unavailable energy is shown in Fig. 5-8. The total area in each of the three rectangular figures represents the heat transferred, or  $1000 \text{ Btu}$ . The cross-hatched areas represent the unavailable portions of the heat transferred. The unavailable energy added from 3 to 4 is decidedly greater than that removed from 1 to 2. This excess of unavailable energy comes from decrease in available energy.

If the illustration is carried still further, and *all* of the 1000 Btu is added at the lowest available temperature, as in process 5-6, then *all* of the available energy becomes unavailable.

As the concept of a reversible process is one in which none of the ability to do work is lost, then heat transfer from a body at high temperature to one at lower temperature is an irreversible process. When the temperature of the body receiving heat is brought closer to that of the one supplying heat, less available energy is lost in the heat-transfer process. In the limiting case (*i.e.*, when heat is transferred from one body to another body at the same temperature), there is no loss of available energy. Such a transfer is a reversible one.

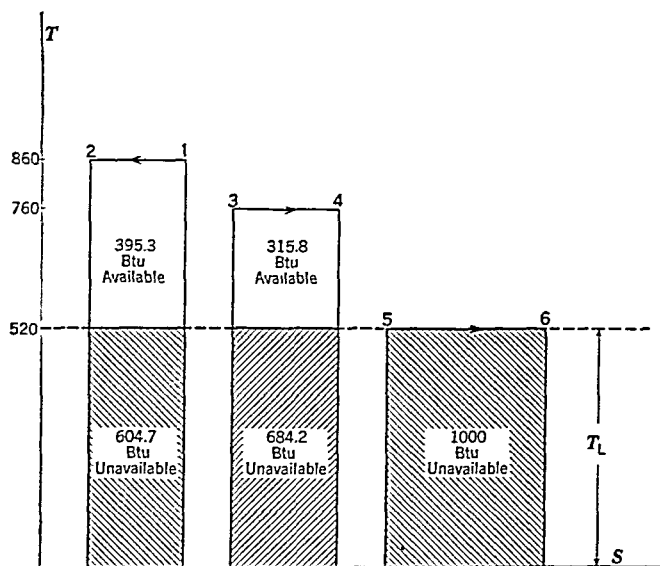


Fig. 5-8. Use of T-S Diagram to Show "Run-Down" of an Energy System

Care must be taken to distinguish between the transfer of heat from one body to another and the actual addition of heat to the second body. If a pan of water is heated over a burner on a stove, the transfer of heat to the pan is very definitely an irreversible process. However, if the thermal conductivity of the pan is so high that its temperature remains substantially the same as that of the water, then the transfer of heat from the pan to the water approaches reversibility. Similar cases may arise in certain heat-engine cycles, such as the Rankine, in which heat may be added to the working fluid in a reversible manner but the transfer of heat from the source



is irreversible. Such a cycle, in spite of its reversibility, will have a lower efficiency than that of the Carnot Cycle for the same source and sink temperatures.

An irreversible heat transfer causes a *net increase* in entropy. Any irreversible action taking place within the body also increases the entropy (see equation 5-10). Because in practice every process is an irreversible one, every change of any sort is ultimately accompanied by an over all increase in entropy.\* This has led to the statement that the entropy of the universe is continuously increasing. Since an increase in entropy means an increase in unavailable energy, in every actual process some of the supply of available energy is becoming unavailable. "The stock pile of available energy of the universe is gradually being used up." Some philosophers have not agreed with this conclusion, claiming that somewhere in the universe a reverse action must be going on in which unavailable energy is being turned into available energy. The existence of such a process is open to question.

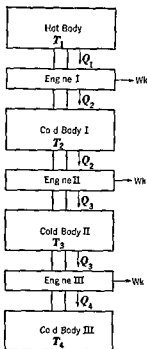


Fig 5-9 Concept for Thermodynamic Scale of Temperature

★5-9 The Thermodynamic Scale of Temperature — Thus far the scale of temperature has been tied up with the physical properties of water. The concept of the Carnot-Cycle engine gives rise to a temperature scale that is independent of the properties of any working substance. This is illustrated in Fig 5-9, as follows.

Take a quantity of heat  $Q_1$  out of the hot body at temperature  $T_1$  and supply it to a Carnot-Cycle engine, as engine I. This engine will produce a quantity of work and will reject the remainder of  $Q_1$  as  $Q_2$  to cold body I at temperature  $T_2$ . The amount of work produced by engine I, for a given value of  $Q_1$ , depends solely on the amount of the temperature difference  $T_1 - T_2$ .

\*It must be remembered that for a single process involving one substance such as a gas there is one way in which to decrease entropy namely to transfer heat from the substance and there are two ways in which to increase entropy one by transferring heat to the substance and the other by having an irreversibility occur. Whenever heat is removed from one substance it must be given to another substance at a lower temperature as a result of the *over-all* change of entropy which is the algebraic sum of a decrease applicable to one substance and a larger increase applicable to another substance there is an ultimate increase in entropy.

Now set up a second Carnot-Cycle engine, as engine II, which receives the quantity of heat  $Q_2$  from cold body I. Adjust the temperature  $T_3$  of the cold body II, to which engine II rejects heat, so that the work produced in engine II is exactly equal to that produced in engine I. The work produced in engine I is

$$(Q_1) \left( \frac{T_1 - T_2}{T_1} \right)$$

and

$$Q_2 = Q_1 - Q_1 \left( \frac{T_1 - T_2}{T_1} \right) = Q_1 \left( \frac{T_2}{T_1} \right)$$

The work produced in engine II is

$$Q_2 \left( \frac{T_2 - T_3}{T_2} \right) = \left( Q_1 \frac{T_2}{T_1} \right) \left( \frac{T_2 - T_3}{T_2} \right) = Q_1 \left( \frac{T_2 - T_3}{T_1} \right)$$

Equating the works of the two engines, we get:

$$Q_1 \left( \frac{T_1 - T_2}{T_1} \right) = Q_1 \left( \frac{T_2 - T_3}{T_1} \right)$$

or

$$T_1 - T_2 = T_2 - T_3$$

Thus, when the two engines produce equal amounts of work, the temperature ranges over which they operate must be equal. By using this concept, a scale of temperature may be set up in which a unit of temperature is that temperature difference necessary to produce a unit amount of work in a Carnot-Cycle engine, the amount of heat supplied to the Carnot-Cycle engine and the temperature at which it is supplied both being fixed.

Two concepts of the absolute zero of temperature have been given, *i.e.*, by the kinetic theory of gases and by Charles' Laws. The Carnot-Cycle engine set-up leads to a third concept. If additional engines and cold bodies be added to the arrangement shown in Fig. 5-9, more and more work will be produced until, when enough engines are added, the summation of the amounts of work produced by all the engines will equal the heat  $Q_1$  removed from the hot body. As it is impossible to produce more work than this, it is inconceivable that there can be a temperature lower than that of the last cold body. Thus, absolute zero of temperature is the cold-body temperature which will enable a Carnot-Cycle engine to transform all the heat it receives into work. Temperatures above this absolute zero of temperature, measured in terms of work of the Carnot-Cycle engine, were first proposed by Lord Kelvin in 1848 and were termed absolute temperatures.

## CHAPTER 6

### VAPORS

**6-1. Vapors Versus Gases.**—Matter is said to exist in three forms, solid, liquid, and gaseous. A gas has no form of its own but fills the entire container in which it is placed. Furthermore, a gas is conceived of as a substance far removed from its liquid state. A vapor, on the other hand, commonly denotes that material given off by a liquid when it is heated. The vapor, like the gas, has no form of its own. As all known gases may be liquefied, it is possible that under certain conditions all so-called gases may fit the concept of a vapor. Hence, there can be no rigid line of demarcation between a gas and a vapor. Common usage, however, has designated as vapors those substances which are ordinarily near the point of liquefaction. On the other hand, those substances are called gases which are ordinarily far removed from their point of liquefaction. Thus, steam is always thought of as a vapor and air is always thought of as a gas. This is a poor policy because, under some conditions, steam may be treated as a perfect gas without appreciable error. On the other hand, under other conditions, a very serious error will be caused by assuming that air is a perfect gas.

From the engineering standpoint a vapor may be taken as any substance which under its given conditions is so close to its liquefaction point that the use of perfect-gas laws causes serious\* error. Two outstanding reasons for failures of the perfect-gas laws are that in vapors molecular attractions are appreciable\* and molecular sizes are appreciable\* in comparison to the total volume. Because perfect-gas laws cannot be used for vapors, it is desirable to review equations used earlier and see which may be used for vapors.

Equation 2-3, which is  $PV = WRT$  or  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ , was derived for a perfect gas and cannot be used for vapors.

When the plot of a process of a vapor is made on the  $P$ - $V$  plane, it has been found that under certain conditions—particularly when the vapor is not very close to condensing—equation 2-4, which is  $P_1 V_1^n = P_2 V_2^n$ , fits the actual curve very closely. On the other hand, if condensation or evapora-

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\* The use of the indefinite words "serious" and "appreciable" points out the impossibility of distinguishing a gas from a vapor. In general, the closer a substance approaches its point of liquefaction, the more it deviates from perfect-gas laws.

tion occurs during the process, there may be a large break in the curve, each section of the curve having its own value of  $n$ . Under these conditions it becomes difficult to use equation 2-4

The equations for the pressure-temperature and the volume temperature relationships, or equations 2-5 and 2-6, will not hold for vapors, as the characteristic equation for a perfect gas was used in their development

Equation 3-1, or  ${}_1Q_2 = U_2 - U_1 + \frac{{}_1Wk_2}{J}$ , must hold for vapors as well as gases, as it is a form of the law of conservation of energy

Equation 3-2, or  ${}_1Q_2 = Wc_v(T_2 - T_1)$ , may be used, but difficulty may be encountered as the value  $c_v$  may not be readily obtainable in many cases

Equation 3-3, or  ${}_1Wk_2 = \int_1^2 P dV$ , is valid for vapors. Difficulty may be encountered in its use, however, because of the inability of obtaining the true pressure-volume relationship for the process in question

Equation 3-4, or  $U_2 - U_1 = Wc_v(T_2 - T_1)$ , holds only for a perfect gas

The equation of steady flow, being a form of the law of conservation of energy, must hold for vapors as well as for gases

Equation 4-3, or  $H_2 - H_1 = Wc_p(T_2 - T_1)$ , holds only for a perfect gas

Equation 5-11, or Entropy Change = Change in Unavailable Energy divided by the Lowest Available Temperature, is a general definition and hence applies to vapors as well as gases

It will be shown in Chapter 14 that, if an accurate equation of state is known for a vapor, it is possible to set up equations to replace those used for perfect gases. However, if accurate results are desired under all conditions, these equations become excessively complex. To minimize this difficulty, it is desirable to have available properties of the vapor in question

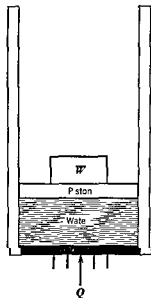


Fig 6-1 Water Being Heated at Constant Pressure

## 6-2 Saturated and Superheated Vapors

Consider a cylinder containing water with a loaded piston resting on the water, as indicated in Fig 6-1. If the water is cold at the beginning of the process, addition of heat will produce a temperature rise. The entropy of the water will also increase, as is shown by the process 1-2 in Fig 6-2. After

sufficient heat is added, the water will start to boil. This boiling takes place at constant temperature. Since much heat is being added, there will be a large increase in entropy during the boiling process, as in process 2-3. When the water is completely vaporized, further addition of heat produces an increase in temperature as well as in entropy, as in process 3-4.

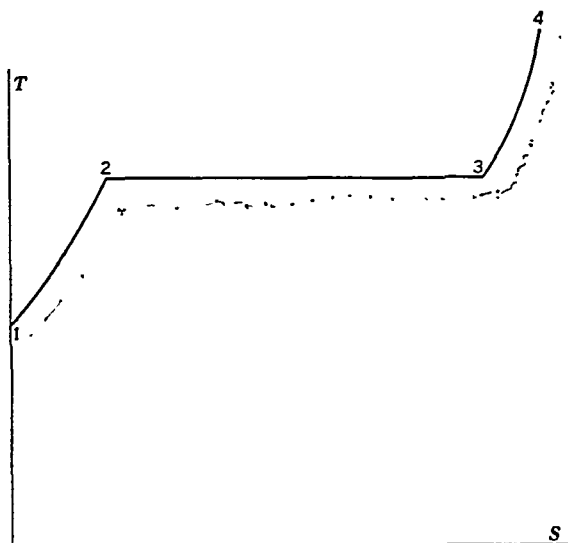


Fig. 6-2. Temperature-Entropy Plot of Heating Water and Steam at Constant Pressure

The temperature at which boiling starts depends on the pressure impressed upon the water. It can be demonstrated that, if the pressure is reduced to 0.08854 psia, water will boil at 32 F. If, on the other hand, the pressure is increased to 3000 psia, water will boil at 695.36 F. A plot of boiling temperature versus impressed pressure for water is shown in Fig. 6-3. Plots of boiling temperatures versus impressed pressures for other liquids are similar in shape.

A boiling liquid is said to be a *saturated liquid*, evidently because it fails to remain in its given phase when heat is added to it. From this, the boiling temperature of a liquid, for its existing pressure, is known as its *saturation temperature*. The pressure impressed upon a liquid when it is boiling at a given temperature is known as the *saturation pressure* for the given temperature. A vapor existing at the saturation temperature is a *saturated vapor*. In the process of boiling, a vapor generally entrains some particles of liquid and carries them away from the boiling liquid. Such a mixture is a *wet saturated vapor*. If the particles of liquid are mechanically

separated from the vapor or are vaporized by addition of heat, the vapor is *dry saturated vapor*, provided it exists at the saturation temperature.

Addition of heat at constant pressure to a dry saturated vapor produces a *superheated vapor*, i.e., a vapor existing at a temperature in excess of the saturation temperature. The difference between the existing temperature and the saturation temperature is called the *degrees of superheat*.

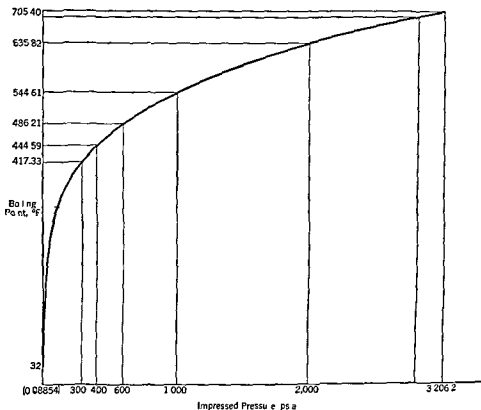


Fig 6-3 Boiling Temperature Versus Impressed Pressure for Water

If the process of heating cold water, described in the first part of this article, be repeated with various weights on the piston, data for a family of curves will be obtained. Such a family is shown in Fig 6-4. The lines representing the heating of the liquid up to its boiling temperature are drawn in an exaggerated position because, in reality, they lie so close to each other that they would appear as one line on a small plot.

A line connecting the points representing the saturated liquid is known as the *saturated liquid line*. This is often shortened incorrectly, to the term "liquid line." In a like manner, a line connecting the points representing the dry saturated vapor is known as the *dry saturated vapor line*. This term is often shortened to "saturation line." It will be noted in

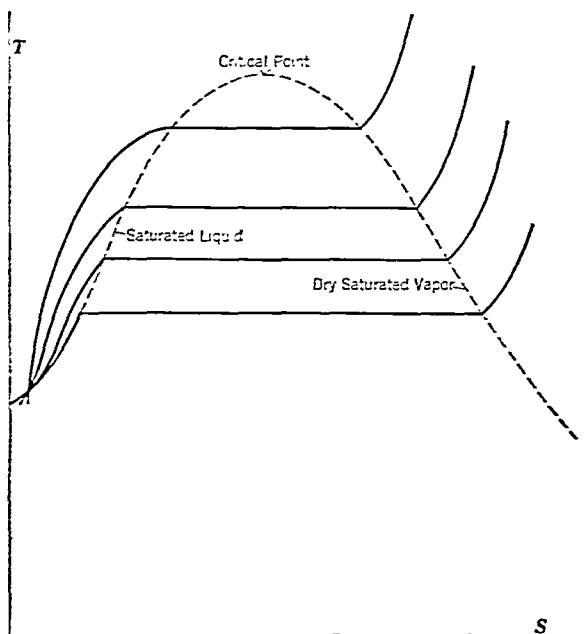


Fig. 6-4. Constant-Pressure Curves on T-S Diagram for Steam

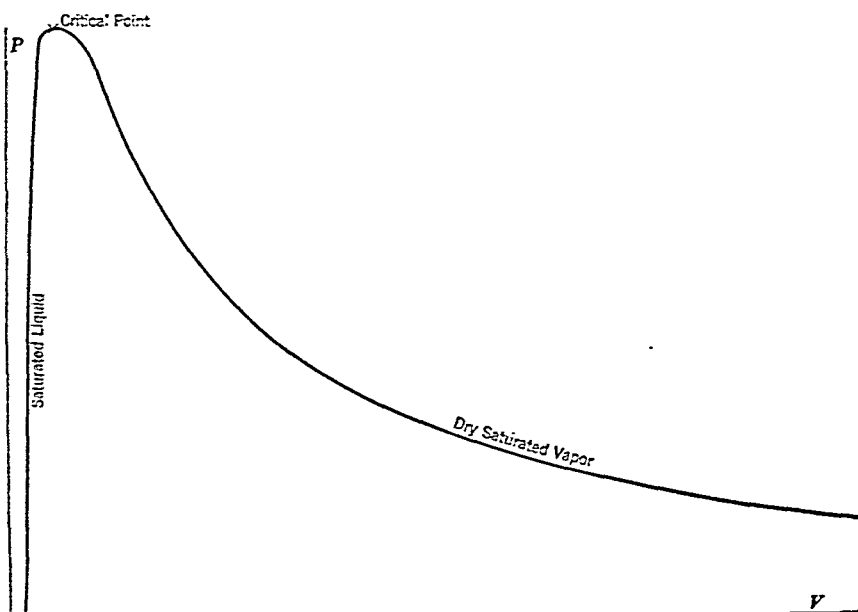


Fig. 6-5. Saturated Liquid Line and Dry Saturated Vapor Line on P-V Plane

Fig 6-4 that these two curves come together at the top and form what is known as the "steam dome" In Fig 6-5 are shown the saturated liquid line and the dry saturated vapor line plotted on the  $P$ - $V$  plane

The point where the saturated liquid line and the dry saturated vapor line meet is known as the critical point, which represents the critical state of the substance As the saturation pressure is increased, the changes in properties during vaporization are decreased, as can be seen in Fig 6-4 and Fig 6-5, until they become zero at the critical point In other words, as the critical state is approached, there is less and less difference between the properties of a liquid and its vapor until, when the critical state is reached, these differences disappear entirely At the critical temperature and at higher temperatures, the substance has all the characteristics of a vapor and no characteristics of a liquid

**6-3. Vapor Tables**—Direct measurements of saturation temperatures for various pressures are made with comparative ease Direct determinations of specific volumes are much more difficult, particularly at high temperatures and pressures Other properties, such as enthalpy and entropy, do not lend themselves to direct experimental determination but must be calculated from other determinations There are many thermodynamic means of correlating the various properties (see Chapter 14) For instance, by means of the Clapeyron equation, the volume change during vaporization at any given temperature is fixed by the heat required to vaporize the liquid (latent heat of vaporization) and the change in the saturation temperature with pressure at the given temperature Thus, to prepare a table of properties of a substance requires a vast number of experimental determinations and extensive calculations and correlations For this reason the existing tables of the properties of most substances are limited in the range covered and are, in many cases not very accurate

The most extensive, as well as most reliable, tables that exist are for steam Since a technique of handling vapors in engineering calculations can be acquired by practice with vapor tables, the study of the construction and the use of steam tables is an exceedingly valuable asset to the student engineer, once he has obtained mastery of the technique of handling steam tables, the engineer can apply this knowledge to the handling of other vapors Many of the other fluids for which the engineer desires tables are classed as refrigerants One of the most common refrigerants is ammonia

Extensive work on the determination of the thermodynamic properties of ammonia was done by the U S Bureau of Standards and published by term "table form" Abstracts of the thermodynamic properties of the following the

This term is { the Bureau of Standards, No 142 *Tables of Thermodynamic Properties*  
Government Printing Office, Washington, D C



most common refrigerants are to be found in the Refrigerating Data Book, published by The American Society of Refrigerating Engineers.

Much work has been done all over the scientific world for many years on the determination of various properties of steam. Several individuals have collected and correlated the existing information and have compiled steam tables. These tables have been superseded by others, as more extended and more accurate information became available. In the United States, early tables were those of Peabody, and these were followed later by those of Marks and Davis and by those of Goodenough. Next came tables by Keenan, and finally those by Keenan and Keyes. As the tables of Keenan and Keyes\* are recognized as the standard in this country, all solutions in this text will be based on them.

Tables 1 and 2 of Keenan and Keyes are similar, both dealing with the properties of saturated water and dry saturated steam. Table 1 is compiled for even values of temperatures, and Table 2 is for even values of pressures. Table 2 contains values of internal energies, whereas Table 1 does not. The arrangement of the properties in Table 2 is as follows:

Abs. Press.		Temp.		Specific Vol.		Enthalpy		
$\frac{\text{lb}}{\text{sq in.}}$		Fahr.		Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor
$p$		$t$		$v_f$	$v_g$	$h_f$	$h_{fg}$	$h_g$

Entropy			Internal Energy			Abs. Press.
Sat. Liquid	Evap	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	$\frac{\text{lb}}{\text{sq in.}}$
$s_f$	$s_{fg}$	$s_g$	$u_f$	$u_{fg}$	$u_g$	$p$

Subscript  $f$  denotes the saturated liquid,  $g$  the dry saturated vapor, and  $fg$  the change in property during vaporization.

Absolute values of enthalpy and entropy do not exist. Even if such values could be determined, they would be of little interest to the engineer. He is interested in obtaining the changes in these properties during a change in state. The changes in these properties can be readily obtained if the values of these properties are known above some datum plane. For steam, both the enthalpy and the entropy of the saturated liquid are taken as zero at 32 F.

\* *Thermodynamic Properties of Steam*, by Keenan and Keyes. John Wiley & Sons, Inc., 1936.

Table 3 of Keenan and Keyes gives values of specific volume, enthalpy, and entropy of *superheated steam* up to pressures of 5500 psia. As the engineer does not deal with non flow processes to a large extent, he has little need for values of internal energies. Hence, these are not tabulated in Table 3. In case of need, internal energy may be calculated from the defining equation

$$H = U + \frac{PV}{J}$$

★6-4 **Properties of Compressed Liquid Water**—Table 4 of Keenan and Keyes gives means of determining properties of non saturated water. If the pressure on boiling water is increased, *while the temperature is kept constant* the specific volume, enthalpy, and entropy will change slightly. These changes in properties are given in Table 4. The use of Table 4 may be illustrated as follows

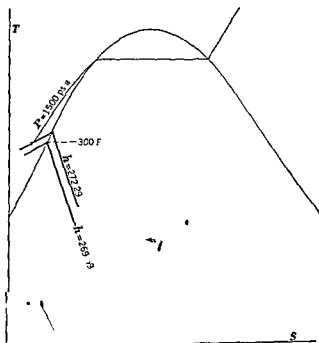


Fig 6-6 Skeleton T-S Diagram for Example 6-1

**Example 6-1**—What is the specific enthalpy of water at 1500 psia and 300 F?

**Solution**—See Fig 6-6. The specific enthalpy  $h_f$  of saturated water at 300 F is 269.59 (psia of Table 4 or Table 1). Table 4 gives the quantity  $(h - h_f)$  as 2.70 Btu at 1500 psia and 300 F. Hence, the enthalpy  $h$  of the non saturated water equals 269.59 + 2.70 = 272.29 Btu. (In Fig 6-6 the enthalpy lines shown are exaggerated in scale to illustrate the difference between the enthalpy of the saturated liquid and that of the compressed, non-saturated liquid.)

It is to be noted from Table 4 that a small increase in pressure upon water, particularly at low temperatures, produces a very small change in its properties. For this reason, for low-pressure work, it is customary to neglect the effect of pressure on properties of a liquid and to assume that the values of its properties are the same as those of a saturated liquid at the same temperature. On the other hand, when working with pressures encountered in the modern high-pressure power plant, the effect of pressure on the properties of water becomes sufficiently important to warrant the use of Table 4.

★6-5. Other Tables.—Table 5 is of particular value in dealing with air-conditioning problems at low temperatures, as it gives the properties of the saturated solid and its vapor from 32 F down to  $-40$  F. Tables 6 and 7 are useful in heat-transfer calculations.

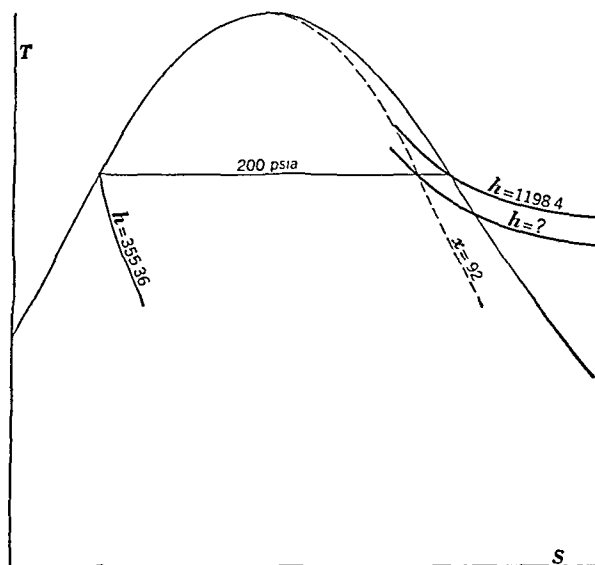


Fig. 6-7. Skeleton T-S Diagram for Example 6-2

6-6. Properties of Wet Saturated Vapor.—When dealing with wet saturated vapors, it is necessary to specify how much of the mixture is vapor and how much is entrained liquid. This may be done by specifying its *quality*,\* which is the weight of the vapor divided by the total weight of the vapor and its entrained liquid. It may also be done by specifying its *moisture content*, which is the weight of the entrained liquid divided by

\* This is a specific definition for wet saturated vapor. In general, the word “quality” of a vapor denotes its condition, *i.e.*, wet or superheated, and how much.

the weight of the vapor and its entrained liquid. The symbol for quality of a wet vapor is  $x$ , quality is expressed as a decimal.

The enthalpy of wet saturated vapor may be expressed in equation form as follows

$$h_x = h_f + x h_{fg} \quad (6-1)$$

or

$$h_x = h_g - (1-x) h_{fg} \quad (6-1a)$$

Equation 6-1a is to be preferred to equation 6-1 when the quality is greater than 50% as it enables one more significant figure to be obtained on the slide rule.

The manner of determining properties of wet saturated vapors will be shown by means of an example.

*Example 6-2* — What is the enthalpy of a pound of saturated vapor at 200 psia? The quality is 92%.

*Solution* — See Fig. 6-7 (the student should always sketch a skeleton  $T$ - $S$  diagram when working problems such as these). From page 38 in Keenan and Keyes, the following values of enthalpy are obtained  $h_f = 355.36$ ,  $h_{fg} = 843.0$ ,  $h_g = 1198.4$ . As addition of sufficient heat to boiling water to turn 92 per cent of it into steam will produce saturated steam with a quality of 92%, the enthalpy of saturated steam will equal the enthalpy of the saturated liquid plus the enthalpy change when 0.92 lb is changed into steam, or  $355.36 + 0.92(843.0) = 1131.0$ . That is, the enthalpy of the 92% quality steam is  $h_x = 1131.0$ .

It is also true that the enthalpy of 92% quality steam is less than the enthalpy of dry saturated steam by 8 per cent of the enthalpy of vaporization. Or,  $h_x = 1198.4 - 0.08(843.0) = 1131.0$ .

Equations for obtaining other properties of wet saturated vapor are similar in form to equations 6-1 and 6-1a. Thus

$$s_x = s_f + x s_{fg} \quad (6-2)$$

or

$$s_x = s_g - (1-x) s_{fg} \quad (6-2a)$$

$$v_x = v_f + x v_{fg} \quad (6-3)$$

or

$$v_x = v_g - (1-x) v_{fg} \quad (6-3a)$$

At low pressures, the volume of the liquid is insignificant in comparison to the volume of the dry saturated vapor. If the value of  $v_f$  be neglected,  $v_{fg} = v_g$  and equation 6-3 then reduces to

$$v_x = x v_g \quad (6-3b)$$

At higher pressures, the volume of the saturated liquid increases and the volume of dry saturated vapor decreases until they become equal at the critical point. See Fig. 6-5. Thus, at the higher pressures the volume of the saturated liquid cannot be neglected if any degree of accuracy is desired. No hard or fast rule can be established for determining the

pressure below which the specific volume of the liquid may be neglected until the desired degree of accuracy of results is known. It is suggested that at first the student use equations 6-3 and 6-3a for all cases, as they are similar in form to the equations for other properties.

**6-7. Determination of Properties in a Given State.**—To evaluate such quantities as heat added, work done, and change in entropy for vapors, it is generally necessary to have a knowledge of the value of more of the properties of vapors at both the start and end of the process than are measured. As pointed out in Art. 3-8, the state of the substance—and hence all its properties—is fixed if two independent properties are fixed. Hence, it becomes necessary to determine and tabulate two independent properties at the start and at the end of the process and from these independent properties to find the other desired properties.

In case the vapor is in the saturation region, the specification of its specific volume, enthalpy, entropy, or internal energy, together with its pressure or with its temperature, permits the determination of its quality and hence the other desired properties. In the superheated region, all properties may be determined from the superheated tables (Table 3) if any two of the following are specified: pressure, volume, temperature, enthalpy, and entropy. In case one of the preceding properties is specified along with the internal energy for superheated steam, the state is truly fixed; but the values of the other properties can be determined only by a trial-and-error solution, as internal energies are not given in the superheated tables.

The determination of properties in a given state may be illustrated by the following examples.

*Example 6-3.*—Steam at a pressure of 100 psia has a specific enthalpy of 1100 Btu per pound. What is the specific entropy?

*Solution.*—By equation 6-1a,

$$h_x = h_g - (1-x)h_{fg}$$

Hence,

$$1100 = 1187.2 - (1-x)888.8$$

and

$$1-x = \frac{87.2}{888.8} = 0.0981 \text{ or } 9.81\%$$

Now see Fig. 6-8 and note the entropy dimensions. By equation 6-2a,

$$s_x = s_g - (1-x)s_{fg}$$

$$= 1.6026 - 0.0981(1.1286) = 1.4919 \text{ units per lb}$$

Ans.

*Example 6-4.*—Steam at a pressure of 2500 psia has a specific volume of 0.127 cu ft per lb. What is its specific enthalpy?

*Solution.*—By equation 6-3a,

$$v_x = v_g - (1-x)v_{fg}$$

Hence,

$$0.127 = 0.1307 - (1-x)0.1021$$

and

$$1-x = \frac{0.0037}{0.1021} = 0.0361 \text{ or } 3.61\%$$

By equation 6-1a,

$$h_x = h_g - (1-x)h_{fg}$$

$$= 1091.1 - 0.0361(360.5) = 1078.1 \text{ Btu per lb}$$

Ans

**Example 6-5**—Steam at a pressure of 140 psia has a temperature of 440 F. What is the specific enthalpy?

**Solution**—As the given temperature of 440 F is higher than the saturation temperature of 353.02 F, the steam is superheated. From the superheat tables for the given conditions of pressure and temperature, the specific enthalpy is given as 1243.3 Btu per lb.

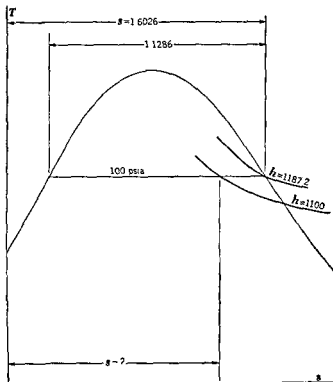


Fig 6-8 Skeleton T-S Diagram for Example 6-3

**Example 6-6**—Steam at a pressure of 210 psia has a specific enthalpy of 1267.8 Btu per lb. What is its specific volume?

**Solution**—As the given specific enthalpy is higher than that of dry saturated steam at this pressure ( $h_g = 1199.0$ ), the steam must be superheated. From the superheat tables the pressure of 210 psia and the enthalpy of 1267.8 Btu per lb are two independent properties which definitely establish a point where the temperature is 500 F. At this point, the specific volume is 2.589 cu ft.

Ans

**6-8 Changes of State of Vapors for Non-Flow.**—As pointed out in the first part of this chapter, it is generally difficult to evaluate the heat added to vapors in terms of weight, specific heat, and temperature change. There are two special non-flow cases where the heat added may be deter-

mined directly from the properties of the vapors. These cases are the constant-pressure and the constant-volume ones.

In Art. 4-3 it was shown that at constant pressure without flow the change in enthalpy equals the heat added. Hence, in the *non-flow constant-pressure process* (and only under *constant pressure*) the heat added may be found by determining the original and final enthalpies and subtracting one from the other.

In the *non-flow constant-volume process*, the general energy equation  $Q_2 = U_2 - U_1 + \frac{{}_1Wk_2}{J}$  shows that the heat added must equal the difference in internal energies as no work is done.

The fundamental expression for *work done* in the non-flow process, or  ${}_1Wk_2 = \int_1^2 P dV$ , may be used in a *constant-pressure process* as it reduces to  ${}_1Wk_2 = P(V_2 - V_1)$ .

The work is often desired in a second non-flow process, the *isentropic*. As no heat is added in this process, the general energy equation reduces to  ${}_1Wk_2 = J(U_1 - U_2)$ .

**6-9. Changes of State of Vapors for Flow.**—In general, the Law of Steady Flow may be applied to a vapor in the same manner as it was to a gas. However, in the case of a vapor, the values of enthalpy at entrance and exit of the device may be determined and substituted directly in the equation of steady flow; this eliminates the necessity of evaluating the difference in enthalpy in terms of temperature and the constant-pressure value of specific heat, as was done for a gas.

A special flow problem is the evaluation of the work done in an isentropic expansion in a theoretical steam engine. Because *no heat is transferred in the engine* in an isentropic expansion, the work done, expressed in Btu, must equal the difference in enthalpy.

**6-10. Vapor Charts.**—To facilitate the solution of problems involving changes in state of vapors, it is desirable to have available charts of vapor properties. In the United States there are three common charts of vapor properties, namely, the temperature-entropy chart, the enthalpy-entropy\* chart, and the enthalpy-volume or Ellenwood chart.

The temperature-entropy chart, in addition to showing the "steam dome," shows lines of constant pressure, constant volume, constant enthalpy, and constant quality. In addition, lines of constant superheat

\* Often called the Mollier diagram. Mollier, however, used other coordinates in addition to enthalpy and entropy.

sometimes are shown. In Fig 6-9 is shown a skeleton temperature-entropy chart. A more elaborate one is shown as Fig 9 in Keenan and Keyes. As most temperature-entropy charts are drawn to a small scale, accurate determinations cannot be made by their use. Furthermore, as lines of constant pressure and lines of constant volume are almost parallel in the superheat and high quality regions of the temperature entropy chart, their intersections are rather indeterminate. The temperature-entropy chart does have the real merit of predicting qualitative results. For this reason, it is suggested that the student trace the process in question on the temperature-entropy chart before making a numerical solution.

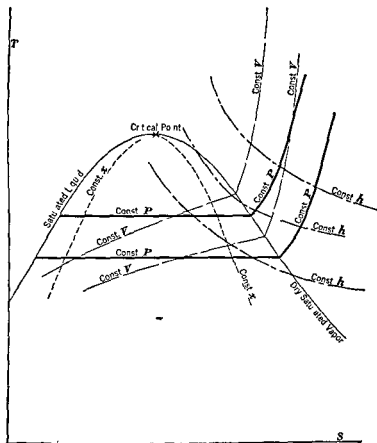


Fig 6-9 Temperature-Entropy Chart for Vapors

As was pointed out in the chapter on steady flow, most problems encountered in thermodynamics are flow problems. Since enthalpy is useful as energy in cases of flow, its determination is very important. Except in the low-pressure region, lines of constant enthalpy on the temperature-entropy plane are curved lines. As these lines are not parallel,



it is difficult to determinate enthalpy accurately from the temperature-entropy chart. On the other hand, as enthalpy is one of the coordinates of the Mollier (enthalpy-entropy) chart, its determination by such a chart is much easier.

In Fig. 6-10 are shown the saturated liquid line and the dry saturated vapor line on the enthalpy-entropy plane, together with lines of constant pressure. It should be noted that lines of constant pressure are straight\* in the saturated region.

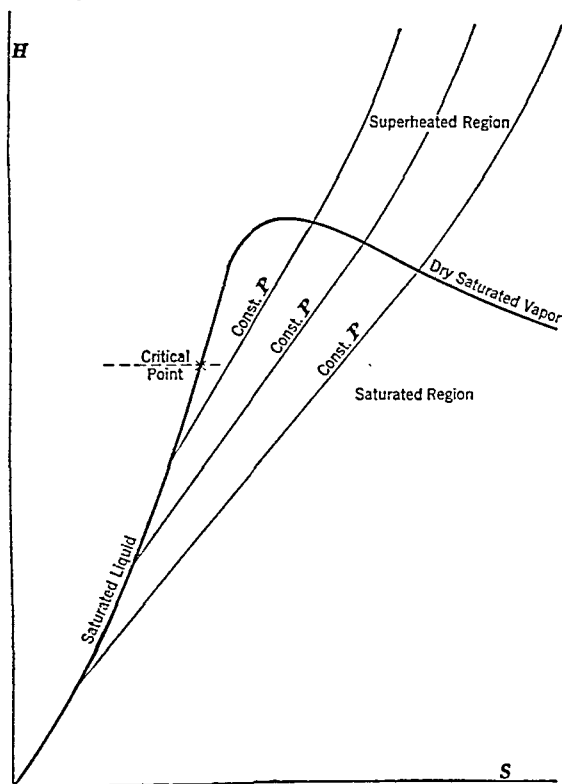


Fig. 6-10. Enthalpy-Entropy Chart for Vapors

There is little need for the properties of very wet steam. To be sure, there is a change from the liquid to the vapor phase in the boiler, and there is a change back from the vapor to the liquid phase in the condenser.

\* The slope of constant-pressure lines is  $\left(\frac{dH}{dS}\right)_P$ . But  $dH = dQ$  at constant pressure and  $dQ = T ds$ . Therefore, slope =  $\frac{T ds}{ds} = T$ . As the temperature is constant in the saturated region when the pressure is constant, constant-pressure lines must be straight in the saturated region.

However, conditions are rarely needed inside either the boiler or the condenser, but are considered rather at exit or entrance. At these points the fluid is either a liquid or a high quality vapor. All other pieces of equipment, the vapor is normally a high-quality one. Hence a Mollier diagram for the high-quality and superheated regions is sufficient for most

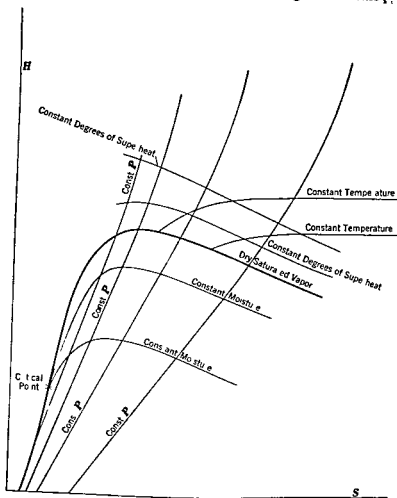


Fig. 6-11 Enlarged Enthalpy Entropy Chart

purposes. This condition permits making the diagram to a much larger scale than would otherwise be possible. Such a diagram in skeleton form is shown in Fig. 6-11. Large scale diagrams are included with steam tables such as those of Keenan and Keyes. It should be noted that constant volume lines are not shown on the Mollier diagram for the following reason. Since they do not have sharp intersections with the constant pressure

lines, particularly in the high-quality and superheated regions, specific volume cannot be determined accurately on the Mollier diagram.

In steam-turbine design work, the determination of specific volumes is a necessity in order to size the nozzles and the blades. A set of charts that readily permits determination of volumes is the enthalpy-volume charts,\* prepared by Ellenwood and known as the Ellenwood Charts.

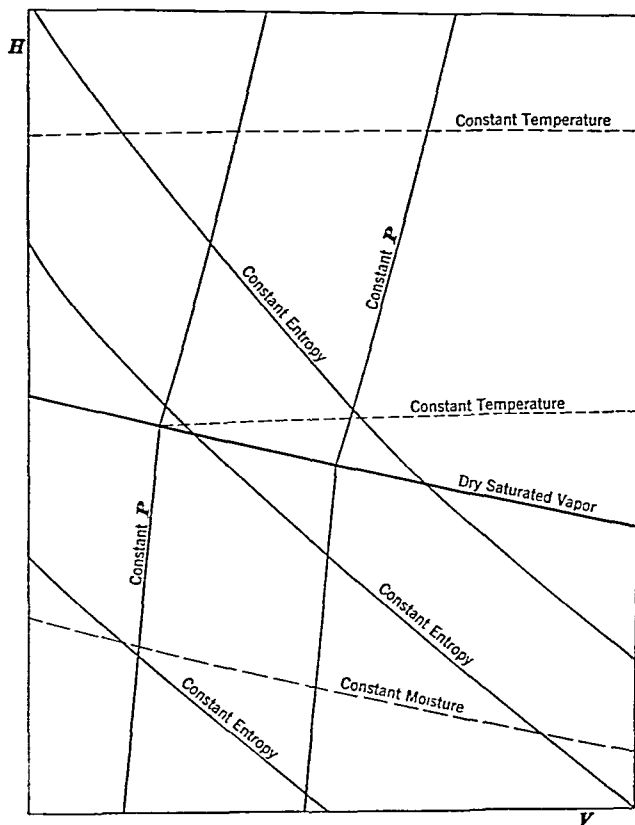


Fig. 6-12. Ellenwood Chart

Ellenwood has broken up one large chart into a series of small charts, each chart being bound as a page. A skeleton of such a page is given in Fig. 6-12. The Ellenwood charts, in addition to having volume lines, permit accurate determination of enthalpies.

6-11. Steam Calorimeters.—Many of our prime movers today are supplied with superheated steam. The enthalpy of the supply steam is

\* *Thermodynamic Charts*, by Ellenwood and Mackey, John Wiley and Sons.

readily determined when its temperature and pressure are known. However, there are many cases in which saturated steam is used. The measurement of its temperature, when the pressure is known, simply confirms the fact that the steam is saturated. In no way does it give any information as to either the quality or enthalpy of the steam.

To aid in the determination of the enthalpy or quality of saturated steam, various types of steam calorimeters have been devised. One of the simplest of these is the *throttling calorimeter*. This calorimeter makes use

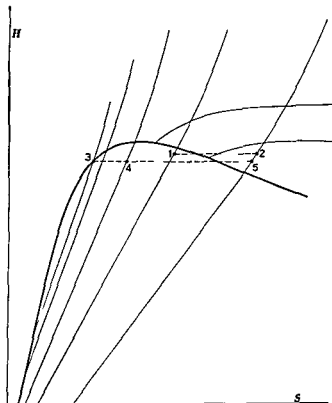


Fig 6-13 Effect of Throttling on High Pressure Steam

of the fact that the enthalpy of dry saturated steam at *low* pressures is considerably less than is the enthalpy of dry saturated steam at higher pressures. Hence, if high-quality steam at moderately high pressures is throttled, it tends to become superheated, as the enthalpy after the throttling equals the original enthalpy. If the steam is throttled sufficiently to produce superheated steam, then an observation of the temperature and pressure of this superheated steam will give sufficient information to permit determination of the final (and hence the original) enthalpy of

the steam. The action of the throttling calorimeter is shown in Fig. 6-13 as process 1 to 2 (dotted because the exact path is unknown).

It should be noted by reference to the Mollier diagram that when steam is under very high pressure (*i.e.*, above about 450 psia) the quality of the steam has a tendency to decrease at first. Further throttling will reverse the tendency and will produce superheated steam if carried far enough,

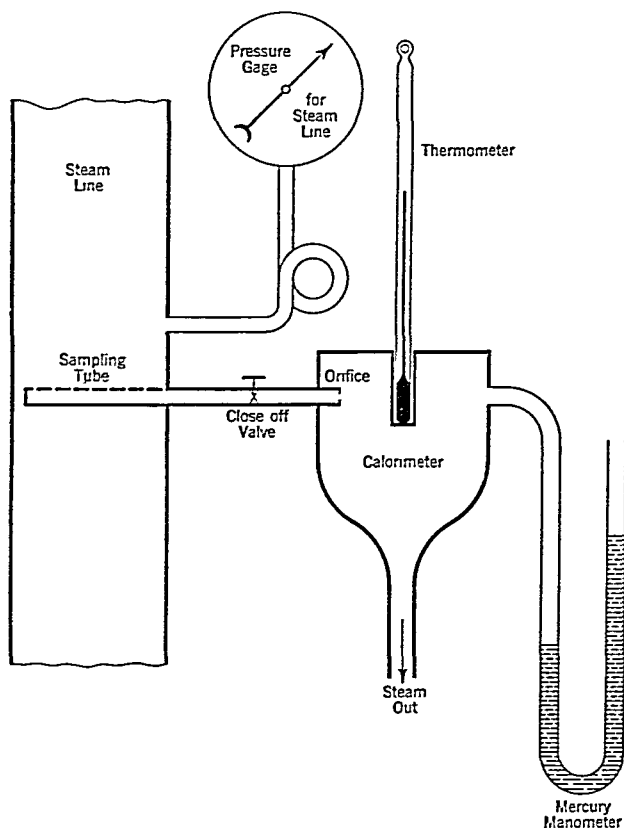


Fig. 6-14. Throttling Calorimeter

as indicated for processes 3 to 4 to 5 in Fig. 6-13. This tendency to produce wet steam when steam under very high pressure is partially throttled is important in modern high-pressure power plants whenever high-pressure steam is throttled down to medium-pressure headers. As wet steam is not a uniform mixture, the water formed by such a partial throttling will have a tendency to separate out of the steam. Traps should be provided to take care of the water thus formed, in spite of the fact that the steam may not originally have contained any moisture.

Because wet steam is not a uniform mixture, the securing of a true sample is difficult. As the water in the steam has a tendency to separate out and flow along the bottom of horizontal pipes, samples should be taken only from vertical pipes. Even in vertical pipes the sample obtained is not a true one unless the sampling tube is located at some distance from horizontal portions of the line. To obtain as representative a sample of steam as possible, the sampling tube extends well across the steam main. The sampling tube is closed at the end and has several small holes in line. See the A.S.M.E. test codes for details. In Fig. 6-14 a sampling tube is shown installed and connected to a throttling calorimeter.

Many throttling calorimeters are nickel plated. Others are heavily lagged. In both cases the heat lost from the calorimeter is minimized. If the orifice is small, the kinetic energies are small at points of state determinations (*i.e.*, at the entrance of the sampling tube and in the calorimeter body at the points where the temperature and pressure are measured). This is particularly true if the area for flow around the calorimeter thermometer is large. If the foregoing conditions are fulfilled then the equation of steady flow as applied to the calorimeter reduces to

$$H_{\text{line}} = H_{\text{calorimeter}} \quad (6-4)$$

When readings of the calorimeter temperature and pressure indicate superheated steam in the calorimeter, then the enthalpy in the calorimeter, and hence in the line, may be determined by use of steam tables or a Mollier diagram. If the quality of the line steam is desired it may be readily determined from its enthalpy. In case the calorimeter readings indicate *saturated steam in the calorimeter* the throttling calorimeter is not of value as there is no way to determine whether the steam in the calorimeter is dry saturated or wet saturated or to determine the quality of wet steam. Because results tend to be erratic with only a slight amount of superheat in the calorimeter, some authorities desire at least 10 degrees of superheat in the calorimeter. In case the throttling in the calorimeter is not sufficient with atmospheric exhaust, further throttling may be obtained by connecting the calorimeter exhaust to a steam condenser, a vacuum pump line, or a water aspirator.

In case the steam cannot be throttled sufficiently in a throttling calorimeter to obtain superheated steam, a *separating calorimeter* may be used. In this device the water is mechanically separated from the steam by abruptly changing the direction of the steam flow. The separated water is retained in the body of the calorimeter, its amount being determined volumetrically. The dry steam leaves the calorimeter through a specially sized orifice. As the amount of dry steam passing through a given orifice per unit of time is a function of the steam pressure at the up-stream side

of the orifice, the pressure gage attached to the calorimeter may be calibrated to read dry-steam flows per unit of time.

The separating calorimeter is not reliable for low steam pressures. Even at high pressure the separating calorimeter will not give perfect separation in the case of steam having high moisture content. In this case, the separating calorimeter is sometimes followed by a throttling calorimeter. The action of the two calorimeters in series will result in superheated steam in the body of the throttling calorimeter unless the steam initially is very wet.

A so-called *barrel calorimeter* is sometimes used in an attempt to determine the enthalpy of a steam sample. In this set-up, steam from the sampling tube is led into a barrel of water mounted on platform scales. The water increases in both temperature and weight as it condenses the steam. From observations of these two quantities, the enthalpy of the steam sample may be determined. Although attempts have been made to refine the process, results from barrels are not reliable, primarily for the following reasons:

- (1) It is difficult to determine the weight of steam accurately.
- (2) The temperature of the water is not uniform.
- (3) Radiation from the calorimeter is appreciable.
- (4) Appreciable vaporization from the surface of the water may take place.

Many attempts have been made to construct a satisfactory *electric calorimeter*. The fundamental concept of the electric calorimeter is that a measurable quantity of electrical energy is added to a steam sample of known weight. Sufficient energy is added to produce superheated steam. Thus, the enthalpy after the addition of electrical energy can be determined. By subtracting the added electrical energy, the original enthalpy of the steam is determined. Most electric calorimeters are neither portable nor rugged. Furthermore, there is some difficulty in determining the true weight of steam flow.

## CHAPTER 7

### STEAM POWER CYCLES

7-1. **Carnot Cycle Unsuitable for Vapor Cycle.**—In Chapter 5, the Carnot Cycle was demonstrated to be a most efficient cycle from a purely thermodynamic point of view, although its shortcomings as a working cycle were soon apparent. Reference to the study of vapors and their properties has already shown that the temperature of a saturated vapor is dependent on its pressure; also, the Carnot Cycle, involving isothermals

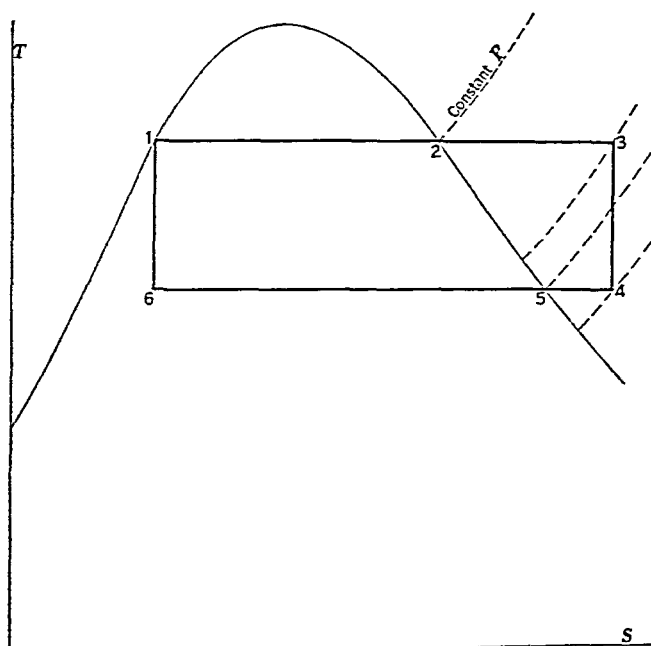


Fig. 7-1. Carnot Cycle for Vapors Below the Critical Point

and isentropics, is independent of the nature of the working substance. Hence, this cycle, which is always rectangular on the  $T$ - $S$  plane, will appear as in Fig. 7-1 when a vapor that is evaporated, superheated, and partly condensed is used as the working substance. Thus, evaporation would take place from 1 to 2, superheating from 2 to 3, isentropic expansion from 3 to 4, cooling with loss of superheat from 4 to 5, condensation to a lower quality from 5 to 6, and isentropic compression from 6 back to 1.



The pressure would be constant from 1 to 2 and likewise from 5 to 6, otherwise, it would vary as indicated

The impracticality of the foregoing cycle is obvious when it is considered that the heat is not supplied to the engine but is furnished *to the working substance* in another piece of equipment, *e g*, the boiler. In its simplest form, the vapor cycle embraces the *flow* of the working substance through a source of heat (boiler), then through a prime mover (engine, reciprocating or turbine), then through a heat sink (condenser), and then back to the heat source (boiler). This return to the boiler must be by a pumping action of some sort, since both the temperature and the pressure are

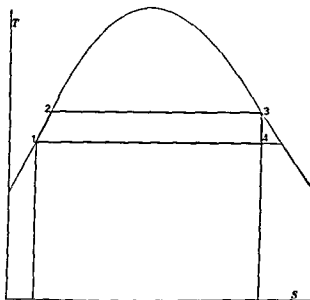


Fig 7-2 Rankine Cycle on T-S Plane,  
Using Saturated Vapor

greater in the boiler than in the condenser. If this pumping action were an isentropic compression, as from 6 to 1 in Fig 7-1, not only would condensation have to be accurately controlled to locate point 6 but also a large machine in the form of a compressor driven by the engine would be required.

**7-2 The Rankine Cycle**—With a view to substituting a theoretical cycle other than the Carnot as a means for judging the performance of vapor cycles, the Rankine Cycle has been established as a standard. In Fig 7-2 is shown an elementary  $T$ - $S$  diagram for this cycle when saturated steam only is used, and a line diagram of the physical layout of the equipment is shown in Fig 7-3.

From point 1 to point 2 in Fig. 7-2 heat is supplied in the boiler to raise the temperature of the water; and heat is supplied from 2 to 3 to evaporate the water. The supplying heat from 1 to 3 is performed at constant pressure regardless of temperature. From point 3, where the working substance enters the prime mover, to point 4, where it leaves, the useful work is performed isentropically. From point 4 to point 1, removal of heat is accomplished by condensation at constant pressure. In this theoretical cycle, the working substance emerges from the condenser at the saturation temperature corresponding to exhaust pressure, and the work of the return of the substance to the boiler (the part played by the feed pump) can be neglected in this preliminary analysis, since the work of the feed pump usually is comparatively small.

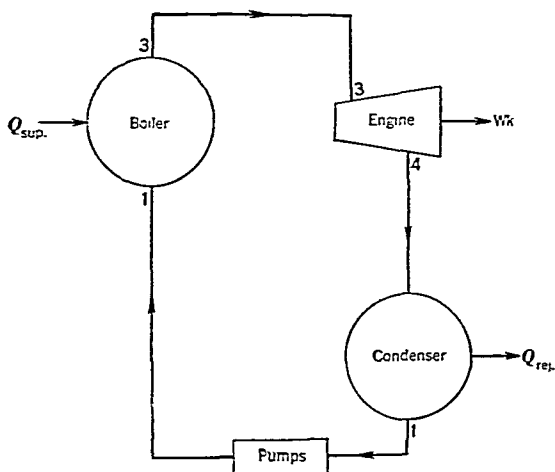


Fig. 7-3. Layout of Equipment for Rankine Cycle

The principle of steady flow may be used to evaluate the heat supplied, the work, and the heat rejected. The heat supplied to the boiler is

$$Q_{\text{sup}} = h_3 - h_1 \quad (7-1)$$

As the heat is supplied in a reversible manner, the heat supplied is represented by the entire area on the  $T$ - $S$  plane under the heat-supplied line.

Because the expansion in the engine is an adiabatic one, the work is

$$\frac{W_k}{J} = h_3 - h_4 \quad (7-2)$$

The heat rejected is then

$$Q_{\text{rej}} = h_4 - h_1 \quad (7-3)$$

It is suggested that the student refer to Chapter 4 and derive equation 7-1, 7-2, and 7-3 from equation 4-1. This heat rejected is represented by the area under process 4-1 on the  $T$ - $S$  plane. The work of the feed pump being neglected, the work of the engine must equal the difference between the heat supplied and the heat rejected. Thus, the area 1234 on the  $T$ - $S$  plane represents the heat equivalent to the cycle work.

The pump work being neglected, the thermal efficiency of the Rankine Cycle is

$$\text{Rankine Cycle Thermal Eff} = \frac{\frac{Wk}{J}}{Q_{sup}} = \frac{h_3 - h_4}{h_3 - h_1} \quad (7-4)$$

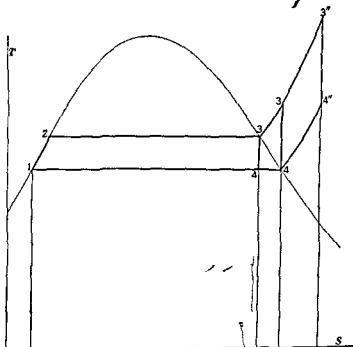


Fig 7-4 Rankine Cycle for Superheated Steam

In Fig 7-2 is shown the Rankine Cycle when saturated steam is used, Fig 7-4 shows the cycle 1-2-3-4, as before, but also shows the cycle 1-2-3'-4' when the steam that enters the prime mover is superheated. It is now obvious that the steam entering the prime mover can be wet, dry, or superheated, and that the steam entering the condenser can also be wet, dry, or superheated, its entropy being the same as that of the steam entering the prime mover. If the steam entering the prime mover is highly superheated, that leaving the prime mover will still be in the superheat region, and the course of the cycle will be 1-2-3''-4'', Fig 7-4.

**Example 7-1.**—Determine the thermal efficiency of a Rankine Cycle operating between the pressure limits of 100 psia and 14.7 psia, if the steam enters the prime mover in the dry saturated condition.

**Solution.**—Reference to Figs. 7-2 and 7-4 shows that one must first determine the enthalpy and entropy at point 3. For dry saturated steam at 100 psia, the steam tables give  $h=1187.2$  and  $s=1.6026$ . This entropy will also be the entropy at point 4, where the pressure is 14.7 psia. The moisture must now be calculated at point 4. For this pressure,  $s_g=1.7566$  and  $s_{f_g}=1.4446$ . Hence,

$$1-x = \frac{1.7566 - 1.6026}{1.4446} = 0.1066$$

The enthalpy at point 4 is then

$$h = 1150.4 - 0.1066(970.3) = 1047.0$$

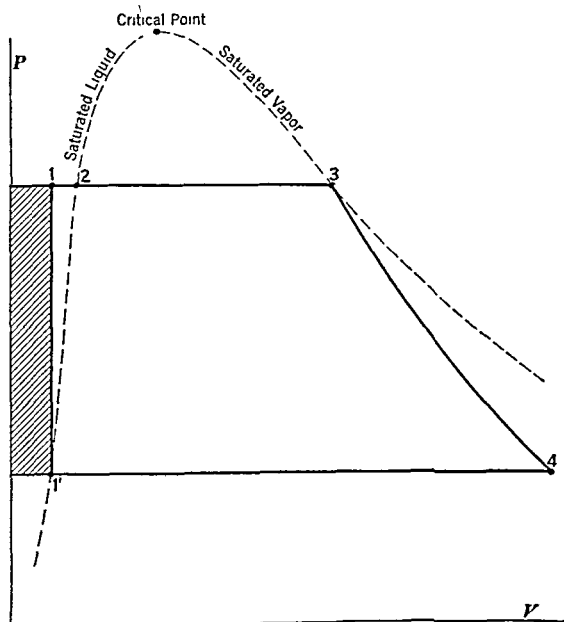


Fig. 7-5. P-V Diagram of Rankine Cycle, Using Saturated Steam

The work per pound is

$$\frac{Wk}{J} = h_3 - h_4 = 1187.2 - 1047.0 = 140.2 \text{ Btu}$$

The heat supplied is

$$Q_{\text{sup}} = h_3 - h_1 = 1187.2 - 180.0 = 1007.2 \text{ Btu}$$

and the thermal efficiency is

$$\text{Eff.} = \frac{140.2}{1007.2} = 0.139, \text{ or } 13.9\%$$

Ans.

★7-3. The Rankine Cycle Including the Work of the Pump.—The moving of the condensate from the condenser to the boiler involves pumps

(see Fig 7-3), there are generally two, but sometimes more. The pump feeding the boiler is the important one to consider, since it must exert on the water a pressure greater than that of the boiler steam in order that the water may flow into the boiler. In Fig 7-5 is shown a  $P$ - $V$  diagram of the Rankine Cycle, numbered to correspond with the cycle in Fig 7-2.

The liquid, initially at point 1, is heated and expands to point 2, the boiling temperature corresponding to the high pressure. From point 2 to point 3, expansion occurs because of evaporation, and from 3 to 4 there is isentropic expansion. Point 4 shows the greatest volume, where the low-pressure steam enters the condenser. Contraction takes place during condensation back to point 1'. If the compressibility of the liquid be neglected, the volume at point 1' can be taken the same as that at point 1, and the area of the cross-hatched part of Fig 7-5 will be

$$Wk = v_1(P_3 - P_4) \quad (7-5)$$

This is the work of the pump, and it reduces the total work of the Rankine Cycle as follows

According to equation 7-3, the total work is  $h_3 - h_1$ . Combining equations 7-3 and 7-5 to determine the net area, we get

$$\frac{Wk}{J} = h_3 - h_1 - \frac{v_1}{J}(P_3 - P_4) \quad (7-6)$$

Then, since thermal efficiency is the net work of the cycle divided by the heat supplied, equation 7-4 becomes

$$\text{Eff} = \frac{h_3 - h_1 - \frac{v_1}{J}(P_3 - P_4)}{h_3 - h_1} \quad (7-7)$$

In equation 7-1 it was assumed that the enthalpy of the liquid entering the boiler,  $h_1$ , was that of saturated liquid at the pressure in the condenser. In equation 7-7 this assumption cannot be made, as we are now assuming that there is a measurable change in enthalpy in the pump. When the steady flow principle is applied to the pump,  $h_1 + \frac{\text{Pump Work}}{J} = h_1$ . Using this in equation 7-7, we get

$$\text{Eff} = \frac{h_3 - h_1 - \frac{v_1}{J}(P_3 - P_4)}{h_3 - h_1 - \frac{v_1}{J}(P_3 - P_4)} \quad (7-7a)$$

The foregoing analysis is dependent on the assumption that the liquid is so nearly incompressible that  $v_1$  is taken as equal to  $v_1$ . However, when

high pressures (over 1000 psia) are to be handled and very accurate calculations are to be made, compressibility should be considered. In Art. 6-4 reference was made to the use of Steam Table 4, *Properties of Compressed Liquid Water*, which is a table of *deviations* from the saturated liquid conditions. When compressibility is thus considered, Fig. 7-5 is superseded by Fig. 7-6, on which the liquid volumes have been exaggerated to show the volume at point 1' much larger than that at point 1.

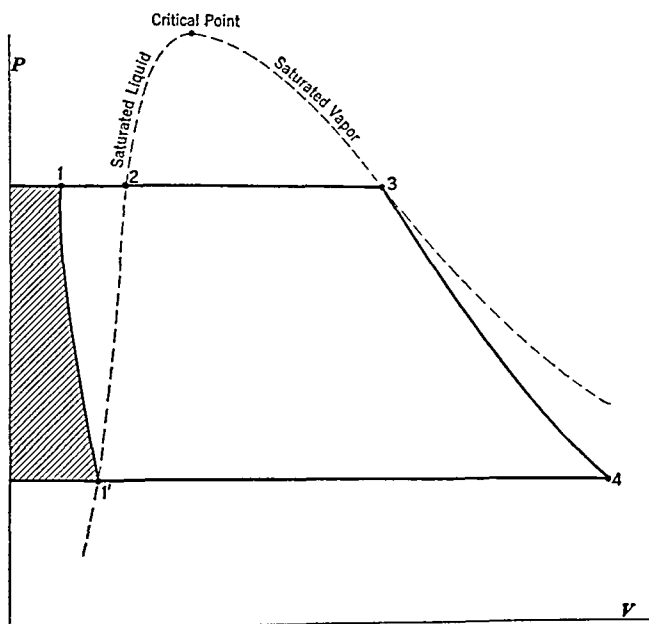


Fig. 7-6. Rankine Cycle, Including Allowance for Compressibility of Liquid

When compressibility of the liquid is considered, as in Fig. 7-6, the shaded area representing the pump work is no longer rectangular but is of a type which is evaluated as  $\int V dP$ . As the process involving this work is generally considered to be adiabatic, this pump work will be equal to  $h_1 - h_{1'}$ . This can be conveniently evaluated by use of the curves in Steam Table 4 entitled *Enthalpy Changes at Constant Entropy*. Refer to this table and note the statement made in conjunction with these curves.

With this modification, equation 7-7 becomes

$$\text{Eff.} = \frac{h_3 - h_4 - (h_1 - h_{1'})}{h_3 - h_1} \quad (7-8)$$

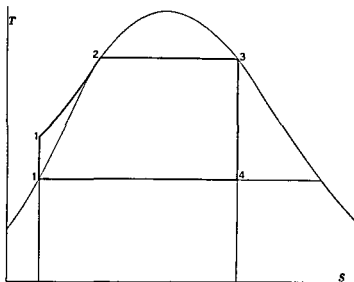


Fig 7-7. T-S Diagram of Rankine Cycle With Allowance for Compressibility of Liquid

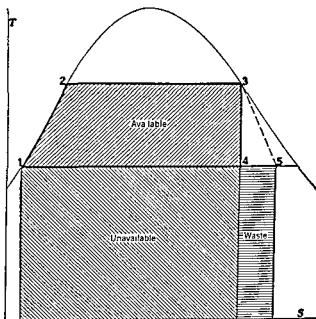


Fig 7-8 Energy Areas in Vapor Cycle

where  $h_1 - h_{1'}$  is the theoretical pump work. Equation 7-8 may be rewritten as follows:

$$\text{Eff.} = \frac{h_3 - h_4 - (h_1 - h_{1'})}{h_3 - h_{1'} - (h_1 - h_{1'})} \quad (7-8a)$$

Compressibility effects on the liquid are shown in Fig. 7-6 and Fig. 7-7.

Condensation from point 4 to point 1' brings the process to the condition of saturated liquid at condenser pressure. Application of pressure to the liquid isentropically, to bring it to boiler pressure, is shown by the vertical line from 1' to 1. The line connecting points 1 and 2 is that of constant pressure, the boiler pressure. The boundary lines for saturated liquid and saturated vapor are drawn lightly.

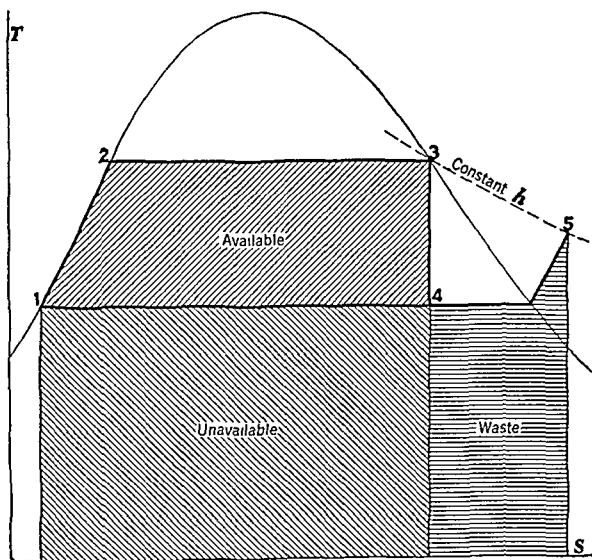


Fig. 7-9. T-S Areas for "Engine of Zero Efficiency"

**7-4. Waste Energy.**—As explained in Chapter 5, the work of a theoretical cycle is known as the available energy. The heat rejected by a theoretical cycle is inherently the unavailable energy. In Fig. 7-8 is reproduced the Rankine Cycle 1-2-3-4 shown in Fig. 7-2, but the areas representing the available energy and the unavailable energy are cross-hatched. In addition, an area designated as waste energy is shown. The concept of this area is necessitated by the fundamental departure of the actual cycle from Rankine conditions. Waste energy is that part of the available energy which is not converted into work.



mover equals its output divided by the minimum amount of heat which must be supplied to the cycle. This minimum amount of heat supplied is the heat which will be added if the rest of the cycle is perfect (i.e., no heat losses in the lines and no subcooling of the condensate).

The thermal efficiency of a well-designed prime mover may still be low, because it is operated over a low pressure range where the cycle efficiency is inherently low. Such an efficiency may be lower than that of a poorly designed prime mover which is operated over a higher range in pressures. Hence, the thermal efficiency of a prime mover, in itself, does not tell whether the prime mover is good or bad. To judge the degree of perfection of an actual prime mover, its performance should be compared to that of a perfect prime mover receiving steam under the same conditions as the actual prime mover and exhausting at the same back pressure. The term that expresses this performance is *engine efficiency*. For a simple prime mover, the A S M E Power Test Codes define the engine efficiency as the ratio of the actual work to the work of the perfect prime mover (i.e., the work done during isentropic expansion from the initial conditions to the final exhaust pressure).

As the actual work can be measured in three places, there are three engine efficiencies: (1) the internal efficiency (for a turbine) or the indicated efficiency (for a steam engine), (2) the brake efficiency and (3) the engine efficiency of the turbine generator.

Fundamentally, engine efficiency is not an efficiency at all. For a simple prime mover, it was conceived of as the ratio of the thermal efficiency of the actual prime mover to the thermal efficiency of a Rankine Cycle. It was therefore known as the Rankine Cycle ratio (R C R). Because the heat added in the Rankine Cycle equals the heat added that is chargeable to the simple prime mover, the Rankine Cycle ratio also equals the actual work divided by the theoretical work. Modern prime movers, because of reheating and regenerative feedwater heating, do not approximate the Rankine Cycle. Hence, the term Rankine Cycle ratio loses its significance and the term *engine efficiency* has come into use.

The foregoing definitions may be summarized as follows:

$$\text{Prime Mover Thermal Efficiency} = \frac{\text{Actual Work of Prime Mover}}{\text{Minimum Heat Supplied}} \quad (7-9)$$

$$\text{Prime Mover Thermal Efficiency} = \frac{2544}{\text{Heat Supplied per hp-hr}} \quad (7-9a)$$

$$\text{Cycle Thermal Efficiency} = \frac{\text{Actual Net Work}}{\text{Heat Supplied to Cycle}} \quad (7-9b)$$

$$\text{Engine Efficiency of Prime Mover} = \frac{\text{Actual Work of Prime Mover}}{\text{Theoretical Work of Prime Mover}} \quad (7-10)$$

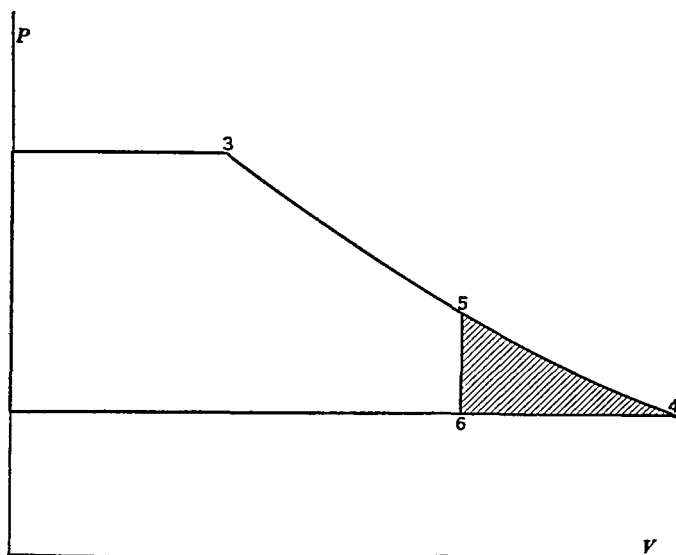


Fig. 7-11. P-V Diagram for Incomplete Expansion Cycle

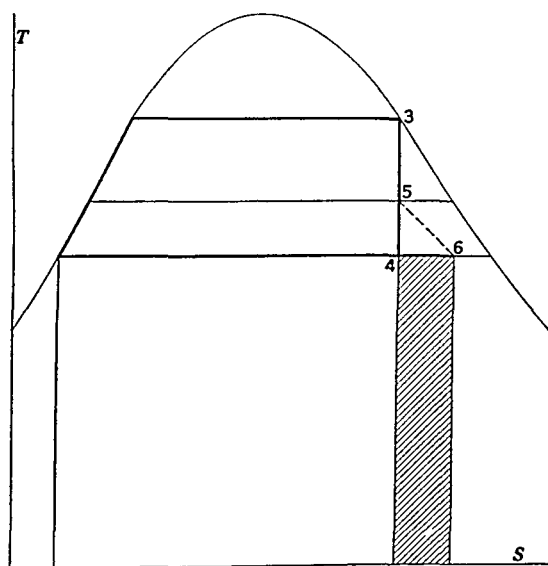


Fig. 7-12. T-S Diagram for Incomplete Expansion Cycle

**Example 7-2**—Steam leaves a steam generator at 200 psia and 600 F. It enters a steam turbine at 194 psia and 590 F. The generator delivers 2100 kw when the turbine receives 29 000 lb of steam per hour and exhausts at 2 in Hg abs. Condensate leaves the condenser at 90 F. Neglect pump work. Determine (a) the thermal efficiency of the cycle, (b) the thermal efficiency of the turbo-generator, (c) the engine efficiency of the turbo-generator.

**Solution**—The net work is 2100×3413 Btu per hr. The heat supplied per pound of steam equals the enthalpy at 200 psia and 600 F (1322.1 Btu) minus the enthalpy of water at 90 F (58 Btu). Then the thermal efficiency of the cycle equals

$$\frac{2100 \times 3413}{29\,000(1322.1 - 58.0)} = 0.1952 \text{ or } 19.52\% \quad \text{Ans.}$$

In finding the thermal efficiency of the turbo-generator, the output per hour is the same. As far as the turbine is concerned, the heat to be added per pound equals the enthalpy at turbine entrance (1317.3 Btu) minus the enthalpy of saturated liquid at 2 in Hg abs (69.1 Btu). Then the thermal efficiency of the turbo-generator equals

$$\frac{2100 \times 3413}{29\,000(1317.3 - 69.1)} = 0.198 \text{ or } 19.8\% \quad \text{Ans.}$$

The work per pound of steam expanding *isentropically* from 194 psia and 590 F to 2 in Hg abs, is 382.3 Btu. The engine efficiency equals

$$\frac{2100 \times 3413}{29\,000 \times 382.3} = 0.645 \text{ or } 64.5\% \quad \text{Ans.}$$

A common method of expressing the performance of steam prime movers is the steam rate. Steam rate is the amount of steam supplied to the prime mover per unit of output per unit of time (*i.e.*, pounds per horsepower hour or pounds per kilowatt hour). Steam rates are useful, as high steam rates mean low thermal efficiencies. However, steam rates do not indicate the relative merits of various prime movers unless these are operating under the same steam conditions.

A common method of expressing the performance of power plants is the heat rate. The heat rate is the heat supplied per unit of output per unit of time (*i.e.*, Btu per horsepower hour or Btu per kilowatt-hour). Thus, in the foregoing example the heat rate of the plant cycle is 17,485 Btu per kilowatt-hour. The student should check this as a simple exercise.

**★7-7. Effects of Incomplete Expansion**—In all that has been said thus far regarding the Rankine Cycle as a theoretical cycle, expansion has been carried *isentropically* from one extreme of pressure to the other, as shown in Fig 7-2 in going from point 3 to point 4. In Figs 7-11 and 7-12 are shown, respectively, the *P-V* diagram and the *T-S* diagram for a theoretical cycle (sometimes called Clausius Cycle) in which the expansion is halted at point 5, before condenser pressure is reached. This causes the pressure to drop to condenser conditions at point 6 in a throttling action, during which the entropy increases so that the cross-hatched area of waste energy on the *T-S* diagram equals the loss of work cross-hatched on the *P-V* diagram. Reference to the *P-V* diagram will demonstrate what is

known colloquially as "cutting off the toe" of the diagram. This situation occurs in reciprocating steam cylinders when the stroke is limited so that point 4 is not reached.

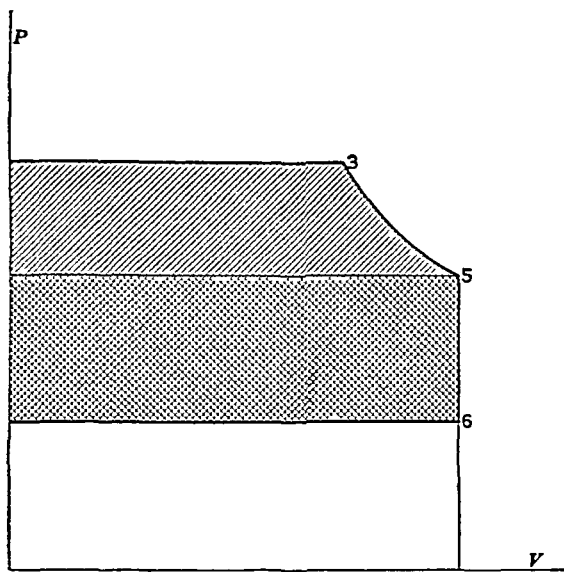


Fig. 7-13. Work Area of Incomplete Expansion Cycle

The work of the cycle can be determined by reference to Fig. 7-13. This is a repetition of Fig. 7-11 with the *cycle work area* cross-hatched. First consider that part of the area with single cross-hatching, or the area bounded by the adiabatic expansion line between points 3 and 5; this area is

$\int_3^5 v dP$  and is therefore equal to  $h_3 - h_5$ . The area of the rectangular part, which is double cross-hatched, is evaluated as  $v_5(P_5 - P_6)$ .<sup>\*</sup> Since  $h_5 = u_5 + \frac{P_5 v_5}{J}$ , adding the two cross-hatched areas gives

$$\frac{Wk}{J} = h_3 - u_5 - \frac{P_6 v_5}{J} \quad (7-11)$$

Hence,

$$\text{Thermal Eff.} = \frac{h_3 - u_5 - \frac{P_6 v_5}{J}}{h_3 - h_1} \quad (7-12)$$

<sup>\*</sup> If steam charts having steam volumes, such as the Ellenwood, are available, the pressure at the end of expansion may be determined, as the specific volume and entropy at this point (point 5) are known. Otherwise, a trial-and-error solution is necessary to obtain this pressure and hence the internal energy.

**7-8 Extending the Limits of the Rankine Cycle**—The Rankine Cycle, like any theoretical or actual cycle, has been shown to be less than 100 per cent efficient, if this were not so, the Second Law of Thermodynamics would be violated. However, the mere fact that the over all efficiency of steam power plants has more than doubled in the last 50 years leads to the conclusion that certain factors affecting the efficiency of the theoretical cycle have contributed to this. Let us see what these factors are

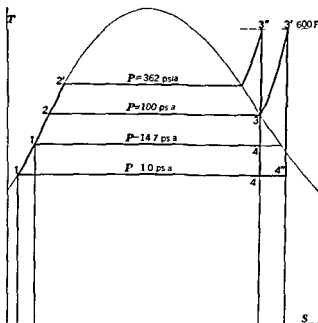


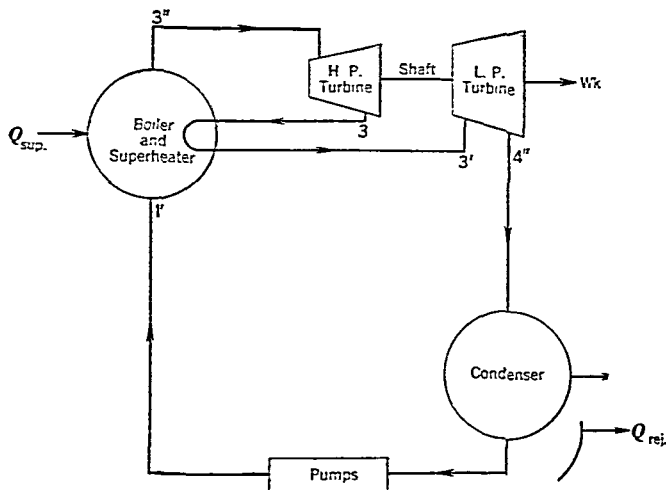
Fig 7-14 Methods of Increasing Efficiency of Rankine Cycle

The example shown in Art 7-2 has depicted a Rankine Cycle operating on saturated steam between the pressure limits of 100 psia and 14.7 psia. The efficiency of this cycle is 13.9% and the cycle is shown in Fig 7-14 as bounded by the figure 1-2-3-4. If the exhaust pressure be reduced to 1 psia by the use of a vacuum condenser, points 1 and 4 will be replaced by the points 1' and 4'. There will be an increase in available energy represented by area 1-4'-4-1', and a corresponding decrease in unavailable energy. This has the immediate effect of increasing the efficiency to 26.2%.

If next the initial temperature of the steam entering the prime mover be increased by superheating to say 600 F, the cycle will be bounded by the figure 1'-2-3-3'-4'-1'. The available energy will increase faster than the unavailable, as indicated by the area 3-3'-4'-4', and the efficiency

will be 27.5%. Thus, the efficiency has been increased and still the initial pressure has remained the same at 100 psia.

A further improvement can be made by raising the pressure at which heat is supplied. Consider that the high pressure is now 362 psia, instead of 100 psia, point 2 being replaced by 2'; and consider that the superheating is again carried to 600 F, so that the steam enters the prime mover at point 3'', instead of at either 3 or 3'. Note how relatively large the available energy has now become by comparison with the unavailable energy. The efficiency of this cycle, which is bounded by the figure 1'-2'-3''-4', is 33.4%. (The student should calculate each of the efficiencies mentioned.)



**Fig. 7-15. Resuperheating Cycle**

To sum up, it may be stated that the efficiency of <sup>ve Vapor</sup> can be increased by the following methods: <sub>ng</sub>

- (a) Lowering the back pressure.  
(b) Raising the initial temperature of the high superheating.  
(c) Raising the pressure at which heat is supplied.
- Modern power plants use all these methods. To take advantage of such practice, calculate the efficiency of a steam engine operating on steam generated at 2200 psia, superheated at an absolute pressure of 1 in. of mercury, and exhausting at an absolute pressure of 1 in. of mercury. The feed-water enters the boiler at 4, and also receives heat from the turbine at 6, and enters the turbine at 6. The condenser at point 6 has a pressure of 1 in. of mercury. The feed-water heater, only one feed-water heater, the bleed steam

★7-9. Modifications of the Fundamental Rankine Cycle in direct contact, and its modifications of the fundamental Rankine Cycle heater and that of the con-

denser All water leaving the heat is pumped to the boiler, where the cycle is repeated

Most power plants use closed feed-water heaters (i.e., heaters where the steam and water do not mix, the heat being transferred from the steam to the water through heat transfer surfaces) For the actual single-heater set-up, the condensate formed in the heater generally is fed to the hot well of the condenser In the theoretical case, it is customary to assume that the condensate is mixed with the feed water leaving the heater Under these conditions, the over-all analysis of the closed feed water heater becomes the same as that of the open feed water heater, and the analysis which follows applies equally well to both types

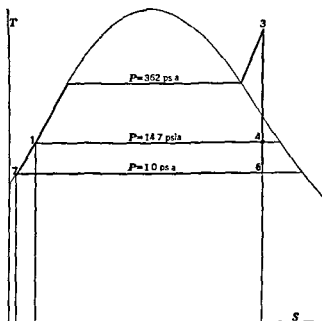


Fig 7-17 T-S Diagram for Regenerative Cycle with Feed Water Heater

An adjusted  $T$ - $S$  diagram of a regenerative cycle is shown in Fig 7-19, but, before an analysis of this cycle can be made on a quantitative basis, a new concept—that of showing variable weights by shifting entropy plots so that the value of  $S$  at any point equals specific entropy times the weight present—must be learned On  $T$ - $S$  diagrams up to now, the entropy has been plotted as either specific entropy or total entropy Since the weight of fluid being studied did not vary, the use of specific entropy was justified If this conventional method is used here, the  $T$ - $S$  diagram of results

In Fig. 7-17, points are numbered to correspond with those of Fig. 7-16. Follow Fig. 7-17 carefully. In the boiler and superheater, heat is supplied from point 1 to point 3; expansion takes place in the high-pressure stages of the turbine from 3 to 4, part of the steam then continuing to expand in the low-pressure stages of the turbine from 4 to 6. That part which does not expand from 4 to 6 in the turbine is withdrawn at 4 and goes to the feed-water heater. The steam that enters the condenser at point 6 is condensed to point 7; this condensate is heated from 7 to 1 in the feed-water heater by the extraction steam giving up heat from 4 to 1. It is seen that the area under 4-1 *should* equal that under 7-1, but in Fig. 7-17 this does not *appear* to be so.

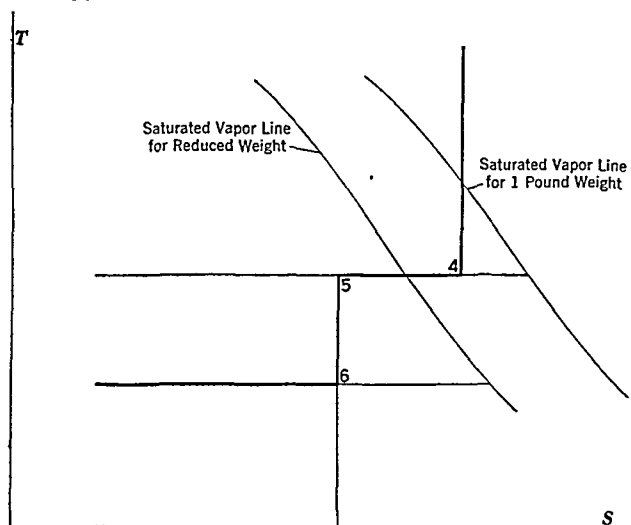


Fig. 7-18. Effect of Extraction on Entropy

Now, let us consider the decrease in weight of vapor due to extraction. If  $Q = \int T dS$ , then any change in entropy for a reversible operation will mean that heat is being transferred. If the change in entropy is considered equal to a change in weight times a specific entropy, the area under the  $T$ - $S$  curve will still represent the heat transferred. Furthermore, the *true state point* of the vapor can be represented.

In Fig. 7-18, steam at point 4 splits into that which goes to the heater and that which expands from point 5 to point 6 in the low-pressure part of the turbine. The important point to keep in mind is that *the condition, or state point, of the steam at points 4 and 5 is the same*; but point 4 represents the *entropy of an entire pound of steam* (all the steam that entered the turbine), while the entropy at point 5 is reduced by that in the fraction of



a pound of steam which is extracted for heating the feed water. To calculate what part of each pound of steam is extracted, consider the effect of the feed-water heater in Fig 7-16. Under steady flow conditions,  $\Sigma wh_{in} = \Sigma wh_{out}$ . Then,

$$w_8 h_8 + w_7 h_7 = u_1 h_1 \quad (7-14)$$

Let  $w_1$  equal 1 lb and  $w_7$  be  $1 - u_8$ . Then, since  $h_8 = h_4$ ,

$$w_8 = \frac{h_1 - h_7}{h_4 - h_7} \quad (7-15)$$

That is, if 1 lb of steam at point 4 splits into  $w_8$  and  $1 - w_8$ , the entropy at points 5 and 6 is equal to the entropy at point 4 multiplied by the weight  $(1 - w_8)$ .

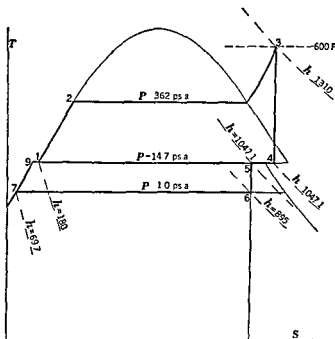


Fig 7-19 Adjusted T-S Diagram for Regenerative Cycle

The entire cycle is shown on the  $T$ - $S$  plane in Fig 7-19. Points 4, 5, and 6 of this figure are the same as those of Fig 7-18. Similarly, point 9 is plotted by multiplying the entropy of point 1 by the weight factor, which is  $(1 - u_8)$ . Now compare the numbering of Figs 7-16 and 7-19, and in each diagram note the state of the working substance at each stage of the process.

For the conditions determined in the previous examples, and the pressures and enthalpies indicated in Fig. 7-19, the fraction extracted at

point 4 is calculated to be 0.113. The thermal efficiency, being work done divided by heat supplied, is

$$\text{Eff.} = \frac{h_3 - h_4 + (1 - w_s)(h_4 - h_6)}{h_3 - h_1} \quad (7-16)$$

or 
$$\text{Eff.} = \frac{1310 - 1047.1 + (0.887)(1047.1 - 895)}{1310 - 180} = 0.352 \text{ or } 35.2\%$$

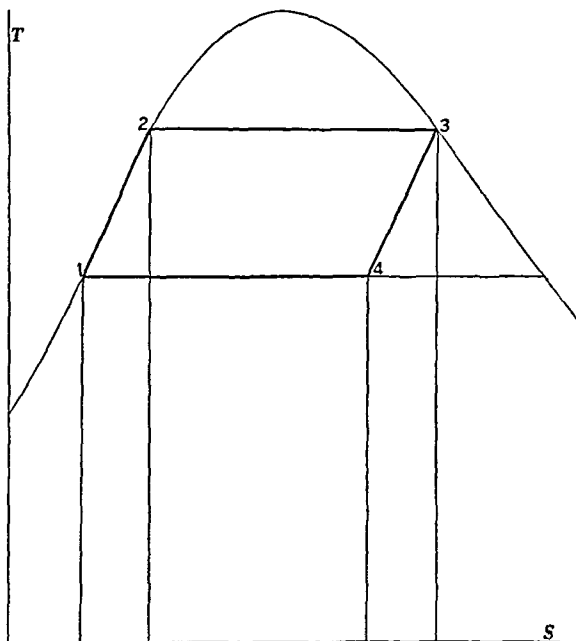


Fig. 7-20. Regenerative Vapor Cycle, Using Infinite Number of Extraction Stages

The foregoing analysis, while theoretical, is the fundamental basis on which extraction feed cycles are based. Many modern power plants employ multi-stage extraction cycles, as many as four stages now being common for plants operating with a pressure range of 1200 psi. Obviously, the number of stages and the pressures at which extraction is selected are governed by the law of diminishing returns. Theoretically, an infinite number of stages (whereby the pressure drops would be infinitesimals) would make a saturated-steam cycle have the same efficiency as the Carnot Cycle. If an infinite number of extraction stages are assumed in Fig. 7-20, heat is removed along the line 3-4 and it is transferred to the liquid along the line 1-2. Therefore, under these conditions external heat is supplied only along the line 2-3, and heat is externally rejected only along the line 4-1.

★7-10 Binary-Vapor Cycles.—In order to obtain high thermal efficiencies from any heat engine, the mean over-all temperature range (i.e., the difference between the mean temperature at which heat is received and that at which it is rejected) must be large. The temperature of the circulating water for the condenser is the controlling factor for the lowest temperature at which heat may be rejected. This temperature is approached in modern steam turbine plants. The upper limit of the temperature at which heat is received is fixed by the temperature which the available metals can stand. Modern practice has increased this

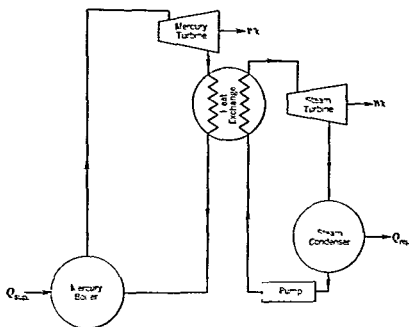


Fig 7-21 Physical Layout for Binary Vapor Cycle

maximum temperature in steam-power plants to a steam temperature as high as 1050 F. However, the mean temperature at which heat is added is much lower than the maximum cycle temperature because water changes state at relatively low temperatures, even though pressures as high as 2000 psi are used. Thus, it is necessary to use a working substance other than water if heat is to be received at a high mean temperature.

The fundamental requirement of a working substance other than water is that it boil at relatively high temperatures for moderate pressures. Unfortunately, those substances that fulfil this requirement condense at relatively high temperatures even though very low pressures are obtained in the condenser. However, a substance that condenses at a high tem-

perature may be used to boil another substance which boils at a lower temperature for a given pressure. The second substance can reject heat at a temperature only slightly higher than that of the circulating water. Thus, by the use of a combination of two fluids, the over-all *mean* temperature range is large and, hence, thermal efficiency is high.

One combination of substances that has been used consists of mercury for high-temperature parts of the binary-vapor cycle and steam for low-temperature parts. Mercury has a higher boiling temperature than water at corresponding pressures; consequently, high evaporation temperatures can be reached with *moderate* pressures.

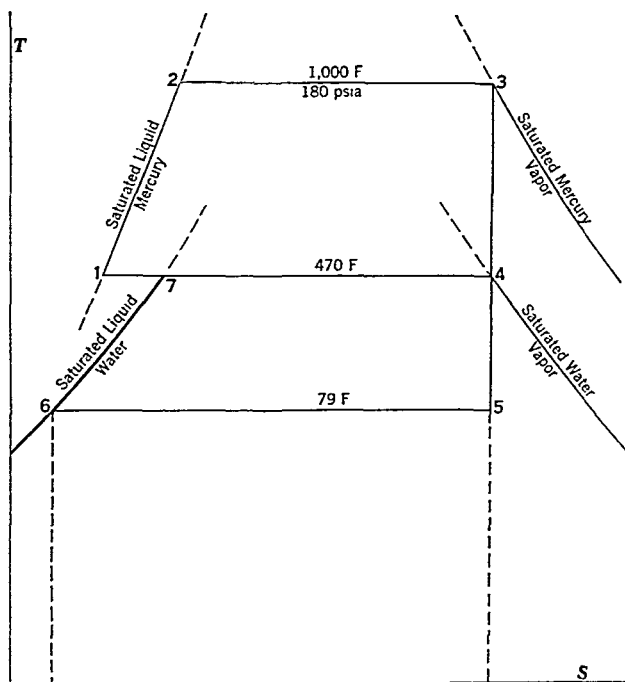


Fig. 7-22. T-S Diagram for Binary-Vapor Cycle

In Fig. 7-21 is shown a line diagram of the physical layout, and Fig. 7-22 shows a T-S diagram of the two theoretical saturated vapor cycles, operating between the extremes of 1000 F and 79 F and joined by a common temperature line at 470 F. The upper cycle uses mercury evaporated at 180 psia. This vapor expands in a mercury turbine to 1.2 psia, and the corresponding condensing temperature of 470 F is the upper temperature limit of the steam. External heat is supplied to the mercury in a boiler along the path 1-2-3, and external heat is transferred from the steam at

79 F in the condenser from point 5 to point 6. These are the external heat transfers. Within the cycle, heat is removed from the mercury while condensing at 12 psia from point 4 to point 1, this heat is supplied to the water and steam at 514.7 psia along the path 6-7-4. In other words, the steam boiler is the mercury condenser.

The cycle as shown is drawn for ideal conditions and has a common temperature of 470 F as the maximum for the steam and the minimum for the mercury. In an actual cycle, this line is split into two temperatures, such as 470 for the mercury and 450 for the steam, in order to promote heat transfer, however, for the purpose of making an ideal analysis, the "gap" between the cycles should be closed as shown in Fig. 7-22.

Because of the low enthalpy values of mercury, it is necessary to use 13 lb of mercury per pound of steam in the particular cycle illustrated. The Rankine-Cycle efficiency of the mercury part alone is 34%, that of the steam part alone is 36.3%, and that of the combined cycle is 58%. As the Carnot Cycle efficiency for temperature limits of 1000 F and 79 F is 63%, the 58% achieved by the binary-vapor cycle appears attractive at first glance. However, only three commercial plants employing the mercury-steam cycle have been built. Serious obstacles to the use of this cycle are the limited supply of mercury, the toxic effect of mercury vapor upon operating personnel, and heat-transfer problems that are inherent with a fluid which does not "wet" the surface. Other substances, such as diphenyl and diphenyl-oxide, have been proposed as a substitute for mercury in binary cycles, but to date such substances have been of academic interest only.

TABLE 7-1  
CONDITIONS AFFECTING EFFICIENCY

Initial Pres psia	Exhaust Pres psia	Initial Temp F	Reheat Pres psia	Extract Pres psia	Theoretical Thermal Eff %
100	14.7	Sat	None	None	13.9
100	10	Sat	None	None	26.2
100	10	600	None	None	27.5
362	10	600	None	None	33.4
362	10	600	100	None	34.0
362	10	600	None	14.7	35.2
2200	0.491	1000	None	None	44.8
2200	0.491	1000	198	193	49.6
Binary Vapor	0.491	1000	*	*	58.0

\* In the binary vapor cycle here considered heat is transferred from mercury to steam at an intermediate temperature of 470 F.

In the fast few years, in spite of greatly improved thermal efficiencies which have been obtained in steam power plants, interest continues in the binary-vapor cycle. It is conceivable that if a more satisfactory fluid than mercury became available, much expansion would take place in the use of the binary-vapor cycle. Furthermore, the engineer must always be on the search for a better way to do things. Casual acceptance of established types of power plants must be avoided.

★7-11. **Recapitulation of Conditions Affecting Efficiency.**—The values in Table 7-1 will indicate the effects of modifications on the efficiency of cycles.

## CHAPTER 8

### STEAM ENGINES

**8-1. Classification.**—Steam engines of the reciprocating type may be classed under several headings, such as class of service, speed, arrangement of cylinders, number and type of cylinders, type of valve design, method of governing, number of expansions, type of exhaust, and approximate type of steam condition. Under class of service can be listed stationary, marine, locomotive, or the use to which the engine is to be put, such as pumping or hoisting. Under speed can be listed high, medium, and low speed, meaning rotative speed. A high-speed engine is usually one designed to operate above 300 rpm; a low-speed one operates below 125 rpm; medium speed is naturally between these limits. By arrangement of cylinders is meant vertical, inclined, or horizontal; or the location of cylinders with respect to one another, as in tandem, at angles with each other, or across from each other; or the position of the cylinder with respect to the shaft. The common form of vertical engine with the cylinders *above* the crankshaft is known as a vertical inverted engine.

The number and type of cylinders is allied closely with the arrangement classification and also with the number of expansions. For instance, a four-legged, triple-expansion engine has its expansion divided into three parts; but, instead of having a high-pressure, a medium-pressure, and a low-pressure cylinder, it has one high-, one medium-, and two low-pressure cylinders, thus having relatively smaller low-pressure cylinders and also being inherently well-balanced dynamically.

Under type of valve design can be listed simple D valve, balanced plate valve, piston valve, riding cut-off valve, poppet valve, Corliss valve, and Uniflow type, the last-named being either with or without auxiliary exhaust valve. Under method of governing, engines can be classed as manual or automatic, throttle or cut-off governing, and centrifugal or inertia type of governor. Under type of exhaust can be condensing, meaning an absolute exhaust pressure below atmospheric, and non-condensing, meaning exhaust pressure at or above atmospheric. Under approximate type of steam conditions can be listed saturated or superheated and high or low pressure, although "high" pressure as now known in turbine work is generally well out of the range of pressures in steam-engine cylinders.

**8-2. Cylinder Performance.**—While the reciprocating steam engine and its counterpart, the direct-acting steam pump, are rapidly becoming

obsolescent, a thermodynamic study of the factors affecting their performance is desirable both from the viewpoint of fundamental engineering knowledge and from that of learning the limitations to which this type of prime mover can be developed. Briefly, the steam engine may be said to be characterized by its ruggedness, by its high torque at slow speed, and by its practical inability to expand steam to a pressure lower than 2 psia in the cylinder because of the huge volume that would be required.

In view of the fact that a physical concept of the construction of such an engine is needed to envisage the behavior of the working substance, Fig. 8-1 has been drawn to show the essential parts of a simple steam engine.

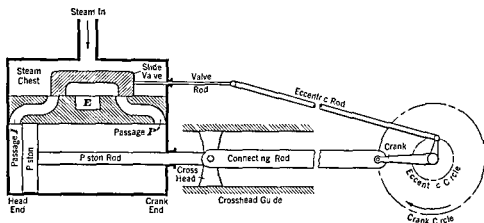


Fig. 8-1 Diagram of Steam Engine

engine having a D slide valve which is kept on its seat by the pressure of the steam in the steam chest. As the crankshaft revolves, the valve slides back and forth, admitting steam from the steam chest to the cylinder through the passages *P* (on the head end) and *P'* (on the crank end), these passages serving both to admit and to discharge steam. A careful study of the figure should be made. Note that, as the position of the small crank (simulating the eccentric) is displaced more than  $90^\circ$  from the position of the large crank, the valve will "lead" the piston in its motion, that is, as the valve admits steam at one end of the cylinder, its motion also permits the hollowed-out part of the "D" to be in contact with a passage and with the exhaust cavity *E*, so that low-pressure steam will be let out of the cylinder and escape to the exhaust.

Fig. 8-2 shows the events of the cylinder's cycle on an indicator diagram. On this diagram, steam is admitted at 1 when the piston is near the end of the stroke. The steam continues its admission to 2, where cut-off occurs, i.e., the valve slides over the port to the passage, and the steam in the cylinder is isolated. As the piston moves to the right-hand side of



Fig. 8-1, work is done by expansion; but, as the end of the stroke is approached, the valve's travel uncovers the passage to the exhaust hollow, and the cylinder pressure drops from point 3, Fig. 8-2 (commonly called *release*) to the exhaust pressure. On the return stroke, the steam will continue to escape through the exhaust until the valve once again covers the port at the passage *P*, Fig. 8-1, and the steam remaining in the cylinder is naturally compressed, starting from 4, Fig. 8-2. This small part of the steam that is compressed is known as "cushion steam," from the effect it has of counteracting the inertia effect of the reciprocating parts near the end of the stroke and promoting smooth running. It also has a thermodynamic effect in that compression raises its temperature—as well as its pressure—and thus aids the new, incoming steam at 1. This is more fully treated later under discussion of initial condensation, which is the major thermodynamic loss in the engine.

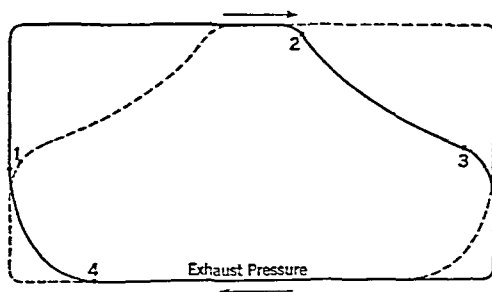


Fig. 8-2. Indicator Diagram for Steam Engine

While the full-time indicator diagram has been drawn in Fig. 8-2 for the head end, a dotted-line diagram has been drawn to show the working events of the cycle for the crank end. Most steam engines are made double acting; hence, the mean force acting on the piston on each side must be considered. As work equals force times distance, the force being pressure times piston area and the distance being the stroke, the horsepower determined in the cylinder with an indicator, or the *indicated horsepower* (per cylinder), is

$$\text{ihp} = \frac{P_H A_H N}{33,000} + \frac{P_C A_C N}{33,000}$$

or

$$\text{ihp} = \frac{(P_H A_H + P_C A_C) L N}{33,000} \quad (8-1)$$

where the subscript *H* denotes the head end and *C* denotes the crank end; *P* is the mean effective pressure,\* in psi; *A* is the effective piston area, in

\* For a previous use of mean effective pressure in general, note Chapter 5, Prob. 4.

sq in (less on the crank end because of the area occupied by the cross-section of the rod),  $L$  is the piston stroke, in feet, and  $N$  is the number of cycles per minute through which each mep operates

The horsepower thus determined is greater than that at the shaft coupling where the useful job is to be done, because engine-friction losses reduce the shaft output, measured by a brake and hence called *brake horsepower* (The terms indicated horsepower and brake horsepower apply to all sorts of machinery, not merely to steam engines, and hence an early familiarity with the means of computing them is essential) The brake horsepower is computed from the time rate of applying the mean torque, or

$$\text{bhp} = \frac{2\pi(\text{rpm})(\text{Torque})}{33,000} \quad (8-2)$$

where the torque is expressed in lb-ft It is measurable by means of mechanical, electrical, or hydraulic dynamometers of either the absorption type or the transmission type, such as the torsionmeter which depends upon the twist of the shaft under torsion, the phase angle being measured \*

The ratio of the brake horsepower to the indicated horsepower is known as mechanical efficiency, or

$$\text{Mech Eff} = \frac{\text{bhp}}{\text{ihp}} \quad (8-3)$$

The thermal-efficiency expressions of Chapter 7 should be recalled for use in the following illustrative example

*Example 8-1*—A single-cylinder steam engine, 8'×10', with a piston rod 1" in diameter, runs at 275 rpm and develops a torque of 638 lb-ft at the shaft coupling If the head end mep is 53 psia and the crank end mep is 54 psia, what is the mechanical efficiency?

*Solution*—As engineering practice always gives the bore ahead of the stroke, the head-end piston area is that of an 8' circle, or 50.26 sq in, this area minus the area of the rod gives the crank-end piston area as 49.48 sq in By equation 8-1,

$$\begin{aligned} \text{ihp} &= \frac{(53 \times 50.26 + 54 \times 49.48) \left( \frac{10}{12} \right) (275)}{33,000} \\ &= 37.06 \end{aligned}$$

By equation 8-2,

$$\text{bhp} = \frac{2\pi \times 275 \times 638}{33,000} = 33.41$$

By equation 8-3,

$$\text{Mech Eff} = \frac{33.41}{37.06} = 0.901, \text{ or } 90.1\% \quad \text{Ans}$$

*Example 8-2*—The engine in Example 8-1 is supplied with dry saturated steam at 100 psia and exhausts at 14.7 psia, and uses 1334 lb of steam per hour What is its engine efficiency based on the ihp?

\* For an intensive study of the details of dynamometers of various types, consult a standard text on testing, such as Shoop and Tuve's *Mechanical Engineering Practice*

*Solution.*—Reference to Chapter 7 for the steam conditions given shows the thermal efficiency of the corresponding Rankine Cycle to be 13.9%. By equation 7-9a, the actual thermal efficiency of the prime mover, based on the ihp, is

$$\text{Actual Thermal Eff.} = \frac{2544 \times 37.06}{1334(1187.2 - 180.0)} \\ = 0.0702 \text{ or } 7.02\%$$

Then,

$$\text{Engine Efficiency} = \frac{7.02}{13.9} = 0.505$$

Ans.

As a matter of interest, it can be noted that the specific steam rate is  $\frac{1334}{37.06} = 36$  lb per ihp-hr.

★8-3. **Factors Affecting Cylinder Performance.**—Example 8-2 shows in a practical way the results which can be expected from an engine of that type. An analysis of the thermodynamics of cylinder losses with a view to determining the reasons for the low engine efficiency based on ihp will now be considered.

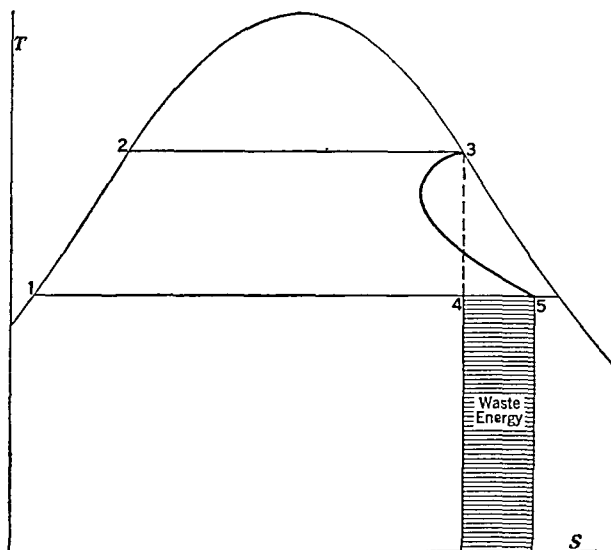


Fig. 8-3. T-S Diagram for Steam Engine Expansion Line

As explained in Chapter 7 in conjunction with the discrepancy between an actual engine and a Rankine one, the expansion in the actual case is not isentropic; the entropy increases, the work is less than the available energy, and the enthalpy of the exhaust steam is higher than it would be under Rankine conditions. Within the cylinder, the energy losses are due to: (a) condensation and reevaporation; (b) incomplete expansion; (c) clearance volume and compression; (d) throttling, leakage, and radiation.

The first of these listed losses is the chief of those resulting from imperfections. When the cylinder has exhausted steam at low temperature, its passages (as  $P$  and  $P$  in Fig 8-1) are relatively cool. As these passages in a slide-valve engine have a comparatively large surface-to-volume ratio, the entering steam is easily condensed on the walls of the passage. The ends of the cylinder also cause some condensation. Effects of condensation are shown in Fig 8-3.

This diagram shows the isentropic expansion line (dotted) of the theoretical Rankine Cycle, and the actual path of the steam expanding in the cylinder (full line) from 3 to 5. It will be noted that, as the line starts down from 3, it proceeds at first to the left, the decrease in entropy is indicative of the withdrawal of heat from the warm steam to the cooler cylinder parts. As the pressure of the steam drops, so does the tempera-

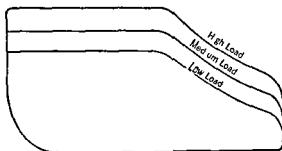


Fig 8-4 Indicator Diagrams for Fixed Cut off, Throttling Governed Engine

ture, until finally the temperatures of the cylinder and the steam are alike and no further heat transfer *from* the steam occurs. However, the steam's temperature continues to drop because of expansion, hence, the cylinder parts—now warmer than the steam—proceed to transfer heat *to* the steam, and the expansion line then proceeds to the right, until point 5 is reached at exhaust pressure. It might be supposed that since heat was given back to the steam, little of it was lost, indeed, the net amount “radiated” by the cylinder to the outside air is often trifling. It might be argued that the steam at first gave energy to the cylinder parts but that, before the expansion was finished, the steam recovered the heat again. This is true enough *but* it must be remembered that, according to the fundamental principles enunciated by Carnot, for high efficiency heat should be supplied at high temperature and rejected at low temperature. Cylinder condensation and reevaporation reverse this condition. The heat is withdrawn from the working substance when it is most needed, and is given back when very little high grade energy remains for work. Study Fig 8-3 again and consider this fact.

Incomplete expansion has been discussed in conjunction with Chapter 7. A more definitive understanding of it can be obtained by realizing that a late cut-off (note point 2 in Fig. 8-2) increases this type of loss while an early cut-off increases the relative cylinder condensation because the little steam admitted expands through a wide range. At this time it might be well to compare Figs. 8-4 and 8-5. The former shows the type of indicator card to be expected from governing an engine by throttling the entering steam and using a fixed cut-off point. The latter shows a constant steam pressure with a variable cut-off point to suit the load conditions.

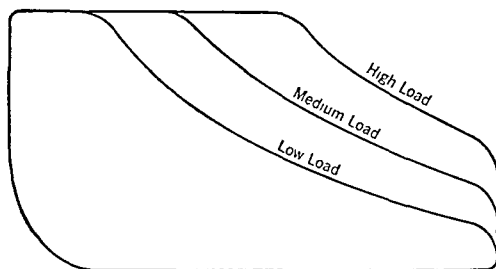


Fig. 8-5. Indicator Diagrams for Variable Cut-off Engine

The method of control by throttling causes an inherent loss of available energy from the irreversibility of throttling, and the relatively late cut-off possessed by a purely throttle-controlled engine also causes a loss due to incomplete expansion. On the other hand, since the steam is throttled before it enters the cylinder, it operates between narrower temperature limits within the cylinder and thus initial condensation is minimized. Furthermore, when the engine is operated on wet steam, the throttling generally tends to dry the steam or even to superheat it slightly, thus combating initial condensation. However, the losses inherent in throttling usually outweigh its good features. For this reason, many automatic engines use cut-off governing. As can be seen from the curve in Fig. 8-5 for light load, an excellently long expansion can be achieved from full high-pressure conditions. The worst loss here is caused by initial condensation. Since the cut-off occurs early in the stroke, a larger per cent of the entering steam is subjected to the cooling effect of the clearance spaces, the surface-to-volume ratio of which is greater than that of the engine cylinder itself.

The fact that the clearance space has a large surface upon which condensation can take place has been the cause of numerous attempts to warm it. A slight warming effect can be obtained by compressing the trapped steam near the end of the stroke. As previously mentioned, this process also has a cushioning effect and enables the incoming steam to have less

of an initial pressure drop. But the compression (note the path from 4 to 1 in Fig 8-2) is work done *on* the steam *by* the engine, and, while most of this energy is returned by reduction of initial condensation of the incoming steam, nevertheless the mep of the engine is somewhat reduced.

In Fig 8-6 is shown a sketch of an indicator card with the clearance volume exaggerated (in appearance) for the purpose of demonstrating certain terminology. Clearance is expressed as a *per cent of piston displacement*, not of total volume, the total volume is the sum of the volume of displacement and the clearance volume. The total volume at any

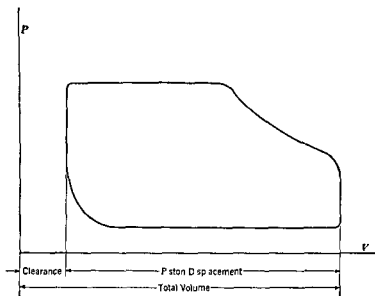


Fig 8-6 Clearance and Piston Displacement

position of the piston must then be the clearance volume plus whatever part of the piston displacement has been reached at that point in the stroke. When this total volume is to be calculated for solving a problem, a sketch similar to Fig 8-6 should always be drawn.

Regardless of the type of governing, there can exist a valve loss from throttling (sometimes called wiredrawing when referred to valves), this can be large in slide-valve engines but is small with Corliss (oscillating) valves and with poppet valves. Leakage of steam is a variable loss the amount of which is dependent on the condition of the engine and not on its type. "Radiation" losses are likewise variable, but can be kept low. These and other minor losses can be as low as 1 per cent.

**★8-4 Methods of Reducing Cylinder Losses**—The cylinder losses just enumerated (particularly those due to condensation and reevapora-

tion) are partly combated by one or several of the following methods: (a) superheating; (b) providing steam jackets; (c) increasing rotative speed; (d) compounding or triple or quadruple expansion; (e) using a Uniflow type engine.

Superheating has been shown (see Chapter 7) to effect some gain in the efficiency of the Rankine Cycle. In an actual engine, the gain over using saturated steam is even more marked because of the decrease in cylinder condensation. The superheated steam's gas film does not transfer heat so rapidly; furthermore, steam that loses some—but not all—of its superheat remains a vapor and is not partly a liquid. Hence, its available

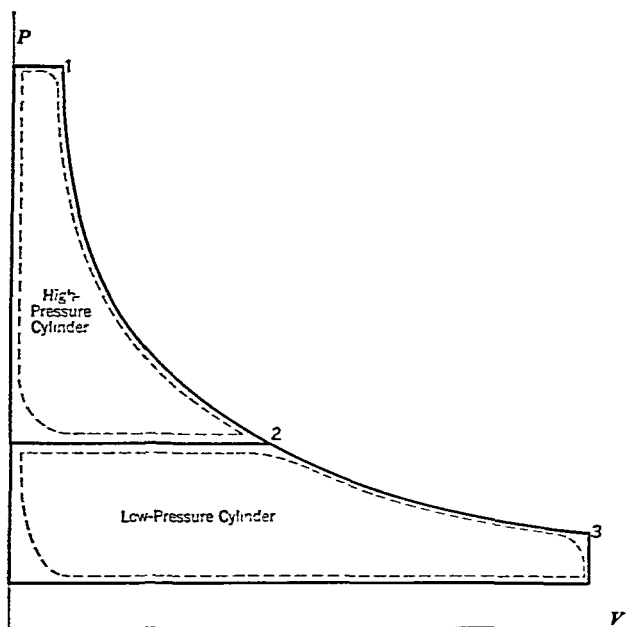


Fig. 8-7. P-V Diagram for Compound Engine

energy is larger. One rule of thumb states that steam consumption is decreased 1 per cent for every 10 degrees of increase in superheat. It should be borne in mind, however, that high superheat increases the difficulty of internal cylinder lubrication, and also that cast-iron slide valves will warp at high temperatures.

Steam jackets to keep the cylinder heads and valve passages hot are hardly new; they were used by James Watt. Claims as high as a 30% saving have been made, particularly in small-cylinder, slow-speed engines or in engines having high initial condensation when unjacketed. With

high-speed engines in general, it would appear that little, if any, improvement is effected by steam jackets

Theoretically, the higher the rotative speed, the less time there is for condensation to take place. For a well designed engine, therefore, a high speed will tend to minimize condensation. This does not necessarily mean that all high-speed engines are efficient and all low-speed engines are not, however, as high-speed engines are often so cheaply constructed that their lack of quality vitiates their inherent advantage due to speed.

Compounding, or the taking of the expansion through several cylinders *in series*, has several advantages. In Fig 8-7 is shown a theoretical  $P-V$

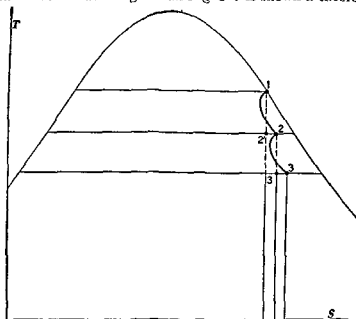


Fig 8-8 T-S Diagram Showing Effect of Compounding

diagram enclosing two actual indicator cards. The steam expands in the high pressure cylinder from point 1 to point 2 and in the low pressure cylinder from point 2 to point 3. Two favorable factors enter into this division of expansion. First, the high-pressure cylinder can have a small surface for condensation because a relatively small volume is sufficient to accommodate point 2 in the diagram, second, the *range* of temperatures through which each cylinder will operate (and hence the fluctuation in the temperature of the cylinder wall and passages) will be less for the over-all range, than with a single expansion. Fig 8 8 shows the corresponding  $T-S$  diagram, on which the expansion line 1-2-3 is traced \*

\* Fig 8 8 shows the actual state point 3 of the steam at exhaust pressure whereas Fig 8 7 shows point 3 at release. However, in a compound engine incomplete expansion is very much minimized.



A significant fact can be noted in conjunction with Fig. 8-8. Let the available energy for the high-pressure stage be denoted by  $h_1 - h_{2'}$ , and that for the low-pressure stage by  $h_2 - h_{3'}$ . Then the increase from  $h_{2'}$  to  $h_2$ , while reducing the possible work of the high-pressure stage, actually increases the available energy of the low-pressure stage. That is, the

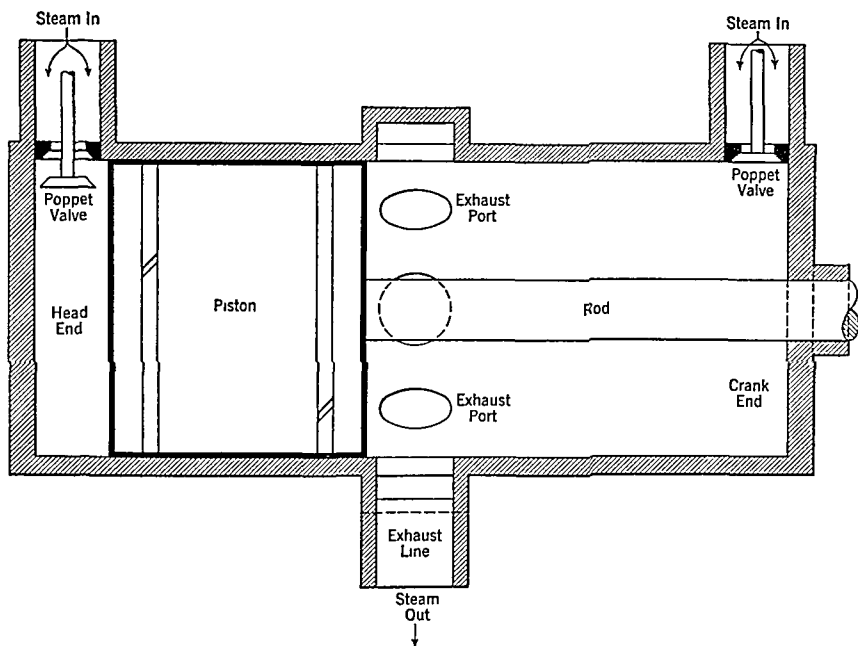


Fig. 8-9. Uniflow Engine

waste energy of the high-pressure cylinder exists at a temperature higher than the lowest available temperature; hence, some of this waste energy may be used in the low-pressure stage. This apparent phenomenon will be met with again later in conjunction with multi-stage turbines under consideration of "reheat factor."

Compounding, as a term, has come to mean the using of two stages of expansion in steam engines; for three and four stages, the terms triple expansion and quadruple expansion, respectively, are used. Very few engines of the last-named type are in use, however, the law of diminishing returns making the triple-expansion engine a better investment.

Since 1907, the Uniflow engine, to which Figs. 8-9 and 8-10 apply, has been used with considerable success, particularly in the low-power field. There, unless high speeds (1800 rpm) are needed, it competes successfully with the auxiliary turbine. As can be seen in Fig. 8-9, the Uniflow engine

has a cylinder that is long in comparison with its stroke, and has a piston about 45 per cent as long as the cylinder. Steam enters at each end and flows one way (literally unflow) toward the middle, where the ring of exhaust ports is uncovered by the piston as it nears the end of the stroke. The long piston thus acts as an exhaust valve. On the return stroke, the piston very soon covers the exhaust ports, so that compression (point *K*

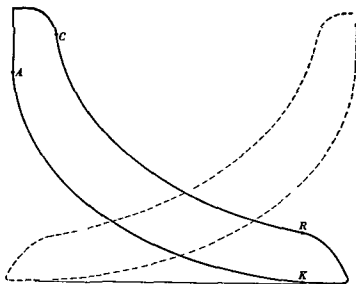


Fig 8-10 Indicator Diagram for Uniflow Engine

in Fig 8-10) starts early and can be carried almost to line pressure. By cleverly using adjustable clearance plugs, an engine of this type can be adjusted to compress the cushion steam to the desired point. Other variations on Uniflow types use auxiliary exhaust valves to prohibit excessive compression. It is obvious that the Uniflow indicator card shown in Fig 8-10 has a lower mep than that of the ordinary counterflow engine operating between the same extremes of pressures, but the gain in R C R more than offsets the disadvantage of lower power per cubic inch of displacement.

#### ★8-5 Theoretical Mean Effective Pressure and Diagram Factor

Fig 8-11 shows a theoretical  $P$ - $V$  diagram for a steam engine without clearance and an enclosed actual indicator diagram. The analysis about to be made is useful to the designer of an engine in estimating the approximate ihp to be expected from certain cylinder dimensions or, conversely, in designing for bore and stroke in accordance with the power to be achieved with given pressures and speed.

Let the volume at the cut-off point be  $V_1$  and that at the end of the stroke be  $V_2$ ; the cut-off ratio  $r$  is then  $\frac{V_2}{V_1}$ . Let the high pressure entering the cylinder be  $P_1$  and the exhaust pressure be  $P_2$ . Then the area of the tall rectangle to the left will be  $P_1V_1$ , and the area of the long, low rectangle at the bottom of the diagram will be  $P_2V_2$ . It has been found that the expansion curve from point 1 to point 2 closely approximates a rectangular

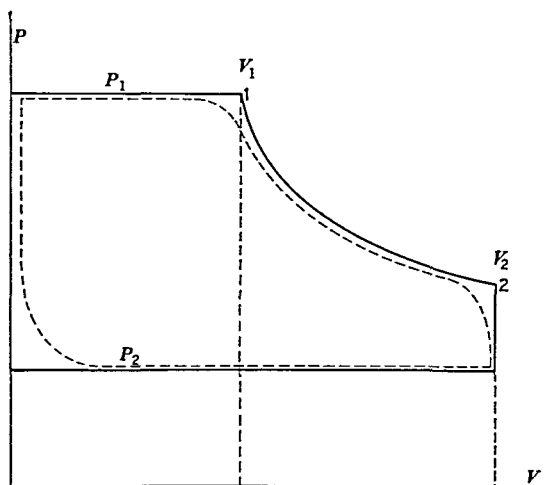


Fig. 8-11. Theoretical P-V Diagram for Engine Without Clearance

(or equilateral) hyperbola; hence, the area under the curve between 1 and 2 integrates to  $P_1V_1 \log_e \frac{V_2}{V_1}$ . (While this expression resembles that for the isothermal-work process with a perfect gas, it must be borne in mind that *this* process is definitely not isothermal.)

The net area between the two bounding pressure lines will be  $P_1V_1 + P_1V_1 \log_e \frac{V_2}{V_1} - P_2V_2$ , and the mean height of this area, or the mean effective pressure, is the area divided by the length. Thus,

$$\text{Theoretical mep} = \frac{P_1V_1}{V_2} \left( 1 + \log_e \frac{V_2}{V_1} \right) - P_2$$

$$\text{or} \quad \text{Theoretical mep} = \frac{P_1}{r} (1 + \log_e r) - P_2 \quad (8-4)$$

The theoretical mep thus obtained is corrected by a "diagram factor," the value of which is based on experience, to bring into line an actual mep

from which an actual ihp can be predicted by the use of equation 8-1. If the piston-rod diameter is unknown and the head and crank ends are assumed to be alike, the designer's formula for predicted ihp is

$$\text{Predicted ihp} = \frac{2 \text{ PLAN}}{33,000} \quad (8-1a)$$

Table 8-1 gives the values of representative diagram factors, by which the theoretical mep of expression 8-4 is multiplied to obtain the actual expected mep.

TABLE 8-1  
DIAGRAM FACTORS AT RATED LOAD

Type of Engine	Factor
Simple D slide-valve engine	0.80
Simple piston-valve engine	0.82
Compound piston-valve engine	0.70
Simple Corliss engine, Releasing	0.90
Compound Corliss engine, Releasing	0.76
Condensing Uniflow, Poppet valve	0.78
Non-condensing Uniflow, Auxiliary exhaust valves	0.76

*Example 8-3* — Calculate the bore and stroke of a simple, two-cylinder piston-valve engine which is to develop 500 ihp per cylinder at 300 rpm when using steam at 250 psia and exhausting at 30 psia to a process steam main, if this power is to be developed when cut-off is at 50 per cent of the stroke. Neglect the piston rod effect. The bore is to be 80 per cent of the stroke.

*Solution* — Cut-off at 50 per cent of the stroke makes the ratio  $r=2$ . By equation 8-4,

$$\begin{aligned} \text{Theoretical mep} &= \frac{250}{2}(1 + \log_e 2) - 30 \\ &= 182 \text{ psi} \end{aligned}$$

If the diagram factor of 0.82 taken from Table 8-1 is used, the actual predicted mep is then  $0.82 \times 182 = 149$  psi.

In expression 8-1a,  $L$  will be  $\frac{D}{0.8(12)}$  and  $A$  will be  $\frac{\pi D^2}{4}$ , where  $D$  is in inches (Check units carefully). Then,

$$\text{ihp} = \frac{(2)(149)(\pi)(D^3)(300)}{(33,000)(8)(12)(4)} = 500$$

When this equation is solved for  $D$ , the bore is found to be 13.1 in., and the stroke is then computed to be 16.4 in. Ans

*Example 8-4* — Suppose that the engine in Example 8-3 is built with the computed bore and stroke, and has a clearance volume of 7%. How many pounds of dry saturated steam at 250 psia will fill the head end of the cylinder at the point of cut-off, which is at 50 per cent of the stroke?

*Solution* — Make a sketch, such as that shown in Fig. 8-12. Place the clearance next to the axis of ordinates, and then place the piston displacement as shown. Calculate the piston displacement from the bore and stroke.

Thus,  $\frac{\pi}{4}(13.1) \cdot \frac{16.4}{1728} = 1.279$  cu ft; the clearance is  $0.07 (1.279) = 0.090$  cu ft; and

the total volume is  $0.090 + 1.279 = 1.369$  cu ft. Since cut-off occurs at 50% of stroke, the volume at this point is  $0.090 + 0.50(1.279) = 0.730$  cu ft. From the steam tables, the specific volume of dry saturated steam at 250 psia is 1.8438 cu ft per lb. Then,

$$\frac{0.730}{1.8438} = 0.396 \text{ lb}$$

Ans.

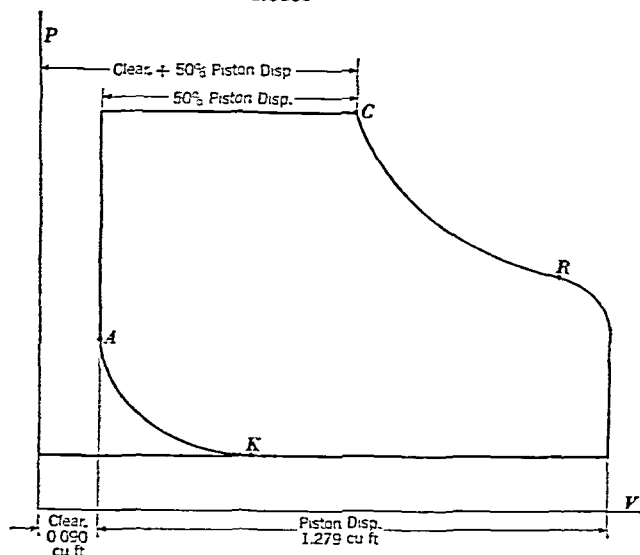


Fig. 8-12. Diagram for Example 8-4

By calculating the amount of cushion steam trapped in the clearance space on compression, and deducting it from the total amount present at cut-off, the cylinder feed can be computed. Such an analysis, however, makes no provision for leakage; an exact determination of the steam consumption to include leakage would involve very accurate flow metering of the steam supplied. A fairly accurate determination that neglects leakage *from* the engine but includes leakage *within* the engine (such as past the piston between the head and crank ends) is made by accurately weighing the exhaust steam after it has been condensed in a surface condenser. This is the method most frequently used for determining steam consumption.

★8-6. Performance Curves.—Fig. 8-13 shows a set of typical performance curves of a reciprocating steam engine under test conditions. Most of the curves are self-explanatory and should be carefully scrutinized by the student who can gain some training in analysis of the variables involved. One salient fact is that the speed curve as shown is practically

horizontal, with a slight drop as the load is increased, this shows good governing. The Willans line as shown is straight, this line is straight when the governing is by throttling with a fixed cut off, and indicates a linear relationship between the total steam consumption, in lb per hr, and the horsepower if the steam pressure is varied to suit the load. This fact

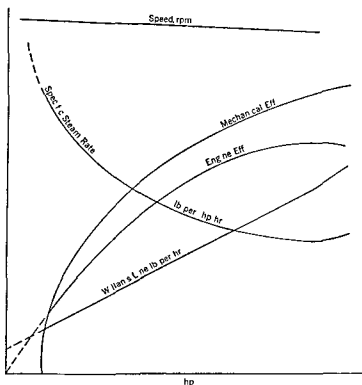


Fig 8-13 Performance Curves of Reciprocating Steam Engine

is also of great advantage in predicting the part-load steam consumption of steam turbines, most of which use throttle governing. If the steam consumption is plotted for an automatic engine with cut-off governing, the line will not be strictly straight, likewise, for steam turbines that use nozzle control, the line will become steeper whenever another nozzle valve is opened to admit additional steam to carry additional load. For further discussion of this control, see Chapter 10.

## CHAPTER 9

### FLOW IN NOZZLES

**9-1. Kinetic Energy in Steady Flow.**—In Chapter 4, the general equation for energy changes during steady flow was discussed in general, and there were mentioned various illustrations of its application to thermodynamic apparatus wherein several of the terms in the equation were predominant and others were of little or no importance. Flow in nozzles is a particular application which is concerned with two energy terms, enthalpy and kinetic energy. The reason is that steady flow through a nozzle, an orifice, an *insulated* pipe or a throttling device of any sort, such as a valve, is an adiabatic operation, though not necessarily an isentropic (reversible) type of adiabatic. In the case of a moving fluid, the energies and properties of the fluid will be considered at two positions, 1 and 2. When flow such as that in a turbine nozzle is encountered, the enthalpy change is so large that, since the flow is adiabatic, the general energy equation for steady flow may be used as follows:

$$\text{Enthalpy}_1 + \text{Kinetic Energy}_1 = \text{Enthalpy}_2 + \text{Kinetic Energy}_2$$

$$\text{or} \quad Wh_1 + \frac{W(\text{Vel}_1)^2}{2gJ} = Wh_2 + \frac{W(\text{Vel}_2)^2}{2gJ} \quad (9-1)$$

For 1 lb of fluid, this equation becomes

$$h_1 + \frac{\text{Vel}_1^2}{2gJ} = h_2 + \frac{\text{Vel}_2^2}{2gJ} \quad (9-1a)$$

This is called by some writers the "equation for the continuity of energy." Frequently, one or the other of the velocities is so low that the quantity  $2gJ$  in the denominator (which equals 50,000) makes the kinetic energy term negligible. As the square root of 50,000 is 223.8, it is seen that a velocity of 223.8 ft per sec gives a kinetic energy of 1 Btu per lb; so lower velocities can usually be neglected by comparison with the change in enthalpy. If, as frequently occurs, the velocity with which a fluid approaches a nozzle is low, equation 9-1a becomes

$$h_1 = h_2 + \frac{\text{Vel}_2^2}{2gJ} \quad (9-2)$$

If the velocity resulting from a decrease in enthalpy is the prime objective, this equation can be conveniently handled in the form

$$\text{Vel}_2 = 223.8 \sqrt{h_1 - h_2} \quad (9-2a)$$

In case the velocity is negligible at point 1, that at point 2 can be readily calculated if the properties of the fluid are known at the two points. Since  $P$  and  $T$  can be measured,  $v$  and  $h$  can be computed and the velocity calculation can then be made.

**9-2 Continuity of Mass Flow**—When an incompressible fluid flows steadily through a closed pipe, both the weight and the volume passing a given point per unit of time are constant. When the fluid is compressible, the weight per unit of time is still constant (otherwise there would be an accumulation or diminution of fluid), but the effects of pressure and temperature on the specific volume cause a variation in the number of cubic feet per second flowing. For a given steady flow, then, of  $W$  pounds per second,

$$W = \frac{A_1 \text{ Vel}_1}{t_1} = \frac{A_2 \text{ Vel}_2}{t_2} \quad (9-3)$$

in which  $A_1$  and  $A_2$  are the areas and  $t_1$  and  $t_2$  are the specific volumes. This equation is often termed the "equation of the continuity of mass." The two continuity equations form the basis of a theoretical approach to a study of nozzle characteristics.

*Example 9-1*—Air moves past point 1 in a pipe with a velocity of 447.6 ft per sec, a pressure of 50 psia, and a temperature of 120 F, at point 2 the pressure is 30 psia and the temperature is 50 F. Determine the velocity at point 2 and the cross-sectional area at each point if the quantity of air flowing is 100 lb per sec.

*Solution*—If it is assumed that  $h = c_p T = 0.24T$  for air, equation 9-1a gives

$$0.24(580) + \frac{(447.6)^2}{50,000} = 0.24(510) + \frac{\text{Vel}^2}{50,000}$$

from which the velocity at point 2 is found to be 1020 ft per sec \* Ans

Applying the characteristic equation and solving for the specific volume at each point we get

$$t_1 = \frac{RT_1}{P_1} = \frac{53.3(580)}{50(144)} = 4.30 \text{ cu ft per lb}$$

and

$$t_2 = \frac{53.3(510)}{30(144)} = 6.29 \text{ cu ft per lb}$$

The cross-sectional areas are found by applying equation 9-3. Thus,

$$W = 100 \text{ lb per sec} = \frac{A_1(447.6)}{4.30} = \frac{A_2(1020)}{6.29}$$

from which  $A_1 = 0.960$  sq ft and  $A_2 = 0.617$  sq ft Ans.

**9-3 Nozzle Efficiency**—As stated in Art. 9-1, the adiabatic flow of a fluid through a pipe or nozzle can be an adiabatic operation that is to a certain extent irreversible, that is, an adiabatic with an increase in entropy. The extreme of this is an operation of throttling, which is completely

\* For accuracy and ease of solution, the continuity of energy equation should be worked by collecting terms, and taking  $h_1 - h_2 = c_p(T_1 - T_2)$ .



irreversible with no change in  $h$ , as shown in Chapters 4, 6, and 7 thus far. The other extreme, or the reversible adiabatic where no change in entropy takes place, would naturally give the maximum change in  $h$ ; therefore, there would also be the maximum change in velocity. Since the usual function of a nozzle in thermodynamics is to effect a change in velocity, the *isentropic* flow is taken as applying to a nozzle that is 100% efficient at changing enthalpy into kinetic energy. Thus,

$$\text{Nozzle Eff.} = \frac{\text{Actual Kinetic Energy at Exit}}{\text{Theoretical Kinetic Energy at Exit}} \quad (9-4)$$

If the entering velocity is negligible, this equation reduces to:

$$\text{Nozzle Eff.} = \frac{\text{Actual Change in Enthalpy}}{\text{Isentropic Enthalpy Change from Original Conditions}} \quad (9-4a)$$

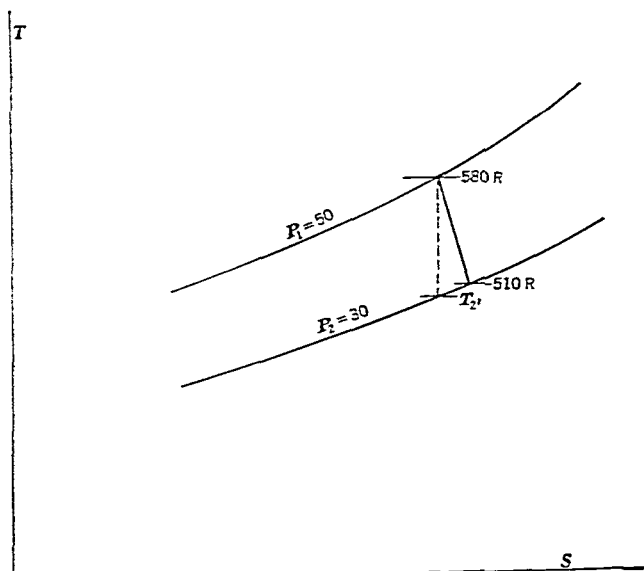


Fig. 9-1. T-S Diagram for Example 9-2

**Example 9-2.**—Considering the pipe in Example 9-1 as a nozzle, determine what velocity would occur at point 2 if the nozzle were 100% efficient. Also determine its efficiency from the true conditions.

**Solution.**—In Fig. 9-1 is shown a  $T$ - $S$  sketch representing the conditions already known. The unknown temperature is that at  $P_2$  which would be reached by isentropic expansion from  $P_1$  and  $T_1$ . Since a reversible adiabatic follows the law  $PV^k = C$ , the temperature  $T_{2'}$  is found from the temperature-pressure relationship. Thus,

$$T_{2'} = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 580 \left( \frac{30}{50} \right)^{0.286} = 501 \text{ R, or } 41 \text{ F}$$

Substituting 501 for 510 in the previous example gives  $Vel. = 1071$  ft per sec, as contrasted with the previously determined value of 1020 Ans

As the kinetic energies are proportional to  $Vel^2$ ,

$$\text{Nozzle Eff} = \left( \frac{1020}{1071} \right)^2 = 0.905, \text{ or } 90.5\% \quad \text{Ans.}$$

By contrast, it should be observed that, had there been *no approach velocity to consider*, equation 9-4a would have given

$$\text{Nozzle Eff} = \frac{0.24(580-510)}{0.24(580-501)} = 0.886, \text{ or } 88.6\%*$$

**9-4. Nozzle Forms**—Up to this point, any use made of equation 9-3 has not considered what variations in area should be made when an entire series of stations are taken in equal increments of pressure change in order to obtain the maximum velocity that a nozzle can produce for the conditions indicated. Let it be required to find the cross-sectional areas of a perfect nozzle expanding 1 lb of steam per second from a position of negligible velocity at 100 psia and 1000 F down to 20 psia, the area being determined where the pressure is a multiple of 10 psi.

From the superheat tables, the initial conditions give  $h=1530.8$ ,  $v=8.656$ , and  $s=1.9193$ . Of these values,  $s$  will remain constant, since this is to be a nozzle with an efficiency of 100%. When this value of  $s$  is used at 90 psia,  $h=1514.1$ ,  $v=9.40$ , and the velocity is computed as follows

$$Vel = 223.8 \sqrt{1530.8 - 1514.1} = 915 \text{ ft per sec}$$

The area is then

$$A = \frac{1(9.4)}{915} = 0.01028 \text{ sq ft}$$

In a similar manner, values are computed for every pressure drop of 10 psi. The results are shown in Table 9-1.

TABLE 9-1  
CALCULATED DATA ON PERFECT NOZZLE

$P$	$h$	$v$	$Vel$	$A$
100	1530.8	8.65	0	
90	1514.1	9.40	915	0.01028
80	1496.2	10.30	1318	0.00782
70	1476.2	11.36	1655	0.00686
60	1453.4	12.95	1968	0.00653
50	1427.7	14.87	2272	0.00634
40	1398.0	17.69	2578	0.00686
30	1361.3	22.09	2912	0.00758
20	1313.9	30.18	3295	0.00915

\*The use of equation 9-4a has become so widespread that many texts do not mention equation 9-1.

The tabulated values have special significance when plotted as in Fig. 9-2. In this plot, pressures have been used as abscissas; and specific volume, velocity, and area have been used as ordinates. Examine the shape of the curves closely. It will be seen that the specific volume increases as the pressure decreases, according to the law  $Pv^k = C$ ; hence, the value of  $v$  increases very slowly at first. Contrary to this, the velocity increases at a rapid rate during the early part of the pressure drop. However, as the pressure continues to drop, the rate of change of specific volume continues to increase, while the rate at which velocity changes soon becomes practically constant. The resultant effect on the area is as follows:

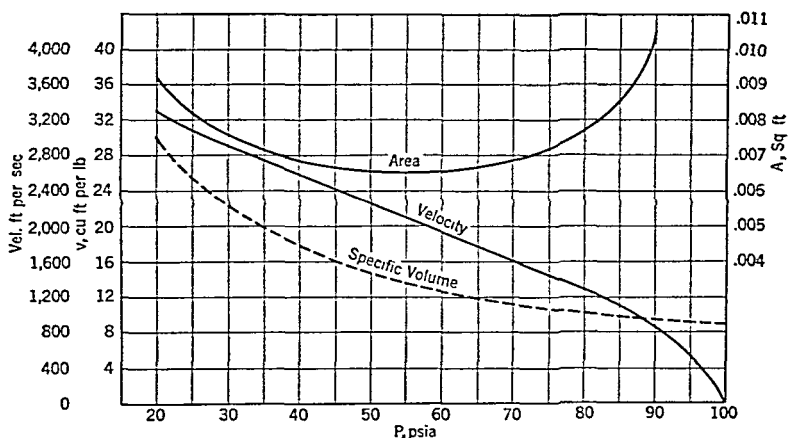


Fig. 9-2. Curves Showing Data of Table 9-1

While the velocity is increasing faster than the specific volume, the area must decrease; *but*, when the specific volume increases faster than the velocity, room must be made and the area increases. Accordingly, when a nozzle is designed with areas as indicated, it is spoken of as "convergent-divergent."

If the steam in the foregoing example had been expanded from 100 psia only to an exit pressure of 60 or 70 psia, the nozzle would have been merely convergent. Note carefully the relationship between the pressure and area in Fig. 9-2, and see where the minimum cross-sectional area occurs. The term "throat" is usually applied to the minimum area of a convergent-divergent nozzle.

If a fluid is scarcely compressible (as is a liquid), there will be no increase in specific volume as the pressure drops; hence, a nozzle for liquids should be only convergent.

In Figs. 9-3 and 9-4 are shown actual nozzle profiles for, respectively, a vapor (or gas) expanding from a high pressure to a low (less than 50 per

cent of high) pressure, and a vapor expanding to a pressure only slightly less than the high pressure. Comparing Figs 9-2 and 9-3, which pertain to the same set of conditions, let it be remembered that in Fig 9-2 area are plotted against equal increments of pressures, while Figs 9-3 and 9-4 are actual nozzle shapes where the ordinates are diameters and the abscissas are lengths. The length is not directly proportional to the pressure along the abscissa because a nozzle laid out for equal pressure drops would have poor fluid dynamic characteristics.

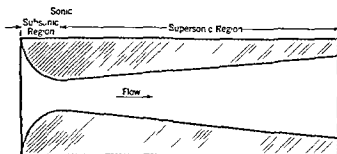


Fig 9-3 Profile of Typical Convergent Divergent Nozzle

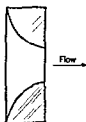


Fig 9-4 Profile of Convergent Nozzle

Both friction, affecting nozzle efficiency and ease of manufacture must be considered, a nozzle with a straight line divergent section can be easily reamed for smoothing. Experience has shown that the divergent part of a nozzle will give the best efficiency if the taper of the sides is about 1 in 10 on diameters, experience has also shown that a minimum friction loss will occur in the rounded entrance section (between high pressure entrance and the minimum cross-section) if this section is made very short, say one-twentieth of the length of the divergent section. Since the velocity in this short entrance section is lower than that in the exit section, an abrupt change of area is permissible.

The cross-section may be circular, elliptical, square or rectangular. For steam turbine work rectangular areas are favored as the steam can be made to conform better to the blade passages into which it is directed, for additional discussion on this feature, see the next chapter.

**9-5 Limitation of Discharge**—Up to this point, no mention has been made of the influence of pressure on rate of discharge, beyond that of calculating the areas at various pressures for a given weight flowing per unit of time. The question now is: What does changing the exit pressure do to the flow? If the exit pressure were to be raised until it equaled the initial pressure there could be no flow at all unless there were an initial velocity. Fig 9-5 shows what a given nozzle, such as the one used in the

previous illustrative example, will do if the back (exit) pressure is varied from the one extreme of equaling the initial pressure to the condition (20 psia) used in the example. To facilitate understanding the principles of this article, the ratio of back pressure to initial pressure, or  $\frac{P_2}{P_1}$ , is used as the abscissa in Fig. 9-5. When  $P_2 = P_1$ , the ratio is unity and flow ceases; when the ratio is a little more than 0.5, the maximum flow is attained, and the flow remains constant as the pressure ratio is further decreased. The

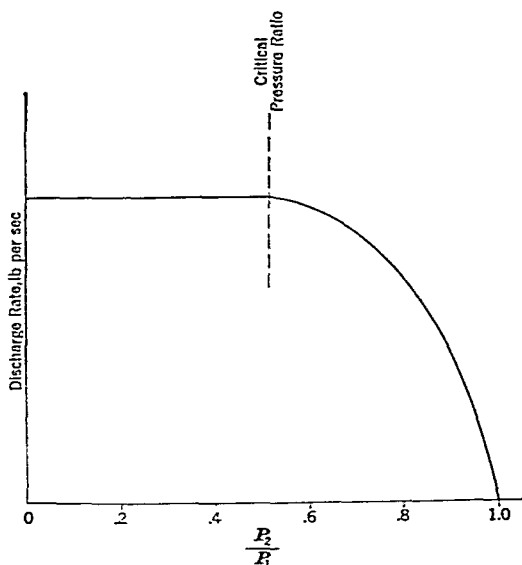


Fig. 9-5. Effect of Exit Pressure on Flow

reason for this is as follows: When the velocity reaches a high value, a pressure\* wave can no longer make itself felt to influence the discharge. A pressure wave is transmitted through a fluid with the velocity of sound; at the throat of the nozzle, the velocity of the moving fluid reaches that of sound, and hence the back pressure cannot send a wave up-stream. The conditions are analogous to those when a man attempts to swim up-stream; if the man's swimming speed in still water is equaled or exceeded by the speed of the stream, he cannot swim up-stream. Hence, the back pressure cannot send an opposing wave up-stream if the stream velocity exceeds that of sound. Note again Fig. 9-3 and the velocity curve of Fig. 9-2, and take cognizance of which part of the nozzle is in the subsonic region

\* With any disturbance in the back pressure, a pressure wave tends to move up-stream in an attempt to equalize pressures.

and which is in the supersonic region. Reference to any standard physics text will give the velocity of sound as

$$\text{Acoustic velocity} = \sqrt{P/\rho} = \sqrt{RTgk} \quad (9-5)$$

where the symbols have the usual significance used in this text.

The pressure at which the acoustic velocity is reached by isentropic expansion is related to the initial pressure as follows:

$$\frac{P_c}{P_1} = \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (9-6)$$

where  $P_c$  is the throat pressure, or nozzle critical pressure; this is called the nozzle critical pressure because at this point the specific volume starts to increase faster than the velocity increases. The derivation of this expression is reserved for a subsequent section of the text.

*Example 9-3*—For the conditions in the example in Art. 9-4, at what pressure should the throat of the nozzle occur, and what should be the acoustic velocity?

*Solution.*—As  $k$  for superheated steam is about 1.3,

$$\frac{P_c}{P_1} = \left( \frac{2}{1.3+1} \right)^{\frac{1.3}{1.3-1}} = 0.545$$

Since  $P_1 = 100$  psia,  $P_c = 54.5$  psia

Ans

According to the curve of specific volume in Fig. 9-2,  $v$  at the throat pressure is 13.9 cu ft per lb. (This could also be found by isentropic  $Pv$  relations.) Also, it is seen from Fig. 9-2 that the minimum cross-sectional area occurs at this pressure. By formula 9-5,

$$\begin{aligned} \text{Acoustic velocity} &= \sqrt{54.5 \times 144 \times 13.9 \times 32.2 \times 1.3} \\ &= 2140 \text{ ft per sec} \end{aligned}$$

Ans

This value also can be read approximately from the velocity curve at the throat pressure.

It is often asked why expansion is carried below the throat pressure if the flow is not increased thereby. It is necessary to remind one's self that the quantity of flow is determined by the throat but that the velocity is increased by expansion to a lower pressure and that the kinetic energy, which is a function of velocity squared, is increased by expansion into the supersonic region.

To reiterate, a nozzle for expansion to a pressure above the critical ratio indicated by equation 9-6 should be merely convergent; the velocity will remain subsonic and the quantity of flow will be affected by the back pressure. A nozzle for expansion to a pressure lower than the critical ratio should be convergent to the throat, and then divergent; the quantity of flow will be determined by the throat pressure and area, not by the final exit pressure.

\* Provided that no appreciable velocity exists at  $P_1$ .

★9-6. Derivation of Nozzle Critical Pressure Ratio.—Since  $h = u + \frac{Pv}{J}$ , differentiating gives

$$dh = du + \frac{P}{J} dv + \frac{v}{J} dP$$

Since conditions are adiabatic,  $dQ = 0$ . Hence,  $du + \frac{P}{J} dv = 0$  and

$$dh = \frac{v}{J} dP$$

If this be integrated with  $Pv^k = C$ , then

$$h_1 - h_2 = \frac{k}{J(k-1)} (P_1 v_1 - P_2 v_2)$$

Substituting in equation 9-2 gives

$$\frac{\text{Vel}_2^2}{2g} = \frac{k}{k-1} (P_1 v_1 - P_2 v_2)$$

from which

$$\text{Vel}_2 = \sqrt{\frac{2gk}{k-1} (P_1 v_1 - P_2 v_2)} \quad (9-2b)$$

Now consider that  $\frac{W}{A} = \frac{\text{Vel}}{v}$ , and that this ratio is a maximum where the area is a minimum, i.e., at the throat. There,  $v = v_c$ ,  $\text{Vel} = \text{Vel}_c$ , and  $P = P_c$ . Then,

$$\text{Vel}_c = \sqrt{\frac{2gk}{k-1} (P_1 v_1 - P_c v_c)}$$

or

$$\begin{aligned} \frac{W}{A} = \frac{\text{Vel}_c}{v_c} &= \sqrt{\frac{2gk}{v_c^2(k-1)} (P_1 v_1 - P_c v_c)} \\ &= \sqrt{\frac{2gk}{v_c^2(k-1)} P_1 v_1 \left(1 - \frac{P_c}{P_1} \frac{v_c}{v_1}\right)} \\ &= \sqrt{\frac{2gk}{v_c^2(k-1)} P_1 v_1 \left[1 - \frac{P_c}{P_1} \left(\frac{P_c}{P_1}\right)^{-\frac{1}{k}}\right]} \end{aligned}$$

$$\text{Since } v_c^2 = v_1^2 \left( \frac{P_1}{P_c} \right)^{\frac{2}{k}},$$

$$\frac{W}{A} = \sqrt{\frac{2gl}{k-1} \frac{P_1}{v_1} \left[ \left( \frac{P_c}{P_1} \right)^{\frac{k+1}{k}} - \left( \frac{P_c}{P_1} \right)^{\frac{k+1}{k}} \right]}$$

Since this relation is for a maximum value of  $\frac{W}{A}$ , the pressure conditions when this maximum will be obtained are found by differentiating, with respect to  $P_c$ , the part of the expression pertaining to  $P_c$  and placing the first derivative equal to zero. Thus,

$$\frac{d}{dP_c} \left[ \left( \frac{P_c}{P_1} \right)^{\frac{2}{k}} - \left( \frac{P_c}{P_1} \right)^{\frac{k+1}{k}} \right] = 0$$

from which

$$\frac{P_c}{P_1} = \left( \frac{2}{k+1} \right)^{\frac{1}{k-1}}$$

As previously stated, this ratio depends on the assumption of no entering velocity and no friction, but, as the velocity at entrance and the friction to the throat generally are small, this ratio can be used without serious error.

**9-7. Flow of Wet Steam; Supersaturation**—Up to this point no mention has been made of the flow of steam which was initially wet or which might be expected to become wet during its expansion. It has been known for many years that such steam does not remain in equilibrium during its expansion. However, the necessity of accounting for the effects of this non-equilibrium expansion on steam-turbine performance was not recognized for some time. Two seemingly incongruous test results pointed out the necessity of studying the expansion of steam when condensation is expected during the expansion. First, it was found that the amount of steam flow, when the steam was initially of very high quality or slightly superheated, was actually greater than that calculated by the assumption of equilibrium expansion in a frictionless nozzle. In addition, it was found that the efficiency of that part of the turbine where there was little or no superheat was a little less than that predicted for equilibrium expansion.

Condensation is a time-taking process. It appears that in the absence of nuclei, such as dust particles, a few molecules of steam must come together to form a nucleus. This nucleus attracts other molecules of steam and grows into a very small droplet. This formation of a droplet may take place in a very short time, but it must be realized that the time



required for steam to pass through a nozzle is only a very small fraction of a second. Thus, condensation will not occur so near the entrance of the nozzle as might be expected. In some cases, steam may not start to condense until after it leaves the nozzle.

Expanding steam should partially condense to remain in equilibrium. If it does not condense at all during its expansion, its increase in kinetic energy as a mass must come from the molecular kinetic energy, because there is no condensation to release molecular potential energy. This increased reduction in molecular kinetic energy is accompanied by a greater

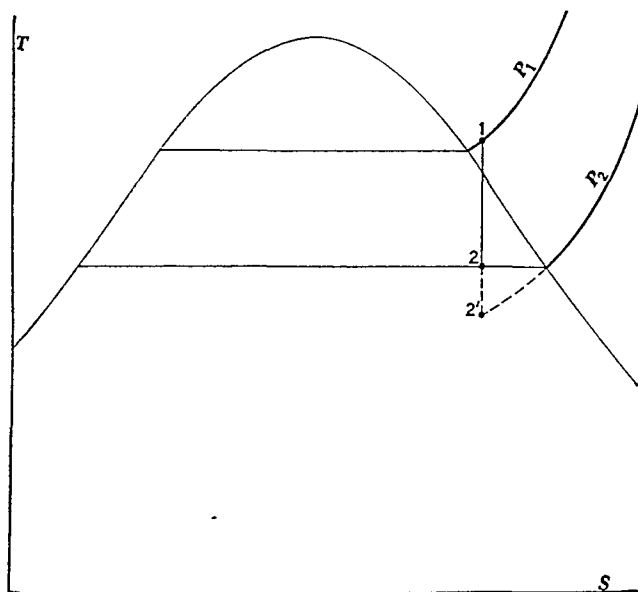


Fig. 9-6. Effect of Undercooling on Steam Undergoing Expansion Under Supersaturation Conditions

reduction in temperature than would have occurred if condensation had taken place. Such steam exists at a temperature below its normal saturation temperature. In this state, it cannot be in equilibrium. It may then be said to be metastable or supersaturated. Because it is at a temperature below its saturation temperature for the given pressure, it is often referred to as undercooled steam.

Steam expanding without condensation behaves like superheated steam, and may be treated as such. A constant-pressure line on the  $T$ - $S$  plane is horizontal in the saturated region. If there is supersaturation, the steam cannot be saturated. Hence, the constant-pressure line is no longer horizontal, but becomes an extension of the curved line in the superheated

region. Note the dotted extension of line  $P_2$  in Fig 9-6. If an isentropic equilibrium expansion occurs between an initial state 1 and a final pressure  $P_2$ , state 2 in Fig 9-6 is reached. However, if the expansion is a metastable one to pressure  $P_2$ , then the state after expansion is 2. In a like manner the metastable expansion shown on the  $h$ - $S$  plane, Fig 9-7, will end at state 2 instead of 2. The final enthalpy at the end of the metastable expansion is higher than that for stable expansion. This smaller change in enthalpy for the supersaturated expansion results from lack of condensation to release molecular potential energy. Because of a smaller change in

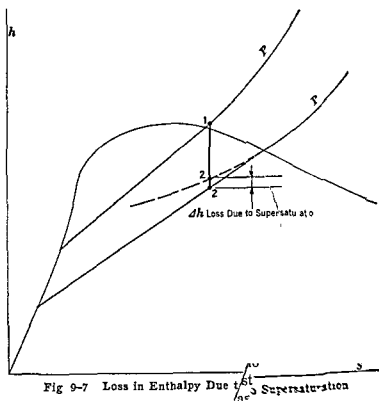


Fig 9-7 Loss in Enthalpy Due to  $\frac{st}{ac}$  Supersaturation

*Example 9-4.*—Steam expands isentropically through a nozzle from initial conditions of 400 psia and 450 F to atmospheric pressure. Calculate the quantity discharged in pounds per second by: (a) equilibrium flow and (b) supersaturation flow. The throat area is 1 sq in.

*Solution.*—If it is assumed that the throat pressure is in each case  $0.545 \times 400 = 218$  psia, Fig. 9-8 shows the throat temperature under each set of conditions. Although saturated steam has a value of  $k$  of 1.135, and hence has a nozzle critical throat pressure ratio of 0.578, it is customary to use the superheated value of 0.545 if the steam is superheated before the expansion starts.

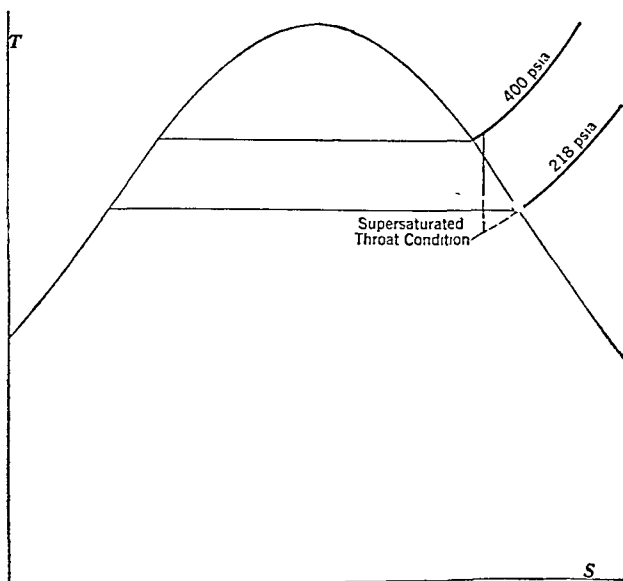


Fig. 9-8. T-S Diagram for Supersaturated Steam

initial set of properties may be taken as follows:  $v = 1.1744$ ,  $h = 1208.8$ , and  
 2. Then the moisture at the throat pressure of 218 psia will be  $\frac{1.5380 - 1.4892}{0.9853} =$   
 and  $v = 2.002$  and  $h = 1158.1$ . Hence,

$$\text{Throat Vel} = 223.8 \sqrt{1208.8 - 1158.1} = 1595 \text{ ft per sec}$$

equilibrium flow,

$$W = \frac{1}{144} \frac{1595}{2.002} = 5.54 \text{ lb per sec}$$

Ans.

supersaturation flow, the specific volume at the throat is

$$v = 1.1744 \left( \frac{400}{218} \right)^{\frac{1}{1.5}} = 1.872 \text{ cu ft per lb}$$

By equation 9-2b,

$$\text{Vel} = \sqrt{\frac{(64.4)(1.3)(144)}{1.3 - 1} [(400)(1.1744) - (218)(1.872)]}$$

$$= 1574 \text{ ft per sec}$$

and

$$W = \frac{1}{144} \frac{1574}{1.872} = 5.84 \text{ lb per sec}$$

Ans

Note that there is little percentage difference between the velocities as computed by the two methods, but that there is considerable difference between the specific volumes

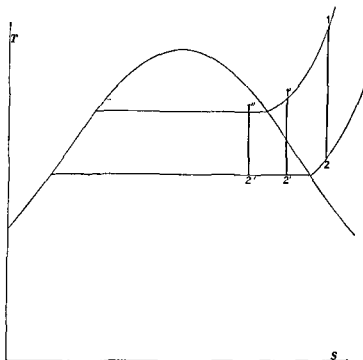


Fig 9-9 Effect of Initial Conditions of Steam on Final Condition

Two questions arise concerning supersaturation in conjunction with turbine design:

1. When must it be taken into account?
2. How can it be taken into account?

The answer to the first question is tied up with the initial state of the steam and with the amount of the expansion. Expansion 1-2 shown in Fig 9-9 is wholly in the superheat region. Hence, the steam cannot be supersaturated. Expansion 1'-2' crosses the saturation line, and as condensation should occur, supersaturation will exist. Expansion 1''-2'' takes

place wholly in the saturated region, and supersaturation will exist.\* In general, supersaturation will exist at that point in the nozzle where condensation is expected.

As the degree of supersaturation† increases (*i.e.*, the further away the steam is from saturation), the greater will be the force tending to restore the steam to equilibrium conditions. When the degree of supersaturation reaches a certain limit, condensation will start. The nature of the recovery process is not fully understood, but it appears both from an analytical study and from experimental work that there is a pressure rise when the very small drops of water are formed. As much of the original work on supersaturation was done by Wilson over fifty years ago, his name has been applied to the line where recovery starts to take place. Various authorities have located the Wilson line as a line on the  $h$ - $S$  diagram approximately paralleling the saturation curve. Yellott‡ places this line between the lines for 3 and 4 per cent of moisture at pressures below 80 psia.

No hard and fast rules can be given for determining when full recovery has been reached in a nozzle. Some authorities believe that complete recovery is never reached in a nozzle, but there is disagreement on this point. Where other information is lacking, the following is suggested.

1. If condensation is expected at the throat of a convergent-divergent nozzle or at the mouth of a convergent nozzle, treat the flow up to that point as supersaturated. Normally, the effects of friction, particularly on specific volume, up to this point are not large. As a close approximation of the true velocity at this point, it is suggested that the velocity calculated for supersaturation be multiplied by 0.98.

2. Assume that equilibrium is restored at the exit of a convergent-divergent nozzle. This assumption is a reasonable one, as the length of the nozzle between the throat and the mouth is large and a relatively long time is available for recovery to take place. The nozzle exit conditions are determined by applying a nozzle efficiency to stable expansion, as was done earlier in this chapter. The nozzle efficiency to be used is for conditions in the nozzle under calculation and includes the effects of the loss due to supersaturation.

\* Some authorities believe that this expansion will be an equilibrium one, because the original drops of water will act as nuclei and will cause immediate condensation. This is not true, since the drops, being at a high temperature, have a tendency to vaporize.

† The ratio of the pressure of the supersaturated vapor to the saturation pressure corresponding to the supercooled temperature is known as the "degree of supersaturation." Thus, in Example 9-4 the temperature of the supersaturated steam at the throat would be  $\frac{T_1 P_2^{1/2}}{P_1^{1/2}} = 791$  R, or 331 F. Since the saturation pressure corresponding to 331 F is 104.5 psia, the degree of supersaturation is  $\frac{218}{104.5} = 2.08$ .

‡ J. I. Yellott, Jr., "Supersaturated Steam," *Trans. A.S.M.E.*, FSP-56-7, 1934.

**Example 9-5**—Calculate the exit area for the nozzle in Example 9-4 if the flow through the throat is as found by supersaturation but assume that recovery to stable conditions is reached at the exit. The nozzle's over all efficiency is 94%.

**Solution**—In Fig 9-10 point 2 shows the actual condition at 14.7 psia and 2 represents the isentropic condition. By isentropic conditions, the moisture at point 2 is  $\frac{1.7566 - 1.4892}{1.4446} = 0.185$ . Hence,

$$h_2 = 1150.4 - (0.185)(970.3) = 970.9$$

The available energy would then be  $1208.8 - 970.9 = 237.9$  but the nozzle efficiency of 94% means that 6 per cent of the available energy is returned to enthalpy. Therefore  $h_2$  is really  $970.9 + 0.06(237.9) = 985.17$  (see Fig 9-11). For this condition the moisture is 17% and hence the specific volume is 22.25 cu ft per lb.

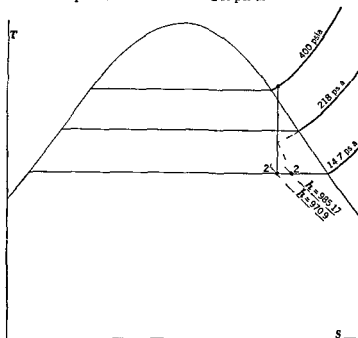


Fig 9-10 Diagram for Example 9-5

Nozzle inefficiency increases specific volume and decreases velocity as could be expected. The exit velocity is then

$$223.8 \sqrt{1208.8 - 985.17} = 3340 \text{ ft per sec}$$

and the area for 5.84 lb per sec is

$$\frac{5.84(22.25)(144)}{3340} = 5.60 \text{ sq in}$$

Ans

The foregoing calculation is typical. Sometimes, instead of quoting nozzle efficiency, there is given a *velocity coefficient* which represents the ratio of the true velocity to that obtained under isentropic conditions. Obviously, the velocity coefficient must be equal to the square root of the nozzle efficiency, since the kinetic energy is proportional to the square of the velocity.

When a nozzle is designed as in the foregoing examples for the conditions specified, the exit area is such that, if the nozzle is operated at a smaller rate of flow or operated to a higher back pressure, the nozzle is too long or is overexpanded. If the initial pressure were raised so that more flow would occur, or if the back pressure were greatly reduced, the nozzle would not be large enough at exit, or would be underexpanded, and there would be a pressure drop in the jet after it left the nozzle. Both overexpansion and underexpansion cause some energy loss; but, when a choice has to be made, overexpansion is far more serious and hence is to be avoided.

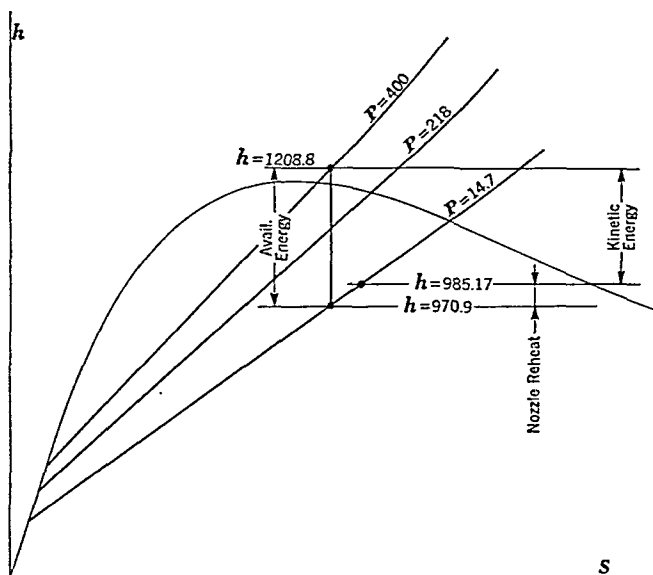


Fig. 9-11. Diagram for Example 9-5

As a result, the nozzle of a steam turbine which is to run at part load a large part of the time is apt to be designed for a certain amount of underexpansion; then, excellent part-load efficiency is obtained.

When moisture is present, the drops of liquid move much more slowly (because of the small change in enthalpy of the liquid) than does the dry saturated part of the vapor. Hence, they cause a loss because the vapor must exert energy on the droplets to push them along. This condition further lowers the velocity coefficient. In fact, the velocity coefficient for wet steam must always be thought of as being due to a combination of nozzle imperfections and their effect on the vapor plus the droplet lag.

In summation, nozzle losses can be classed as follows

(a) Friction between the nozzle and the fluid itself, which is affected by the nozzle's surface finish, its fluid dynamic characteristics, and its design in general—proportions, length, material, overexpansion or underexpansion, etc

(b) Supersaturation, which is certain to be encountered at the throat if saturated steam is used

(c) Droplet lag

Supersaturation and droplet lag are well avoided by the use of superheated vapors that remain in the superheat region

In further summation, the nozzle critical pressure ratio for air (and other gases for which  $k=1.4$ ) is 0.528, for steam initially superheated it is 0.545 and for steam initially wet it is 0.578

**★9-8 Jet Force**—If a nozzle is used as a jet for the purpose of producing a force, and complete expansion takes place, the force can be found as follows. Force equals mass times acceleration, and acceleration is the time rate of change of velocity. Then, for  $W$  lb per sec undergoing a velocity change of  $\Delta(\text{Vel})$ ,

$$F = \frac{W}{g} \Delta(\text{Vel}) \quad (9-7)$$

For a nozzle that is underexpanded (frequently in a jet-propelled airplane),

$$F = \frac{W}{g} \Delta(\text{Vel}) + A(P_i - P_o) \quad (9-8)$$

where  $A$  is the jet area and  $P_i - P_o$  is the pressure drop outside the nozzle



## CHAPTER 10

### STEAM TURBINES

**10-1. Field of Service.**—The steam turbine is supreme in the field of electric power generation. More electric power is generated in the United States by means of the steam turbine than by all other types of prime movers combined. Furthermore, with the exception of the aircraft field, it appears that the development of atomic power will be dependent on the steam turbine as the prime mover.

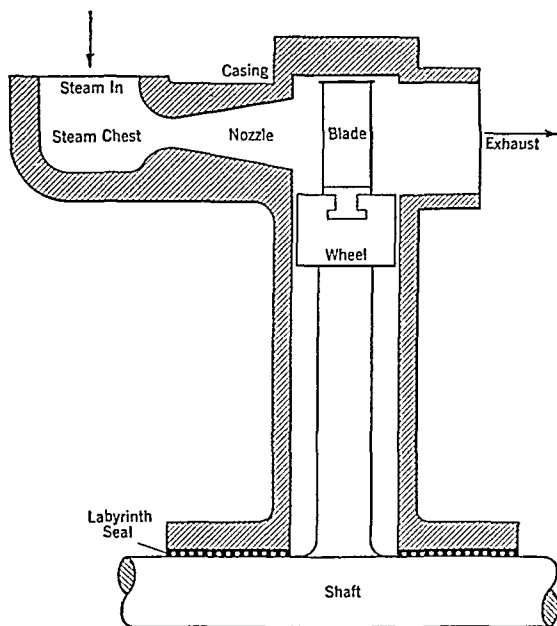


Fig. 10-1. Simple Steam Turbine

Steam turbines are made in sizes ranging from a few horsepower to units of approximately 275,000 horsepower. Steam turbines are used to drive many high-speed pieces of machinery, such as pumps, blowers, fans, and centrifugal compressors. The prime field of service, however, is for stationary electric generators and for large-size marine propulsion. In these fields, steam turbines have little competition from diesel engines and gas turbines for units above 5,000 hp, because the thermal efficiency of the large steam-turbine power plant approaches that of a diesel, and its boiler

burns a cheaper fuel The high speed of the large steam turbine means a compact unit with a low initial cost

Even in smaller sizes (up to 10,000) the steam turbine is used very extensively. Although these steam turbines are inherently less efficient than

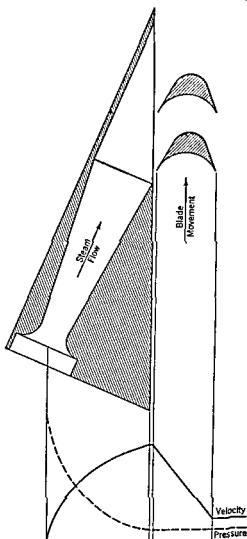


Fig 10-2 Nozzles and Blades of Simple Steam Turbine

diesel engines, the diesel engines cannot compete with steam turbines when the major portion of the turbine exhaust steam can be used for heating purposes

**10-2. Classification and Description**—In the case of a steam turbine there is transformation of enthalpy into kinetic energy in the nozzles

(See Chapter 9 for a discussion of this transformation.) The high-velocity steam issuing from the nozzle is directed onto so-called blades or buckets. Here, by virtue of a change in velocity of the steam, a force is exerted on the blades and work is done. The blades are mounted on disks or wheels to which is transmitted the energy received from the steam. From the wheels the energy passes to the shafts and thence away from the turbine.

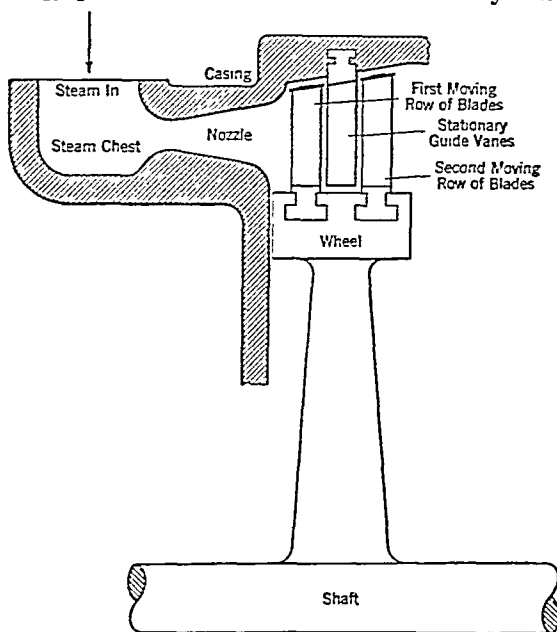


Fig. 10-3. Velocity-Compounded Steam Turbine

A diagram of a simple steam turbine, with the principal parts designated, is shown in Fig. 10-1. In Fig. 10-2 is shown a cross-section of the nozzles and blading for this simple turbine, together with a plot of the variations of the pressure and velocity of the steam as it passes through the turbine. This type of turbine is known as an impulse turbine. Specifically, an *impulse turbine* is one in which the pressure drop occurs in stationary nozzles. As will be shown later, the efficiency of the simple impulse turbine is very low. The efficiency may be improved by using either velocity compounding or pressure compounding or by using both.

The velocity-compounded turbine, often called the Curtis, is illustrated in Fig. 10-3; and in Fig. 10-4 is shown a cross-section of the nozzle and blades with a plot of the variations in pressure and velocity in the turbine. It should be noted that the pressure drop occurs wholly in the nozzles, the velocity being built up to its maximum value. The high-velocity

steam does work in the first row of blades and then enters the stationary blades (or guide vanes) There, the steam direction is reversed and the steam enters the second moving row (mounted on the same wheel as the first moving row) where additional work is done The turbine illustrated

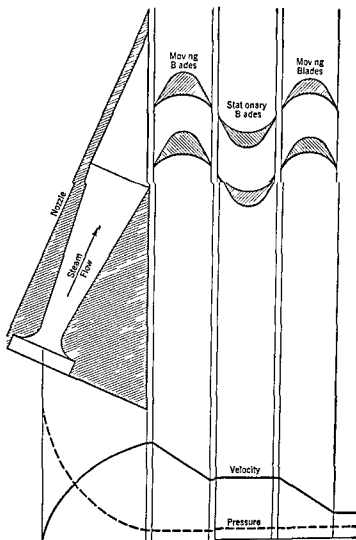


Fig 10-4 Nozzle and Blades of Velocity-Compounded Turbine

is a two-velocity-row turbine (*i.e.*, the velocity set up is utilized in two steps or rows), three-velocity-row turbines were tried out, but they had too many drawbacks, particularly that of low efficiency

Fig 10-5 shows a pressure-compounded turbine Only two stages of the turbine are shown, normally the turbine would have 6 to 15 or more stages Fig 10-6 shows the cross-section through the nozzle and blades,

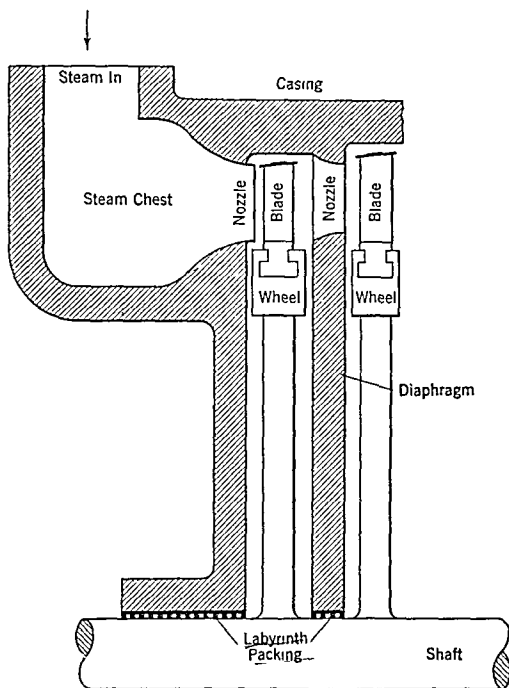


Fig. 10-5. Pressure-Compounded Steam Turbine

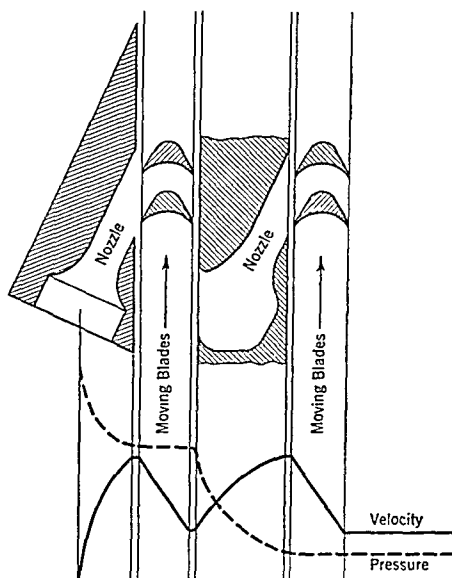


Fig. 10-6. Nozzles and Blades of Pressure-Compounded Turbine

as well as the variations in pressure and velocity. Each of the two pressure stages shown is a single-velocity stage. Such a turbine is called a Rateau turbine.

Many turbines have as a first stage a two-row velocity stage. This is followed by several pressure stages, each pressure stage being a single-velocity stage. When the exhaust steam is to be used for heating purposes and a compact turbine is desired, two or three pressure stages, each of which is a multi-velocity stage, are used in series.

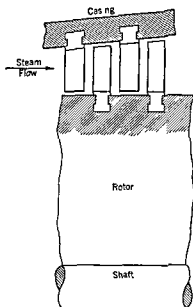


Fig 10-7 Mounting of Blades of Reaction Turbine

In a reaction turbine, there are no nozzles as such, but there is a series of blade rows. The passages between the blades are in the form of convergent nozzles and act as such. Every other row of blades is a stationary one. As indicated in Fig 10-7, the moving blades are mounted on a drum, rather than on wheels. The combination of a stationary row and a moving row of blades is known as a stage\*. In a reaction turbine, there are generally 30 to 40 or more stages. Fig 10-8 shows a cross-section of the blading, together with the pressure and velocity variations. Many reaction turbines have a two-row Curtis impulse stage followed by the reaction stages.

Because of the existence of steam at various pressures in a turbine, steam may be extracted from a turbine part way in the expansion from inlet pressure to exhaust pressure. The extracted steam may be used for process work in a plant, for building heating, or for regenerative feed-water heating. Such turbines are known as extraction bleeder turbines. If the exhaust steam is to be used for any heating purpose, the turbine is called a back-pressure turbine. In recent years, there have been many installations of a high-pressure (1400 to 1500 psi) turbine which exhausts at 200 to 300 psi and supplies steam to an existing turbine designed to take steam at such pressures. This high-pressure turbine is known as a superposed or "topping" turbine.

For constructional reasons, as well as to permit reheating, the full expansion of the steam may not be completed within one casing. Such

\* This is a general definition directly applicable to Rateau and reaction stages. For Curtis stages it should be interpreted as meaning a row of nozzles and the rows of blades moving and fixed, which utilize the velocity generated in those nozzles.

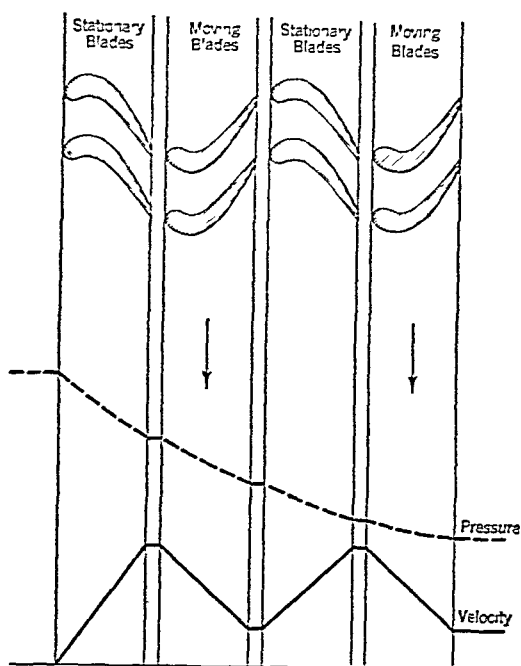


Fig. 10-8. Cross-Section of Blades of Reaction Turbine

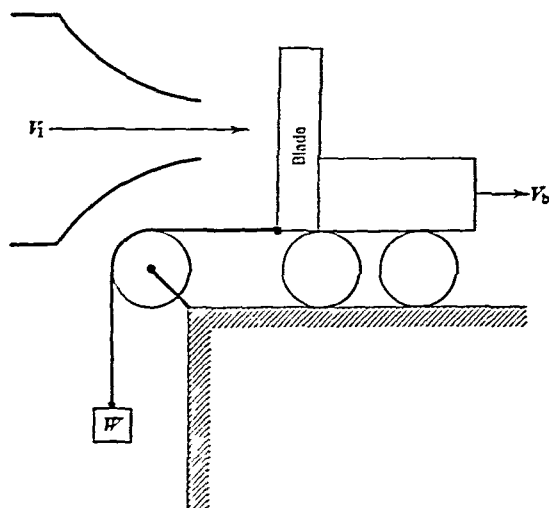


Fig. 10-9. Force Exerted by Steam Striking a Flat Blade

a turbine is called a compound one, it is a tandem-compound machine in case the shaft extends through the two casings, and is a cross-compound turbine if the casings are side by side with individual shafts

When a turbine receives large quantities of steam, particularly at low pressures, the volume per unit of time becomes so great that the steam flow is divided up, each part going to a separate casing. Such a turbine is a double-flow turbine. The double-flow principle is also used in reaction turbines to minimize end thrust.

**10-3. Theory of Impulse Blading**—Work is done on the blades of a steam turbine by virtue of the force exerted on them by the steam as it changes in velocity.\* This force equals

$$F = \frac{W'}{g} a = \frac{W'}{g} \frac{dV}{dt} = \frac{W'}{g} \frac{\Delta V}{\Delta t} = \frac{\Delta W'}{\Delta t} \frac{\Delta V}{g} = \frac{W \Delta V}{g} \quad (10-1)$$

where  $F$  is the force in pounds,  $\Delta W'$  is the weight of steam, in pounds, supplied in a time  $\Delta t$ ,  $W$  is the rate of steam flow, in pounds per unit of time, and  $\Delta V$  is the change in the velocity of the steam in a plane parallel to the motion of the blade on which the steam acts.

In Fig. 10-9 the steam is shown striking a flat blade at right angles to the surface of the blade. The steam leaves the nozzle with a velocity  $V_1$ , and the blade velocity is  $V_b$ . It is assumed that the steam leaves the blade in a direction parallel to the surface of the blade and that its velocity in the direction of motion of the blade is therefore  $V_b$ . Hence for a flat blade,

$$F = \frac{W(V_1 - V_b)}{g}$$

As work equals force times distance, the work in ft-lb per sec is

$$Wk = \frac{W(V_1 - V_b)}{g} V_b$$

and the work in ft-lb per lb of steam is

$$Wk = \frac{V_1 - V_b}{g} V_b \quad (10-2)$$

As the velocity of the steam leaving the blade is high, this steam possesses considerable kinetic energy which has not been utilized. To obtain more work, a 180-deg blade of the type shown in Fig. 10-10 (a) may be used. If the 180-deg blade is held stationary, the steam directed on the blade will follow it around. Friction being neglected, the velocity

\* In this chapter only,  $V$  will be used as the symbol for velocity. This is to avoid the difficulty of writing  $Vel$  on the many velocity diagrams.



of the steam coming out of the blade numerically equals that at the blade entrance.

If now the blade is allowed to move, but the steam velocity *measured in relation to the blade* (i.e., the relative velocity) be made the same as previously, then the exit velocity *measured in relation to the blade* will equal the previous exit velocity. This is illustrated in Fig. 10-10 (b).

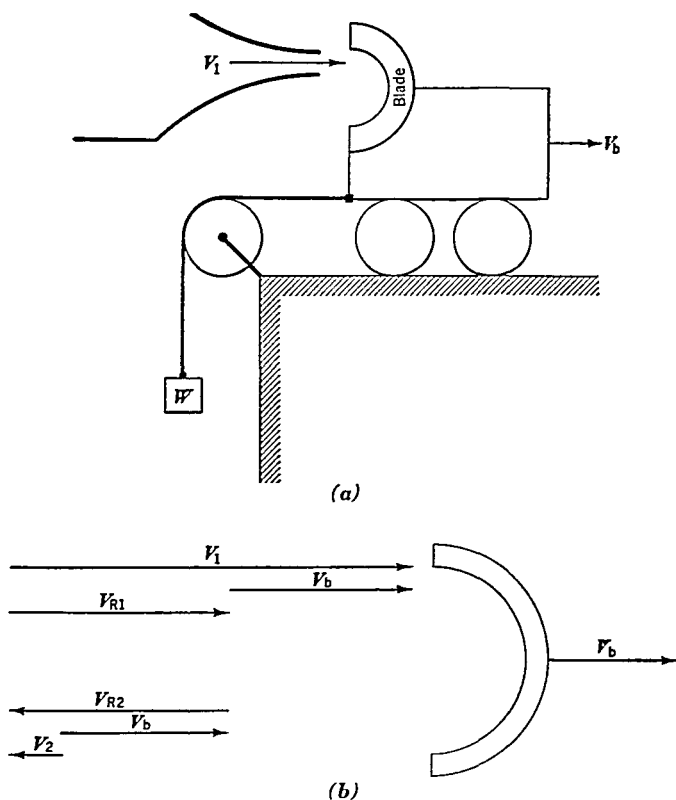


Fig. 10-10. Force Exerted by Steam Striking Curved Blade

Subscript 1 designates entrance conditions, subscript 2 designates exit conditions, and subscript  $R$  refers to relative velocities. Velocities to the right being denoted as positive, the change in steam velocity equals  $V_1 - V_2$ . If blade friction is neglected,  $V_{R1} = V_{R2}$ . The change in steam velocity becomes

$$V_1 - V_2 = V_1 - (V_b - V_{R2}) = V_1 - (V_b - V_{R1}) = V_1 - [V_b - (V_1 - V_b)] = 2(V_1 - V_b)$$

Therefore, for the 180-deg blade, without friction, the work in ft-lb per lb of steam is

$$W_k = \frac{2(V_1 - V_b)}{g} V_b \quad (10-2a)$$

If equation 10-2a is differentiated with respect to  $V_b$  and the result is equated to zero, the proper blade speed will be obtained for maximum efficiency. For the 180-deg blade, this value of  $V_b$  equals  $\frac{1}{2}V_1$ .

The function of a blade is to turn as much as possible of the kinetic energy into work. Blade efficiency equals the blade work divided by the kinetic energy of the entering steam. If the proper blade speed is used for a 180-deg blade without friction, the efficiency is found to be 100%.

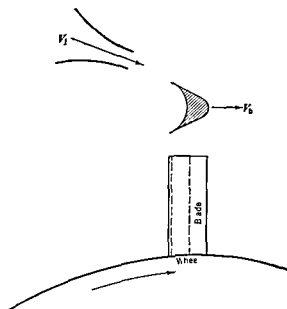


Fig 10-11 Absolute Velocities of Steam and Blade

Because of the necessity of mounting the blades on wheels and of having rotation, the nozzle must be placed at an angle to the path of the blades. In Fig 10-11 is shown one blade with steam issuing from the nozzle with a velocity  $V_1$  and at the nozzle angle  $\alpha$ , Fig 10-12. As far as the blade is concerned, the steam enters it at a velocity  $V_{R1}$  and at an angle  $\beta$ , as indicated in Fig 10-12. If the steam is to enter without shock, the blade entrance angle must be made equal to  $\beta^*$ . The steam follows the blade around and leaves with a relative velocity  $V_{R2}$  at the blade exit angle.

\* The actual direction of the steam is at angle  $\alpha$  in order to intercept the moving blade at angle  $\beta$  just as an anti aircraft gunner is taught to lead the target—in this case the blade.

The steam leaves with an absolute velocity  $V_2$  and at an absolute angle  $\delta$ . If there is no friction, the two relative velocities are equal. Actually, friction causes the relative velocity  $V_{R2}$  at exit to be less than the velocity  $V_{R1}$  at entrance. The ratio of  $V_{R2}$  to  $V_{R1}$  is known as the blade velocity coefficient, and is denoted by the symbol  $k_b$ .†

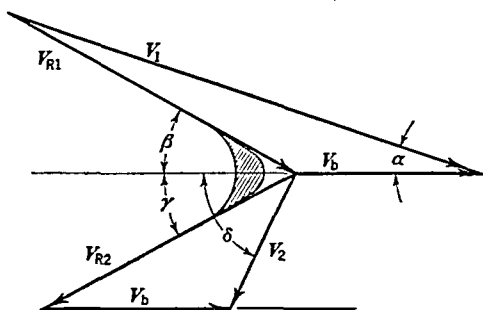


Fig. 10-12. Vector Diagram of Velocities of Steam and Blade

The force exerted on the blade depends on the change in that component of the steam velocity which is parallel to the blade motion. This change equals  $V_1 \cos \alpha - V_2 \cos \delta$ . Care must be taken with the sign of  $V_2$ , as it becomes negative if the exit steam travels in the opposite direction from the entering steam. *This trouble may be avoided if relative velocities are used.* Since  $V_1 \cos \alpha = V_{R1} \cos \beta + V_b$  and  $V_2 \cos \delta - V_b = V_{R2} \cos \gamma$ , the force on the blades, in pounds, is

$$\frac{W}{g}(V_{R1} \cos \beta + V_{R2} \cos \gamma)$$

The work in ft-lb per lb of steam is

$$Wk = (V_{R1} \cos \beta + V_{R2} \cos \gamma) \frac{V_b}{g} \quad (10-3)$$

Equation 10-3 is a working equation. The relations which follow are derived to show the possibilities of actual blades, particularly for maximum conditions, and are not working equations.

To find the value of  $V_b$  for maximum work and efficiency, rearrange equation 10-3 as follows: Since  $V_{R2} = k_b V_{R1}$ ,

$$\begin{aligned} V_{R1} \cos \beta + V_{R2} \cos \gamma &= V_{R1} \cos \beta \left(1 + k_b \frac{\cos \gamma}{\cos \beta}\right) \\ &= (V_1 \cos \alpha - V_b) \left(1 + k_b \frac{\cos \gamma}{\cos \beta}\right) \end{aligned}$$

† A discussion of the factors affecting  $k_b$  is beyond the scope of this text. However, it is desirable to call attention to some of these factors, such as relative steam velocity, blade angles, blade material and workmanship, blade height, blade shape, blade width, nozzle angle, and cross-sectional shape of stream of fluid leaving the nozzle.

Then the work per lb is

$$W_k = \frac{(V_1 \cos \alpha - V_b) \left(1 + k_b \frac{\cos \gamma}{\cos \beta}\right)}{g} V_b$$

If, as  $V_b$  varies, the velocity coefficient remains substantially constant and the angles  $\gamma$  and  $\beta$  remain constant (or the ratio of  $\cos \gamma$  to  $\cos \beta$  remains constant), then the work varies as the quantity  $(V_1 \cos \alpha - V_b)V_b$  varies. Then,

$$\frac{d(V_1 V_b \cos \alpha - V_b^2)}{dV_b} = V_1 \cos \alpha - 2V_b = 0$$

and

$$V_b = \frac{V_1 \cos \alpha}{2} \quad (10-4)$$

In practice, the steam velocities for given blade speeds are higher than those given by equation 10-4. This condition reduces the number of stages and hence the length of the turbine. In case the turbine is a small one, it is not economical to use many stages. Under these conditions, the value of the steam velocity greatly exceeds that given by equation 10-4. However, the blade efficiency is much lower than the theoretical efficiency possible for the given nozzle angle.

For these conditions, the maximum efficiency equals

$$\frac{\frac{1}{g} \left( V_1 \cos \alpha - \frac{V_1 \cos \alpha}{2} \right) \left( 1 + k_b \frac{\cos \gamma}{\cos \beta} \right) \left( \frac{V_1 \cos \alpha}{2} \right)}{\frac{V_1^2}{2g}} = \frac{\cos^2 \alpha}{2} \left( 1 + k_b \frac{\cos \gamma}{\cos \beta} \right)$$

If there is no friction (i.e.,  $k_b = 1.0$ ) and if  $\gamma = \beta$  then the maximum efficiency equals  $\cos^2 \alpha$ . If the blade outlet angle  $\gamma$  is made smaller than the correct blade inlet angle  $\beta$ , then there is an improvement in the blade efficiency. For this reason blade outlet angles generally are made smaller than the inlet angles. The minimum blade outlet angles will be discussed in a subsequent article.

**Example 10-1**—Steam leaves a nozzle with a velocity of 1600 ft per sec at an angle of  $15^\circ$ . The blade velocity coefficient is 0.90. The blade speed is 600 ft per sec and the blade outlet angle is  $20^\circ$ . Calculate the work per pound of steam and the blade efficiency.

*Solution.*—As indicated in Fig. 10-12,

$$\begin{aligned} V_{R1} &= \sqrt{(V_1 \cos \alpha - V_b)^2 + (V_1 \sin \alpha)^2} \\ &= \sqrt{(1600 \cos 15^\circ - 600)^2 + (1600 \sin 15^\circ)^2} \\ &= \sqrt{945^2 + 414^2} = 1032 \text{ ft per sec} \end{aligned}$$

Then,

$$\begin{aligned} V_{R2} &= 0.90(1032) = 929 \text{ ft per sec} \\ V_{R2} \cos \gamma &= 929 \cos 20^\circ = 873 \text{ ft per sec} \\ V_{R1} \cos \beta &= V_1 \cos \alpha - V_b = 945 \text{ ft per sec} \end{aligned}$$

The blade work per lb of steam, by equation 10-3, is

$$\begin{aligned} \text{Wk} &= \frac{(945 + 873)(600)}{32.2} = 33,870 \text{ ft-lb} && \text{Ans.} \\ \text{Efficiency} &= \frac{\text{Wk}}{\text{K.E.}} = \frac{33,870}{\frac{1600^2}{2 \times 32.2}} = .852 \text{ or } 85.2\% && \text{Ans.} \end{aligned}$$

*Example 10-2.*—Make an energy balance to show what happens to the kinetic energy leaving the nozzle in the preceding example.

*Solution.*—The kinetic energy leaving the nozzle goes partly to work, partly to reheat (*i.e.*, increase in enthalpy due to blade friction), and partly to kinetic energy at blade exit. If the blades are stationary, the decrease in kinetic energy in the blade passages is the blade reheat. If the blades are in motion, the blade reheat will be the same as when they were at rest, provided that the steam velocities measured in relation to the blades are the same. Thus, the blade reheat equals

$$\frac{V_{R1}^2 - V_{R2}^2}{2gJ} = 4.0 \text{ Btu per lb or } 3112 \text{ ft-lb per lb}$$

The exit velocity is

$$\begin{aligned} V_2 &= \sqrt{(V_{R2} \cos \gamma - V_b)^2 + (V_{R2} \sin \gamma)^2} \\ &= \sqrt{273^2 + 319^2} = 419 \text{ ft per sec} \quad \text{21} \end{aligned}$$

Then the exit kinetic energy = 2736 ft-lb per lb.

The total of the work, exit kinetic energy, and blade reheat equals

$$33,870 + 2,736 + 3,112 = 39,718 \text{ ft-lb per lb}$$

The original kinetic energy =  $\frac{1600^2}{2g} = 39,730 \text{ ft-lb per lb}$ .

*Example 10-3.*—Calculate the proper blade entrance angle for the data in Example 10-1.

*Solution.*—The proper blade inlet angle is

$$\beta = \sin^{-1} \left( \frac{V_1 \sin \alpha}{V_{R1}} \right) = \sin^{-1} \frac{414}{1032} = 23^\circ 39' \quad \text{Ans.}$$

*Example 10-4.*—Calculate the axial thrust per lb of steam per sec for data in Example 10-1.

*Solution.*—The axial thrust is caused by a change in the component of the steam velocity perpendicular to the blade motion. This change equals

$$\begin{aligned} V_1 \sin \alpha - V_2 \sin \delta &= V_1 \sin \alpha - V_{R2} \sin \gamma \\ &= 414 - 319 = 95 \text{ ft per sec} \end{aligned}$$

and the thrust is

$$F = \frac{W(\Delta V)}{g} = \frac{1(90)}{32.2} = 2.80 \text{ lb per lb per sec} \quad \text{Ans.}$$

**10-4 Compounding**—It has been shown that in order to obtain a high thermal efficiency there must be a high over all temperature range. For a steam power plant this means that the over all pressure range must be large. If the expansion takes place completely in one group of nozzles from boiler to condenser pressure, velocities of the order of 3,000 to 4,000 ft per sec or even higher will result. Equation 10-3 shows that for maximum efficiencies the blade speed of a simple impulse stage equals  $\frac{V_1 \cos \alpha}{2}$ . As the limiting value of the speed of the blade tips is about

1,000 to 1,100 ft per sec, it may be seen that the simple impulse turbine cannot utilize efficiently the velocity produced by the complete expansion.

The two methods of compounding have been discussed in Art 10-2 which should be re-read. It may be shown that for maximum efficiency

the blade velocity of a two-row Curtis stage, Fig 10-3, is  $V_b = \frac{V_1 \cos \alpha}{4}$ .

Because the proper blade velocity  $V_b$  for a simple turbine blade is  $\frac{V_1 \cos \alpha}{2}$ ,

for the same blade velocity the velocity of steam leaving the nozzle of a two-row Curtis stage is twice that of the simple turbine. As the kinetic energy, and hence the work done, is a function of the square of the velocity, the two-row Curtis stage theoretically will produce four times as much work per pound of steam as is produced in a simple turbine having the same blade speed.

Although the two-row Curtis stage produces much work per pound of steam, the steam velocities are excessive. Because of the necessity of abrupt changes in directions of flows, particularly in the blade passages, these high velocities are accompanied by high frictional losses. As a Curtis stage of three or more rows will have still higher velocities (and hence more losses), such stages are not used.

When high efficiency is desired, pressure compounding of the Rateau type, Fig 10-5, is used. In this type of compounding the pressure drop is divided into a sufficient number of steps to permit the kinetic energy set up in any row of nozzles to be efficiently utilized by the blades which follow. Because the velocity in the Rateau type of turbine is relatively low, the efficiency is higher than that of the Curtis type.

In spite of its lower efficiency the two-row Curtis stage is used extensively for several reasons. In the first place it is much more compact than the four Rateau stages it can theoretically replace. Because of the much

higher pressure drop in the nozzles in the Curtis stage, the maximum temperature and pressure of the steam coming in contact with the blading and casing are much lower than when Rateau stages are used. Because the steam flow to the first stage can be readily controlled by controlling the nozzles in use in the first stage, the use of a Curtis stage as the first stage of a turbine will permit the nozzle-governing of a large portion of the energy to the turbine. This is more efficient than to throttle the steam supply at part load. If there is use for the entire amount of the exhaust steam of a turbine, its thermal efficiency is not important. Under these conditions, the use of one or more Curtis stages results in a compact turbine.

★10-5. Blade Exit Angles and Blade Heights.—As was discussed in the previous article, blade outlet angles are made smaller than the blade inlet angles in order to obtain higher blade efficiency. The limiting factor on the extent to which the outlet angle may be decreased is the area required for the flow of steam. The effective area for flow at the blade exit equals the product of the blade height  $h$  and the effective width of the blade passage. As indicated in Fig. 10-13, the effective width equals  $(p \sin \gamma - t)$  where  $p$  equals the blade pitch,  $\gamma$  is the blade outlet angle, and  $t$  is the blade exit thickness. Therefore, the effective

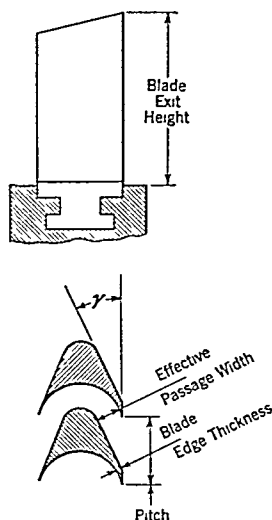


Fig. 10-13. Dimensions at Blade Exit

exit area is  $h(p \sin \gamma - t)$ . The required exit area is  $A = \frac{Wv}{V_{R2}}$ , where  $v$  is the specific volume at blade exit. By equating the effective area to the required area, the minimum blade exit angle may be determined.

*Example 10-5.*—Steam leaves a row of nozzles at 80 psia and 320 F and with a velocity of 1800 ft per sec. The nozzle exit angle is  $14^\circ$ . The nozzle arc is 9 in. in length. The blade pitch is 0.75 in., the blade exit height is 2.0 in., and the blade thickness at exit is 0.02 in. The blade speed is 650 ft per sec and the blade velocity coefficient is 0.89. The steam flow is 8 lb per sec. Calculate the minimum blade exit angle.

*Solution.*

$$V_{R1} = \sqrt{(V_1 \cos \alpha - V_b)^2 + (V_1 \sin \alpha)^2} = \sqrt{1097^2 + 435^2}$$

$$= 1180 \text{ ft per sec}$$

$$\text{and } V_{R2} = 0.89(1180) = 1050 \text{ ft per sec}$$

The blade reheat is

$$\frac{V_{R1}^2 - V_{R2}^2}{2gJ} = \frac{1180^2 - 1050^2}{50,000} = 5.8 \text{ Btu per lb}$$

stage equals  $1314.7 - \frac{215.0}{2} = 1207.2$  Btu, and the pressure as determined by use of a Mollier diagram or the Ellenwood Charts\* is 101 psia.

The stage reheat equals  $(1 - 0.65) \frac{215.0}{2} = 36.3$  Btu. Then the enthalpy at stage exit, or  $h_4$ , is  $1207.2 + 36.3 = 1243.5$ . An isentropic expansion from point 4 to a pressure of 25 psia shows an enthalpy at point 5 of 1130. Then the isentropic enthalpy drop in the second stage equals  $h_4 - h_5 = 1243.5 - 1130.0 = 113.5$  Btu. This differs somewhat from the allocated isentropic enthalpy drop in the first stage of 107.5 Btu.

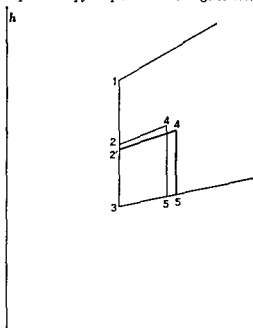


Fig. 10-15 Diagram for Example 10-6

The sum of the isentropic enthalpy drops for the stages is  $107.5 + 113.5 = 221.0$  Btu. If half of this quantity is put in the first stage, the enthalpy at the exit of the isentropic expansion, or  $h_2$ , equals  $1314.7 - \frac{221.0}{2} = 1204.2$  Btu. The pressure equals 97 psia.

As a check on the available energy in the second stage, proceed as before. The first stage reheat equals  $(1 - 0.65) \frac{221.0}{2} = 38.7$  Btu. The enthalpy at stage exit, or  $h_4$ , is  $1204.2 + 38.7 = 1242.9$  Btu. Isentropic expansion from point 4 to 25 psia shows a final enthalpy  $h_5 = 1132$  and the isentropic enthalpy drop for the second stage, or  $h_4 - h_5$ , is  $1242.9 - 1132.0 = 110.9$  Btu. This is sufficiently close to the isentropic enthalpy drop of 110.5 Btu in the first stage. Thus, the proper pressure at exit of the first-stage nozzles is 97 psia. Ans

In case the reheat factor, which equals  $\frac{221.0}{215.0} = 1.027$ , was known ahead of time by previous experience, then point 2 could have been located in the following manner:  $h_2 = 1314.7 - \frac{215.0(1.027)}{2} = 1204.2$ . Isentropic expansion from point 1 to an enthalpy of 1204.2 shows a proper pressure of 97 psia. Ans

\* See Chapter 6



★10-7. **The Reaction Turbine.**—As shown in Fig. 10-7 and Fig. 10-8, and as discussed in Art. 10-2, the reaction turbine consists of both moving blades and stationary blades. The passages between the blades are so shaped that they form convergent nozzles. Thus, there is a pressure drop of the steam in the moving-blade passages as well as in the stationary-blade passages. This means that two actions are going on simultaneously in the moving-blade passages. One of these, the pressure drop, tends to increase the velocity; the other, the conversion of kinetic energy into work, tends to decrease the velocity. This dual action makes the analysis of the reaction turbine a little more involved than that of the impulse turbine.

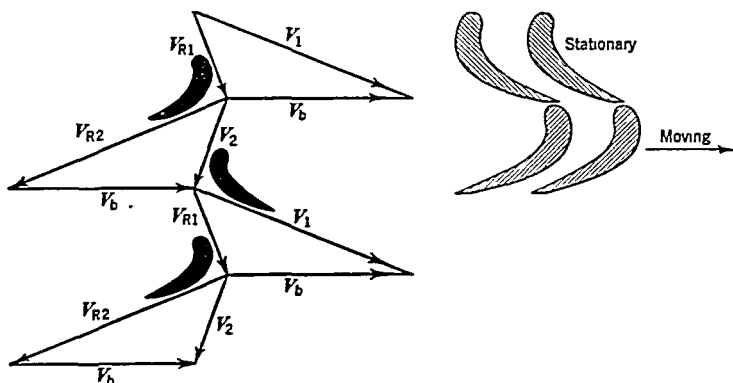


Fig. 10-16. Vector Diagram for Group of Reaction Blades

The analysis of the reaction turbine may be simplified by selecting a group of stages made up of similar blades (*i.e.*, all blades have the same inlet angles and all blades have the same outlet angles). The blade heights will increase in direct proportion to the increase in specific volumes caused by pressure drops through the stages, and yet the changes in blade heights will not be sufficient to cause an appreciable increase in the mean blade velocity. Under these conditions, a vector diagram such as that in Fig. 10-16 may be drawn for the group. Steam leaves a moving row of blades with an absolute velocity  $V_2$ . As the blade rows are close together, it may be assumed that the steam enters the stationary row with the same velocity,  $V_2$ . Because of the pressure drop, steam leaves the stationary row and enters the moving row with the greater absolute velocity,  $V_1$ .

The relative entrance velocity  $V_{R1}$  may be found in the same manner as for impulse blades. The pressure drop in the moving row causes the steam to leave with a relative velocity  $V_{R2}$  and an absolute velocity  $V_2$ . By comparing the two triangles, because of equality of the blade angles and of one side,  $V_b$ , it may be seen that the two triangles are equal. Hence

the statement may be made that the relative velocity  $V_{R*}$  of the steam leaving the moving row equals the absolute velocity  $V_1$  of the steam leaving the stationary row, and the relative velocity  $V_{R1}$  of the steam entering the moving row equals the absolute velocity  $V_2$  of the steam leaving the moving row and entering the stationary row

Consider now a stationary row. Losses in the clearance space being neglected, the kinetic energy entering this stationary row is  $\frac{V_2^2}{2gJ}$  Btu per lb

If there is no loss in the blade passage, this kinetic energy will be carried through the stationary row and will appear at exit. Actually, because of losses in the blade passages, the carry-over of this kinetic energy is not perfect. That portion of the entering kinetic energy which is carried

through the stationary blade row is  $\epsilon_{co} \frac{V_2^2}{2gJ}$  Btu per lb, where  $\epsilon_{co}$  is the

carry-over efficiency. In addition to the kinetic energy carried through the stationary row, kinetic energy is set up by a pressure drop, so that the

total kinetic energy at the exit of the stationary row is  $\frac{V_1^2}{2gJ}$  Btu per lb. Of

this, the amount  $\epsilon_{co} \frac{V_2^2}{2gJ}$  Btu per lb was carried through the stationary row from the previous moving row and the remainder, which is  $\frac{V_1^2 - \epsilon_{co} V_2^2}{2gJ}$

Btu per lb, was set up by nozzle action (i.e., a pressure drop). Because of equality of triangles, it may be said that an equal amount of kinetic energy is set up by nozzle action in the moving passages.

Because of imperfect nozzle action, the kinetic energy actually set up is less than the theoretical energy, which is the isentropic drop between the given pressures. The ratio of the kinetic energy actually set up to the theoretical energy is called the nozzle efficiency\*. These statements may be expressed in equation form as follows:

The kinetic energy set up by nozzle action, per row, is

$$\text{Kinetic Energy Set Up} = \frac{V_1^2 - \epsilon_{co} V_2^2}{2gJ} \quad (10-6)$$

The isentropic enthalpy drop, per row, is

$$\text{Isentropic Enthalpy Drop} = \frac{V_1^2 - \epsilon_{co} V_2^2}{2gJ\eta_n} \quad (10-7)$$

where  $\eta_n$  is the nozzle efficiency

\* This also means that nozzle efficiency is the ratio of the actual enthalpy drop to the theoretical drop caused by nozzle action. This definition, which is commonly used in reaction turbine work, is the same as the restricted definition given for nozzles in equation 9-4a.

Since the relative velocities in and out of the moving row are the same as the absolute velocities in and out of the stationary row, the enthalpy drops in the moving row are the same as those in the stationary row. Because of this, the enthalpy drops per reaction stage are twice those per row which are given in equations 10-6 and 10-7.

The work of a reaction stage can be found in the manner described for the impulse stage. Thus, the work in ft-lb per lb is

$$W_k = \frac{(V_{R1} \cos \beta + V_{R2} \cos \alpha) V_b}{g} = \frac{(V_1 \cos \alpha - V_b + V_1 \cos \alpha) V_b}{g}$$

or 
$$W_k = \frac{(2V_1 \cos \alpha - V_b) V_b}{g}$$

To find the value of  $V_b$  for maximum work and maximum efficiency, the procedure is as follows:

$$\frac{d(2V_1 \cos \alpha - V_b) V_b}{g} = 2V_1 \cos \alpha - 2V_b = 0$$

or 
$$V_b = V_1 \cos \alpha$$

Thus, for maximum efficiency with a given blade speed, the steam velocities in a reaction turbine are one-half those in a Rateau turbine. Because of this, the enthalpy drops per Rateau stage are twice those of a reaction stage for the same conditions. Hence, there must be many more stages in a reaction turbine than in a Rateau; and the fact that blade speeds are low in a reaction turbine means even more stages. However, as reaction stages are compact, the ratio of the size of the reaction turbine to that of the Rateau does not depend directly on the number of stages. Because of the low velocities involved, the efficiency of the reaction-turbine blading tends to be high and approaches the theoretical efficiency, which is  $\cos^2 \alpha$ , more nearly than does the Rateau stage.

As a result of the difference in pressure existing across the blades, there tends to be a flow around the blade tips. Although methods have been found to minimize this flow, the leakage around the blade tips may be a serious proportion of the total flow when the blades are short, as they are in the high-pressure end of the turbine. For this reason, a two-row Curtis stage is often used as the first stage of a reaction turbine, as all the steam must flow through the blades. In addition, the Curtis stage has the advantages that were mentioned previously.

*Example 10-7.*—The blade speed in a reaction turbine is 400 ft per sec. The nozzle efficiency is 90% and the carry-over efficiency is 60%. The ratio of the blade speed to the steam speed is 0.8. Calculate the blade work per pound of steam and the combined nozzle and blade efficiency, if the blade outlet angles are  $20^\circ$ .

*Solution* — In Fig 10-16,  $V_1 = \frac{400}{0.8} = 500$  ft per sec Then,

$$V_1 = V_{R1} = \sqrt{(500 \cos 20^\circ - 400)^2 + (500 \sin 20^\circ)^2} \\ = \sqrt{70^2 + 171^2} = 185$$

The blade work per lb is

$$[(V_1 \cos \alpha - V_b) + (V_{R2} \cos \beta)] \frac{V_b}{g} =$$

$$[(500 \cos 20^\circ - 400) + (500 \cos 20^\circ)] \frac{400}{32.2} = 6700 \text{ ft-lb}$$

Ans

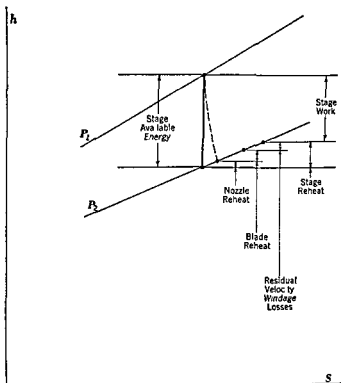


Fig 10-17. *h-S* Diagram for Turbine Stage

The isentropic enthalpy drop per stage, from equation 10-7, equals

$$2 \frac{V_1^2 - \epsilon_\infty V_2^2}{\eta_n (2gJ)} = 2 \times \frac{500^2 - 0.6(185)^2}{0.9(50,000)} = 10.2 \text{ Btu per lb}$$

The combined nozzle and blade efficiency equals

$$\frac{\text{Blade Work}}{\text{Isentropic Enthalpy Drop}} = \frac{6,700}{10.2 \times 778} = 0.844, \text{ or } 84.4\%$$

Ans.

**10-8. Steam-Turbine Performance.**—The efficiency of the nozzles was discussed in Chapter 9, and that of the blades was considered in this

chapter. The wheels, on which the blades are mounted, must rotate in the steam which surrounds them. This action between the wheel and the steam consumes some of the energy delivered to the wheel by the blades, particularly when the steam is dense and the peripheral velocity of the wheel is high. The energy delivered by the wheel to the shaft is the stage output; the summation of stage outputs is called the internal turbine output. In Fig. 10-17 is shown an  $h$ - $S$  diagram of a turbine stage (or a single-stage impulse turbine) with the energy quantities labeled. Some of the energy delivered to the shaft is lost in the turbine bearings and also in driving the oil pump and the governor; the remainder is delivered by the turbine and is known as the shaft output. The mechanical efficiency of a turbine is the ratio of the shaft horsepower to the internal horsepower. For a steam turbine, this is very high, being generally close to 99% except for small steam turbines.

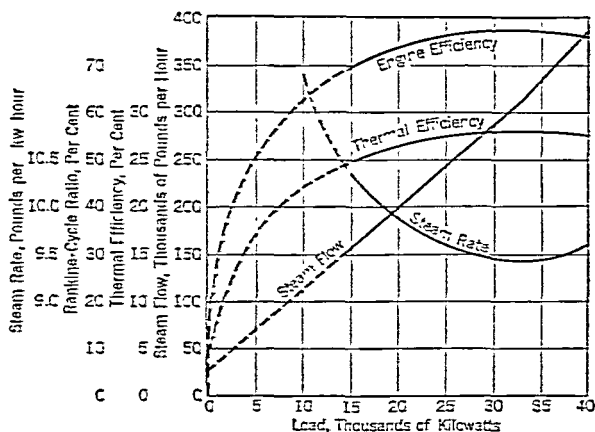


Fig. 10-18. Performance Curve for Steam Turbo-Generator

In Fig. 10-18 is shown a typical performance curve for a large condensing steam turbo-generator. The thermal efficiency of a turbine falls off at light loads. At light loads, there is a large degree of over-expansion in the nozzles. At part load, the ratio of blade speed to steam speed in the blades is not proper; in addition, the steam will enter the blades at the wrong angle, with an attendant loss in efficiency. It is to be noted that at 33,500 kw there is a break in each of the curves. This occurs because the overload valve starts to open at this point.

Just before the overload valve starts to open, there is full steam flow through the nozzles of the first stage. Because the area of the second-stage nozzle exceeds that of the first stage, it follows that, if boiler steam

decreased in volume) For such cases fans serve the purpose of moving the air to the desired location In other cases particularly in drying work, there is appreciable resistance to the flow of air and a compressor of some sort is required to build up sufficient pressure to overcome the resistance to flow

Compressors are widely used in industry to create and maintain vacua ranging from one at a pressure slightly below atmospheric to almost a perfect vacuum Gas compressors are essential in the liquefying of gases as the first step in the common method is to highly compress the gas Compressors, known as superchargers are used extensively to increase the power output of internal combustion engines by forcing more charge into the cylinders

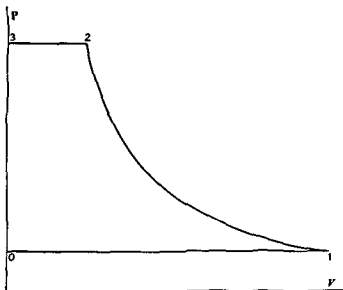


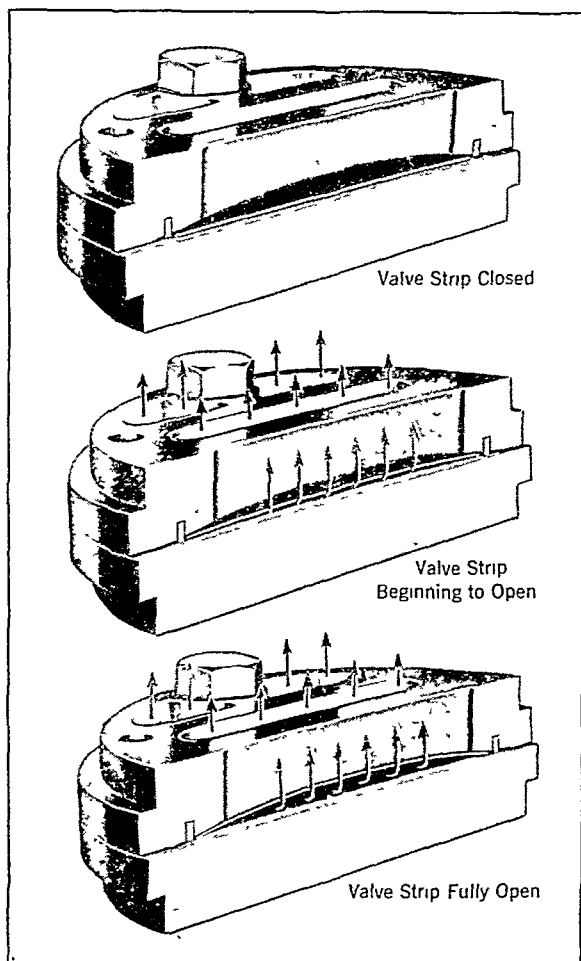
Fig 11-2 P V Diagram of Compressor Without Clearance

**11-2 Classification**—Compressors may be classed as reciprocating, rotary, and centrifugal compressors The rotating and centrifugal compressors will be discussed in a later article

Reciprocating compressors are generally used for pressures above 40 or 50 psi gage For pressures above 80 to 100 psi gage it is common practice to carry out the compression in two, three, or even four steps (called stages)

There are many possible arrangements of the compressor cylinders, as true of steam engine cylinders (See various cylinder arrangements appendix)

Most gas compressors are double-acting, although some very small compressors and some multi-stage compressors are single-acting. A single-stage, double-acting compressor is shown in Fig. 11-1.



(Courtesy of Worthington Pump & Machinery Corp.)

Fig. 11-3. Pressure-Operated Discharge Valve

11-3. Compressors Without Clearance.—If an air compressor having no clearance is considered, the analysis is simplified as all the air compressed is delivered. Fig. 11-2 shows a theoretical  $P$ - $V$  diagram for such a compressor. Air is drawn into the cylinder, as indicated by line 0-1. Compression takes place from 1 to 2. At point 2, the discharge valve

opens, and air is pushed out of the cylinder until the end of the stroke at point 3. As the piston starts its forward stroke again, the discharge valve closes and the inlet valve opens. It should be noted that most compressor valves are pressure operated. Common valves consist of thin plates of steel held in place on their seats by springs. Such a valve is shown schematically in Fig. 11-3. The valve shown is a discharge valve. The valve plates remain on their seats until the pressure in the cylinder becomes sufficiently in excess of the pressure in the discharge line to overcome the slight spring pressure acting on the valve plates. Thus, the position of point 2 on the compression line in Fig. 11-2 depends on the pressure in the discharge line.

The nature of the compression line depends on the design and operation of the compressor. One extreme theoretical case is that of isentropic compression, the other extreme is the isothermal compression. In the actual case, when little heat is removed during the compression, the exponent for the compression curve may exceed that for the isentropic compression.

Using the general case of polytropic compression, the net cycle work, which is the summation of the work of the individual processes, equals

$${}_0W_{k_1+1}W_{k_2+2}W_{k_3} = P_1(V_1 - V_0) + \frac{P_2V_2 - P_1V_1}{1-n} + P_2(V_3 - V_2)$$

As  $V_0$  and  $V_3$  are 0, the net work is

$$W_k = P_1V_1 + \frac{P_2V_2 - P_1V_1}{1-n} - P_2V_2$$

From this, the net cycle work for polytropic compression is

$$W_k = \frac{n}{n-1}(P_1V_1 - P_2V_2) \quad (11-1)$$

To avoid the necessity of solving for  $V_2$ , it is desirable to eliminate that volume from the equation. This may be done as follows

$$\text{Net cycle work} = \frac{n}{n-1}P_1V_1\left(\frac{P_1V_1}{P_1V_1} - \frac{P_2V_2}{P_1V_1}\right)$$

$$\text{Since } \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = \left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}},$$

$$\frac{P_2V_2}{P_1V_1} = \frac{P_2}{P_1}\left(\frac{P_2}{P_1}\right)^{-\frac{1}{n}} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$



Then the net cycle work is

$$Wk = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \quad (11-2)$$

In the case of isothermal compression,  $P_1 V_1$  equals  $P_2 V_2$  and the net cycle work reduces to the work of compression. Then the net cycle work for isothermal compression is

$$Wk = P_1 V_1 \log_e \left( \frac{V_2}{V_1} \right) \quad (11-3)$$

These expressions for net cycle work also may be derived by integrating the area 0-1-2-3 on the  $P$ - $V$  plane in Fig. 11-2, or by finding  $\int V dp$ .

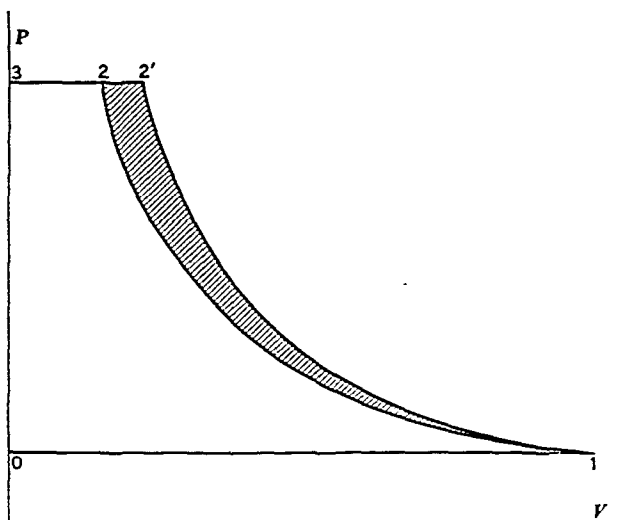


Fig. 11-4. Comparison of Isentropic and Isothermal Compression

The net cycle work can be materially reduced if the air is cooled during its compression. Fig. 11-4 shows both isentropic and isothermal compression for the same range in pressure, the saving in work by using isothermal compression being indicated by the cross-hatched area.

The isothermal compression, while desirable, is difficult to approach—let alone to attain. The air is normally cooled by surrounding the cylinder with a water jacket. Although compressors run at relatively low speeds, still the time used during compression is so short that little heat transfer can take place. Thus, the exponent for the compression line of a water-jacketed air compressor seldom is less than 1.3 and generally is closer to

1.35 Some attempts have been made to cool with water injection during the compression, but mechanical difficulties have prevented the common use of this method

*Example 11-1* — It is desired to compress 500 cu ft of air per min at 14.0 psia and 70 F to 90.0 psia. Calculate the theoretical horsepower required if the compression is (a) isentropic, (b) isothermal, (c) polytropic with  $n$  equal to 1.35

*Solution* — (a) From equation 11-2 the isentropic horsepower required is

$$\frac{\frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]}{33\,000} = \frac{1.4 (144)(14.0)(500)}{0.4 \cdot 33\,000} \left[ 1 - \left( \frac{90}{14} \right)^{\frac{0.4}{1.4}} \right] = -75.2 \text{ hp Ans}$$

The negative sign indicates that the work is done on the air

(b) From equation 11-3 the isothermal horsepower required is

$$\frac{P_1 V_1 \log_e \frac{V_2}{V_1}}{33\,000} = \frac{-P_1 V_1 \log_e \frac{P_2}{P_1}}{33\,000} = \frac{-144(14)(500) \log_e \left( \frac{90}{14} \right)}{33\,000} = -56.8 \text{ hp Ans}$$

(c) From equation 11-2 the polytropic horsepower required is

$$\frac{\frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]}{33\,000} = \frac{1.35 (144)(14.0)(500)}{0.35 \cdot 33\,000} \left[ 1 - \left( \frac{90}{14} \right)^{\frac{0.35}{1.35}} \right] = -72.9 \text{ hp Ans}$$

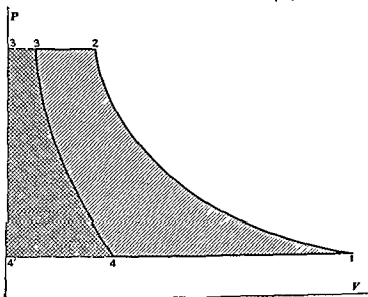


Fig 11-5 P-V Diagram for Compressor With Clearance

**11-4. Compressors With Clearance** — For constructional reasons, an air compressor must be designed with clearance. In Fig 11-5 is shown a P-V diagram for a compressor with clearance. Compression starts at point 1 and continues until the pressure in the cylinder, at point 2 exceeds

the pressure in the discharge line, causing the valve to open. Discharge of air takes place between points 2 and 3. As the piston starts to move back after the discharge, the drop in pressure in the cylinder causes the discharge valve to close. The clearance air expands until the cylinder pressure, at point 4, drops below that in the suction line. At this point, the inlet valve opens and air is drawn into the cylinder between points 4 and 1.

The net cycle work of a compressor with clearance is area 1-2-3-4 in Fig. 11-5. This area equals area 1-2-3'-4' minus area 4-3-3'-4'. By following the method in Art. 11-3, it is found that

$$\text{Area 1-2-3'-4'} = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

and

$$\text{Area 4-3-3'-4'} = \frac{n}{n-1} P_4 V_4 \left[ 1 - \left( \frac{P_3}{P_4} \right)^{\frac{n-1}{n}} \right]$$

The values of  $n$  for the compression and expansion parts of cycles generally are not the same. However, particularly with a small clearance and a low pressure range, area 4-3-3'-4' is such a small per cent of the area 1-2-3'-4' that its value need not be determined accurately. For simplification, the two values of  $n$  are assumed to be equal. As  $P_3 = P_2$  and  $P_4 = P_1$ , the net cycle work is

$$\text{Wk} = \frac{n}{n-1} P_1 \left[ V_1 - V_4 \right] \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \quad (11-4)$$

By comparing equations 11-4 and 11-2, it may be seen that there is only one difference, the term  $V_1$  in equation 11-2 being replaced by the term  $(V_1 - V_4)$  in equation 11-4. The term  $V_1$  in equation 11-2 represents the volume of air drawn into the cylinder, *measured under cylinder conditions*. Likewise, the term  $(V_1 - V_4)$  in equation 11-4 represents the volume of air drawn into the cylinder, *measured under cylinder conditions*. Thus, it may be said that, theoretically, the horsepower required for a compressor with clearance is exactly equal to that without clearance, when the amounts of air drawn into the cylinders occupy equal volumes under the same conditions.

**11-5. Volumetric Efficiency.**—It is desirable for an air compressor to deliver the maximum amount of air possible for a given piston displacement. Theoretically, the compressor should draw into itself a volume of air, measured under conditions approaching the compressor, equal to the piston displacement. Furthermore, the compressor should compress and

deliver all of this air. There are several reasons why the compressor does not deliver the amount of air it should. Some of these are

- (1) The clearance air expands and takes up part of the piston displacement, leaving less room for the incoming air
- (2) Pressure drops in the intake system cause the air to expand, thus decreasing the amount of air drawn in
- (3) The warmer cylinder walls may heat the incoming air slightly, causing a small expansion of the incoming air
- (4) Air leaks from one side of the piston to the other side
- (5) Air leaks around the piston rod to the outside

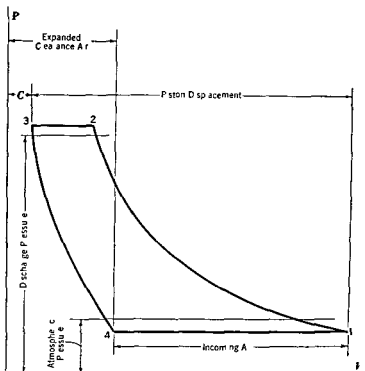


Fig 11-6 Effect of Clearance Factor

A measure of the effectiveness of a compressor in delivering air is its volumetric efficiency. Volumetric efficiency is defined as the ratio of the volume\* of the delivered air, expressed under conditions approaching the compressor, to the piston displacement of the compressor.

A compressor commonly draws air from the atmosphere. The atmospheric air approaching the compressor is called "free air." For com-

\* Some authorities express volumetric efficiency in terms of weights. However it is felt that as volumetric efficiency infers a volumetric quantity, it should be expressed in terms of volumes.

pressors drawing air from the atmosphere, the volumetric efficiency becomes the ratio of the volume of free air delivered to the piston displacement. (The term "volume of free air delivered" means that volume of free air which is compressed and delivered.)

As indicated on the  $P$ - $V$  diagram in Fig. 11-6,  $V_1$  represents the total cylinder volume;  $V_3$  is the clearance volume, designated as  $C$ ; and the piston displacement, which is abbreviated P.D. in the following explanation, equals  $V_1 - V_3$ . The clearance volume may be conveniently expressed as a *per cent of the piston displacement*. Since the inlet valve is open from point 4 to point 1, air is drawn into the cylinder during this part of the cycle. Hence, the volume of air drawn into the cylinder, *measured under cylinder conditions*, is equal to the difference between the volumes at point 1 and point 4, or  $V_1 - V_4$ . If leakages are neglected and it is assumed that the specific volume of the air does not change as the air comes into the cylinder, the theoretical volumetric efficiency may be determined from the relation

$$\text{Theoretical Volumetric Efficiency} = \frac{V_1 - V_4}{\text{P.D.}} \quad (11-5)$$

This theoretical volumetric efficiency is often called the *clearance factor*. Its value depends on the physical dimensions of the compressor and on the pressures under which it is operating.

In an actual compressor there are pressure drops through the valves and there may be a measurable heating of the incoming air by the warmer cylinder walls. Both of these actions cause the air to expand as it comes into the cylinder. As a result a smaller amount of outside air occupies the space  $V_1 - V_4$  in the cylinder. This condition therefore decreases the volumetric efficiency. The air volume in the space  $V_1 - V_4$  can be corrected to outside conditions by the simple relations of pressures, volumes, and temperatures. Thus, the maximum volumetric efficiency (*i.e.*, the volumetric efficiency without leakages) equals

$$\text{Maximum Volumetric Efficiency} = \frac{V_1 - V_4}{\text{P.D.}} \frac{P_i T_o}{P_o T_i} \quad (11-6)$$

where the subscript  $i$  refers to conditions in the cylinder at the end of the suction and the subscript  $o$  refers to conditions when the air is approaching the compressor.

**Example 11-2.**—Calculate the theoretical volumetric efficiency of a compressor receiving air at 14.2 psia and discharging it at 107.4 psia. The compressor has a clearance volume of 4 per cent of the piston displacement, there is a pressure drop of 0.4 psi through the suction valves, and there is a pressure drop of 3.0 psi through the discharge valves. Neglect the heating effects during suction and take the value of  $n$  for both compression and re-expansion as 1.35.

*Solution*—Refer to Fig 11-6 From the data,

$$P_4 = 14.2 - 0.4 = 13.8 \text{ psia, and } P_3 = 107.4 + 3 = 110.4 \text{ psia}$$

Also the total cylinder volume  $V_1$  is equal to  $P D + 0.04 P D = 1.04 P D$ , and the volume  $V_4$  occupied by the expanded clearance air, as determined from the volume and pressure at point 3 is

$$V_4 = V_3 \left( \frac{P_3}{P_4} \right)^{\frac{1}{n}} = 0.04 P D \left( \frac{110.4}{13.8} \right)^{\frac{1}{1.35}} = 0.1866 P D$$

The volume inside the compressor cylinder that is occupied by the incoming air is equal to

$$V_1 - V_4 = 1.04 P D - 0.1866 P D = 0.8534 P D$$

The volume of the free air taken in is computed by converting this air volume to outside conditions. Thus it is

$$0.8534 P D \left( \frac{14.2 - 0.4}{14.2} \right) = 0.829 P D$$

Since the volumetric efficiency is the ratio of the volume of free air delivered to the piston displacement

$$\text{Maximum Volumetric Efficiency} = \frac{0.829 P D}{P D} \text{ or } 82.9\% \quad \text{Ans}$$

Any leakage will make the actual volumetric efficiency less than this

★11-6 **Multi-Stage Compression**—As was pointed out in Art 11-3, a small saving in work may be made by water-jacketing the compressors. In addition, water jacketing prevents the discharge temperature from rising to as high a value as it would otherwise. However, when the air is highly compressed, particularly if the compressor is large, the discharge temperature will be so high that the lubricating oil may possibly vaporize

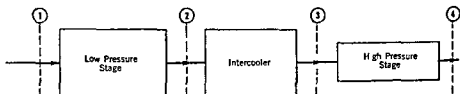


Fig 11-7 Line Diagram for Compound Compressor

and explode. To prevent this, in high pressure compressors, the air is removed after partial compression and put through an intercooler. The intercooler is generally a cylindrical shell containing many small tubes through which water is circulated. The air flowing over the outside of the tubes becomes well-cooled, as there is much heat transfer area. After it leaves the intercooler, the air is further compressed. In case the air is to be compressed to pressures of about 500 psi or higher, three or more stages of compression may be used, with intercoolers between the various stages.

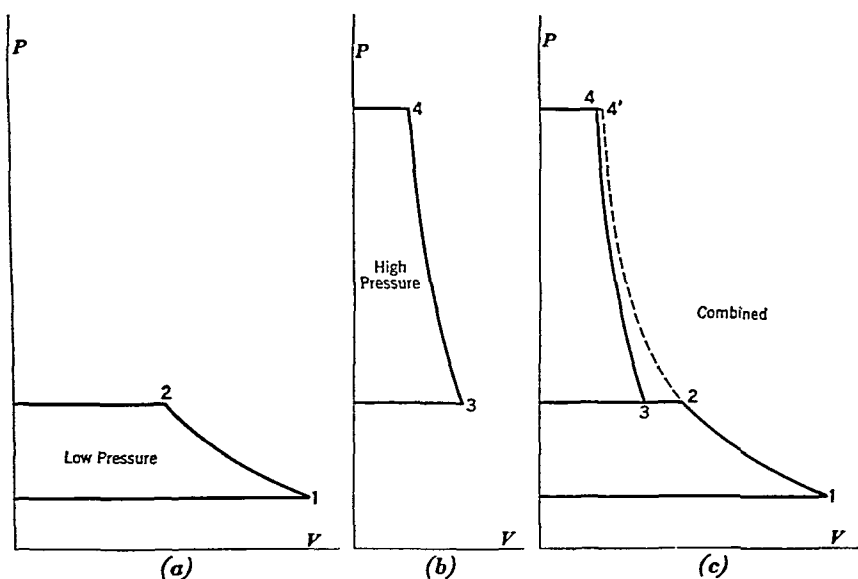


Fig. 11-8. P-V Diagram for Compound Compressor

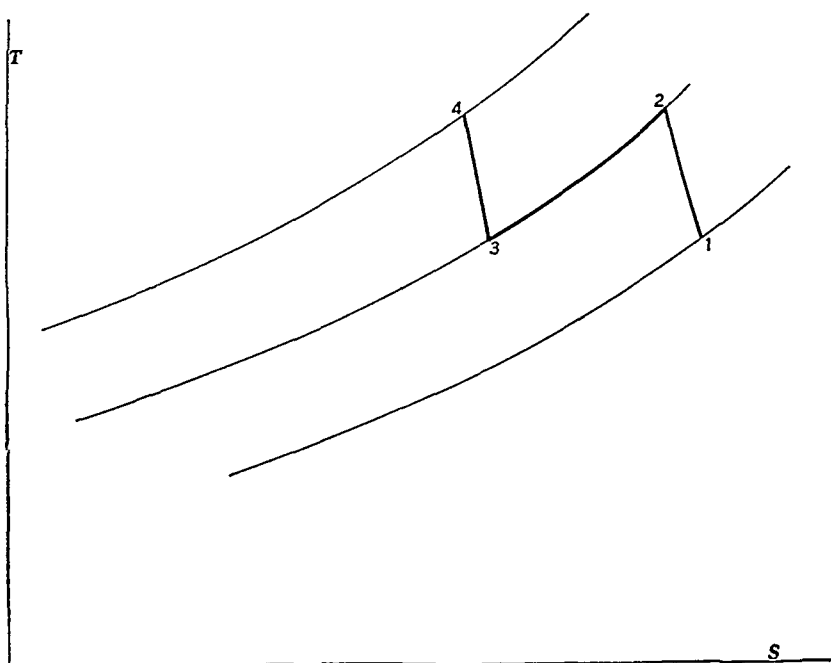


Fig. 11-9. T-S Diagram for Compound Compressor

In Fig 11-7 is shown a line diagram of a two-stage air compressor (commonly called a compound compressor). Also, in Fig 11-8 is shown the  $P$ - $V$  diagram for a compressor without clearance and in Fig 11-9 is shown the  $T$ - $S$  diagram, both being numbered to agree with the line diagram. By cooling the air, its volume is so decreased that the work of the high-pressure cylinder is reduced. Area 2-4'-4-3 in Fig 11-8 (c) shows the saving in work due to the use of an intercooler.

The work required varies with the intercooler pressure. The value of the intercooler pressure for minimum work may be found as follows.

The total work of the two stages equals  $W_{kL} + W_{kH}$ , or

$$W_k = \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{n}{n-1} P_3 V_3 \left[ 1 - \left( \frac{P_4}{P_3} \right)^{\frac{n-1}{n}} \right]$$

In the theoretical case it is assumed that there is perfect intercooling (i.e., the air is cooled down to the temperature at which it entered the low pressure cylinder). In this case,  $T_3 = T_1$  and  $P_3 V_3 = P_1 V_1$ . Furthermore, in the theoretical case it is assumed that there is no pressure drop through the intercooler (i.e.,  $P_3 = P_2$ ). Under these conditions, the total work is

$$\begin{aligned} W_k &= \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_4}{P_2} \right)^{\frac{n-1}{n}} \right] \\ &= \frac{n}{n-1} P_1 V_1 \left\{ 2 - \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_4}{P_2} \right)^{\frac{n-1}{n}} \right] \right\} \end{aligned}$$

The total work varies as the quantity  $\left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_4}{P_2} \right)^{\frac{n-1}{n}} \right]$  varies. As

this quantity decreases, the work also decreases. To ascertain the value of the intercooler pressure  $P_2$  for minimum work, it is necessary to differentiate this quantity with respect to the variable  $P_2$  and equate the derivative to zero. Thus,

$$\frac{d \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left( \frac{P_4}{P_2} \right)^{\frac{n-1}{n}} \right]}{dP_2} = 0$$

Solving this expression for  $P_2$ , we obtain

$$P_2 = \sqrt{P_1 P_4} \quad (11-7)$$



For this value of  $P_2$  the same amount of work is done in each of the two stages. It may be shown that, for a three-stage compressor, the proper low-pressure intercooler pressure  $P_2$  is

$$P_2 = \sqrt[3]{P_1^2 P_4^*} \quad (11-7a)$$

and the high-pressure intercooler pressure  $P_{2'}$  is

$$P_{2'} = \sqrt[3]{P_1 P_4^2} \quad (11-7b)$$

*Example 11-3.*—It is desired to compress 500 cu ft of air per min at 14.0 psia and 70 F to 140 psia. Assume the value of  $n$  for the compression process to be 1.35. Also assume the proper intercooler pressure for minimum work and perfect intercooling. Calculate the theoretical horsepower required with compound compression and compare it with that for single-stage compression. Calculate also the maximum temperatures in the two cases and the heat to be removed in the intercooler per minute.

*Solution.*—From equation 11-7, the intercooler pressure is

$$P_2 = \sqrt{(14.0)(140)} = 44.3 \text{ psia}$$

Based on this value of  $P_2$ , the horsepower of the low-pressure cylinder equals

$$\begin{aligned} \text{hp} &= \frac{n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] \\ &= \frac{1.35}{0.35} \frac{(144)(14)(500)}{33,000} \left[ 1 - \left( \frac{44.3}{14.0} \right)^{\frac{0.35}{1.35}} \right] = -41.0 \end{aligned}$$

The total horsepower is  $-2 \times 41.0 = -82.0$  hp for compound compression. Ans.

An alternate solution for compound compression follows: The expression for the work of compound compression is

$$\frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right] + \frac{n}{n-1} P_1 V_1 \left[ 1 - \left( \frac{P_4}{P_2} \right)^{\frac{n-1}{n}} \right]$$

When the value  $\sqrt{P_1 P_4}$  is substituted for  $P_2$  and the result is simplified, the horsepower for compound compression becomes

$$\frac{2n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{P_4}{P_1} \right)^{\frac{n-1}{2n}} \right] = \frac{2(1.35)}{0.35} \frac{(144)(14)(500)}{33,000} \left[ 1 - \left( \frac{140}{14} \right)^{\frac{0.35}{2 \cdot 1.35}} \right] = -82.0 \text{ hp} \quad \text{Ans.}$$

For single-stage compression, the horsepower equals (refer to Fig. 11-8)

$$\frac{n}{n-1} \frac{P_1 V_1}{33,000} \left[ 1 - \left( \frac{P_4}{P_1} \right)^{\frac{n-1}{n}} \right] = \frac{1.35}{0.35} \frac{(144)(14)(500)}{33,000} \left[ 1 - \left( \frac{140}{14} \right)^{\frac{0.35}{1.35}} \right] = -96.0 \text{ hp} \quad \text{Ans.}$$

For compound compression, the maximum temperature is

$$T_4 = T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (460 + 70) \left( \frac{44.3}{14.0} \right)^{\frac{0.35}{1.35}} = 714 \text{ R, or } 254 \text{ F} \quad \text{Ans.}$$

---

\* In general, stage pressure ratio  $= \left( \frac{P_{\text{final}}}{P_{\text{initial}}} \right)^{\frac{1}{\text{number of stages}}}$  and  $\frac{P_2}{P_1} = \left( \frac{P_4}{P_1} \right)^{\frac{1}{3}}$

For single-stage compression the maximum temperature is

$$T_4 = T_1 \left( \frac{P_4}{P_1} \right)^{\frac{n-1}{n}} = 962 \text{ R, or } 502 \text{ F} \quad \text{Ans.}$$

The heat removed in the intercooler is  $H_2 - H_3 = W c_p (T_3 - T_2)$

$$W = \text{Weight of air per min} = \frac{PV}{RT} = \frac{144(14.0)(500)}{53.3(460+70)} = 35.7 \text{ lb}$$

$$\text{Heat removed} = 35.7(0.24)(204 - 70) = 1157 \text{ Btu per min} \quad \text{Ans.}$$

**11-7 Compression Efficiency, Mechanical Efficiency, Compressor Efficiency**—The indicated horsepower (horsepower delivered by the face of the piston to the air) of the actual compressor differs from that of the ideal compressor, even for the same value of  $n$ , for the following reasons

- (a) Suction and discharge processes are not constant pressure ones
- (b) There are leakages in the actual compressor
- (c) The value of  $n$  for the expansion process may not equal that for the compression process

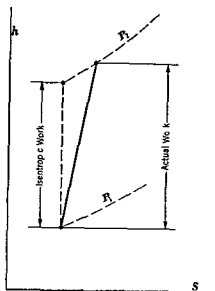


Fig 11-10 Increase in Entropy During Compression

For these reasons, it is difficult to predict the actual indicated horsepower of a compressor. The indicated horsepower of the compressor may be predicted if the so-called compression efficiency is known. This is defined as follows

$$\text{Compression Efficiency} = \frac{\text{Theoretical hp}}{\text{Actual ihp}} \quad (11-8)$$

There are as many theoretical horsepowers as there are values of  $n$ . For reciprocating compressors, particularly those that are water jacketed, it is possible to use the isothermal horsepower as the theoretical horsepower. In this case, the compression efficiency is termed the *isothermal compression efficiency*. When dealing with rotary and centrifugal compressors, in which the air passes through the compressor so rapidly that it loses little heat, it is customary to use the isentropic horsepower as the theoretical horsepower. In such cases, because of the irreversibility of the compression process, there is an increase in entropy during the compression. See Fig 11-10. For such compressors, the compression efficiency should

be termed the *isentropic compression efficiency*. By common usage, however, it is termed—incorrectly—the *adiabatic compression efficiency*. If the compression is adiabatic, it is seen by application of the steady-flow equation that the work input equals the difference between the enthalpies entering and leaving the compressor. Under these conditions, the adiabatic compression efficiency equals the ratio of the isentropic change in enthalpy to the actual change in enthalpy for the same pressure range.

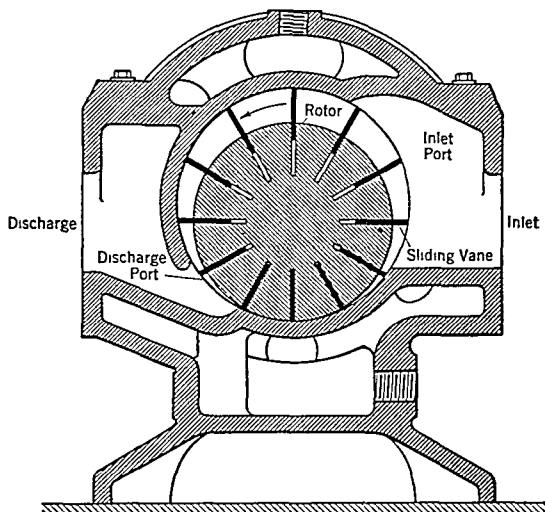


Fig. 11-11. Sliding-Vane Rotary Compressor

Because of mechanical losses in the compressor, the shaft horsepower input must exceed the indicated horsepower. The measure of the perfection of the compressor in transmitting mechanical energy is its mechanical efficiency. In general, the mechanical efficiency is the ratio of the mechanical output to the mechanical input. For an air compressor,

$$\text{Mechanical Efficiency} = \frac{\text{ihp}}{\text{Shaft hp}} \quad (11-9)$$

From equations 11-8 and 11-9, the product of the compression efficiency and the mechanical efficiency equals the compressor efficiency. The compressor efficiency may be either the isothermal compressor efficiency or the adiabatic compressor efficiency. Some writers apply the general term "engine efficiency" to both turbines and compressors.

*Example 11-4.*—Calculate the shaft horsepower required to compress 300 cu ft of air from a pressure of 14.2 psia to a pressure of 150 psia. The isothermal compression efficiency is 74% and the mechanical efficiency is 88%.

*Solution* —The isothermal horsepower equals

$$\frac{-P_1 V_1 \log_e \left( \frac{P_2}{P_1} \right)}{33,000} = \frac{-144(14.2)(300) \log_e \left( \frac{150}{14.2} \right)}{33,000} = -43.8 \text{ hp}$$

$$\text{ihp} = \frac{-43.8}{0.74} = -59.2 \text{ hp}$$

$$\text{Shaft hp} = \frac{-59.2}{0.88} = -67.3 \text{ hp}$$

Ans

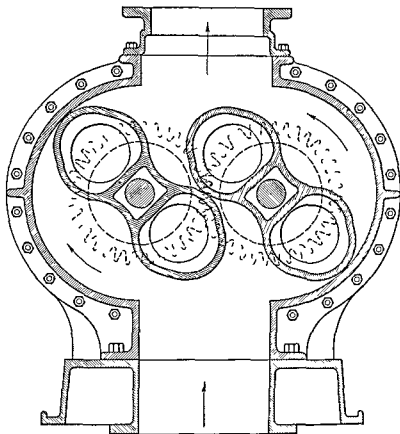


Fig 11-12 Roots Compressor

**11-8 Rotary and Centrifugal Compressors**—As with all types of reciprocating machinery, the volume of fluid which can be handled per unit of time by a reciprocating compressor is limited. In a rotating compressor, as in most rotating machinery, there are no reciprocating parts, hence, speeds can be high and large volumes of fluid can be handled per unit of time. Rotating compressors may be classified as rotary compressors and centrifugal compressors.

Two common types of rotary compressors are the sliding-vane type and the Roots type. The vane type is illustrated in Fig. 11-11. As the rotor is eccentric to its casing, rotation causes the vanes to move in and out of their slots in the rotor, centrifugal force holding them against the casing. In some types, the vanes may be moved in and out by means of an eccentric. For a given compressor there is a definite amount of compression, because there is a definite change in the volume of the space between any two vanes as they are rotated from the inlet position to the discharge position.

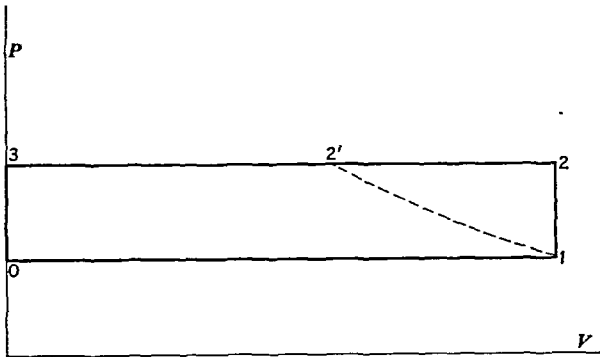


Fig. 11-13. P-V Diagram for Roots Compressor

For conditions other than design ones, the vane type of compressor is theoretically less efficient than the reciprocating type. Because of the inertia of the vanes, the speed of the vane type is not so high as that of some other types; but it is sufficiently high to be able to handle much more air than does the reciprocating compressor of the same bulk. There is a tendency toward air leakage around the vanes and toward excessive friction between the vanes and the rotor and between the vanes and the casing. In spite of these drawbacks, the vane type is used to a large extent for compression of air in amounts up to 2,000 cfm and for pressures up to 30 psi gage or a little higher.

The Roots type of compressor is illustrated in Fig. 11-12. Air is trapped between the impeller and the casing, and is carried around and discharged. Since there is no change in the volume of the space between the impeller and the casing, the air is not compressed as it is carried around. When the top edge of the impeller passes the edge of the discharge space, back flow of air from the discharge line will occur, until the pressure in the space between the impeller and the casing reaches the discharge pressure. As this building up of pressure occurs at a constant total volume, the  $P$ - $V$  diagram for the Roots type of compressor is rectangular, as indicated in

Fig 11-13 \* Here, area 1-2-3-0 represents the work required for the Roots type of compressor, and area 1-2'-3-0 represents the work for the reciprocating type of compressor. As the Roots type of compressor is used for low pressures (i.e., up to 10 to 15 psi gage), the excess of work of the Roots type compared with the reciprocating type is not a serious drawback. The Roots type is used extensively for compressing moderate to large quantities of air to low pressures. It is used to a large extent for supercharging diesel engines.

All the compressors discussed thus far have been of the so-called positive displacement type, that is, a volume of air is trapped and, leakages

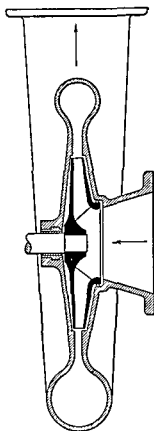


Fig 11-14 Single-Stage Centrifugal Blower

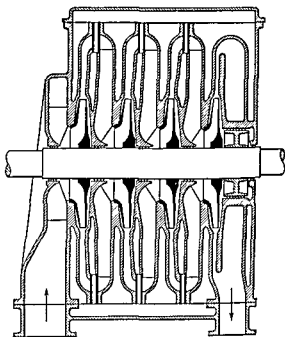


Fig 11-15 Multi Stage Centrifugal Blower

being neglected, all of it is pushed out by a definite movement of a part of the compressor. A non-positive displacement type of compressor is the centrifugal compressor. A single-stage centrifugal compressor is shown in Fig 11-14. By centrifugal action air is whirled from the center outward, more air coming to the void created. Some pressure is built up by centrifugal action. Additional pressure is obtained when the high-velocity air leaving the rotor is slowed down in a nearly reversible manner in the sta-

\* The rectangular card is a theoretical  $P$ - $V$  diagram. Inertia and throttling effects modify the actual diagram somewhat.

tionary part of the compressor. As high peripheral speeds are essential to build up an appreciable pressure, the rate of flow is high and large volumes are handled in a small space. In Fig. 11-15 is shown a multi-stage centrifugal compressor which may be used to obtain pressures as high as 30 to 35 psi gage, or even higher. Centrifugal compressors have a wide field of use, from supercharging aircraft engines to supplying air for blast-furnace work. When a centrifugal compressor is carefully designed and is operated under design conditions, its compression efficiency is high.

For supplying air for combustion in connection with gas turbines, axial-flow compressors are generally employed. These may be of either the positive or non-positive displacement type. When axial-flow compressors are carefully designed, their compression efficiency exceeds that of the centrifugal compressors. For additional discussion of these compressors, see Chapter 18.

## CHAPTER 12

### REFRIGERATION

12-1. **Introduction.**—Refrigeration is an integral part of our present civilization. One large field of use for refrigeration is that of preservation of food. This involves cooling and perhaps freezing the food and keeping it at the desired temperature. Another large field of use is the air-conditioning field. Here refrigeration is used not only to lower the air temperature to the desired point but also to dehumidify the air by cooling it to a still lower temperature. Refrigeration is also used in the preservation of materials other than foods, in the liquefaction of gases, such as air, oxygen, and nitrogen, and in many special manufacturing processes.

Originally, refrigeration was obtained by the use of natural ice. This was not only inconvenient but very inadequate. With the development of mechanical refrigeration, the use of natural ice has become insignificant.

Mechanical refrigeration is obtained by using a system which is essentially a reversed heat engine. Thus, work is brought in from the outside and put into the engine. This enables the engine to abstract heat  $Q_R$  from the cold body (*i.e.*, the place where refrigeration is desired). This heat taken from the cold body, together with the energy introduced in the form of work, is delivered to the hot body as  $Q_H$ . The operation of this system is exactly the reverse of that of a heat engine, which takes heat from the hot body and turns part of it into work and rejects the remainder of it to the cold body.

12-2. **Coefficient of Performance; Tonnage.**—A reversed heat engine performs two functions at once. It abstracts heat from a cold body and it supplies heat to a hot body. This must be true, whether or not it is the desire of the operator. Although it is conceivable that a reversed engine may be desired for both its cooling and heating effects, normally it is used either to produce a given cooling effect or a given heating effect. If the desired purpose of the reversed heat engine is that of cooling, it is called a refrigerating machine; if the desired purpose is that of heating it is termed a heat pump or warming engine.

Thermal efficiency is used to express the effectiveness of a heat engine. The effectiveness of a *reversed* heat engine is expressed by its *coefficient of performance (c.p.)*. This is defined as the desired effect divided by the work required to produce this effect. For a refrigerating machine, as the desired effect is the refrigerating effect, the c.p. is:



$$c_p = \frac{\text{Refrigerating Effect}}{\frac{W_k}{J}} \quad (12-1)$$

As the desired effect of a heat pump is the heating effect, the  $c_p$  is

$$c_p = \frac{\text{Heating Effect}}{\frac{W_k}{J}} \quad (12-2)$$

As the earlier refrigerating machines replaced natural ice, the refrigerating effect of these machines was compared with the refrigeration produced by ice. The unit settled on was the refrigeration produced by the melting of a ton of ice in 24 hours. As the latent heat of ice is about 144 Btu per pound, a refrigerating machine which could produce refrigeration at the

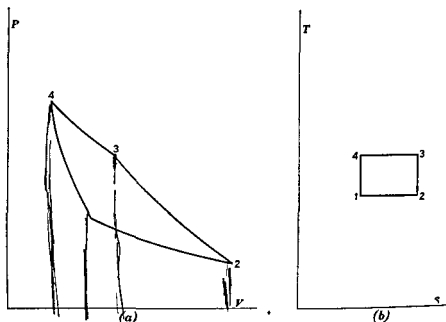


Fig 12-1 Reversed Carnot Cycle

rate of 288,000 Btu per 24 hours was rated as a 1-ton machine. Thus, a ton of refrigeration is a *rate of production of refrigeration*. The capacity of a machine, in tons, may be determined by dividing the refrigeration produced, in Btu per min, by 200, or dividing the refrigeration, in Btu per hr, by 12,000, or dividing the refrigeration, in Btu per 24 hr day, by 288,000.

**12-3. The Reversed Carnot Cycle**—A heat-engine cycle that is composed entirely of reversible processes has the highest thermal efficiency for

given conditions. Likewise, if the cycle of a reversed heat engine is composed entirely of reversible processes, its c.p. will be the highest possible for the given conditions.

When a gas is used for the working substance, the reversed Carnot Cycle has the same drawback as does the straight Carnot Cycle, namely, a very small quantity of energy per unit of cylinder volume. Nevertheless, such a reversed cycle is valuable as an indication of the highest c.p. attainable for given conditions.

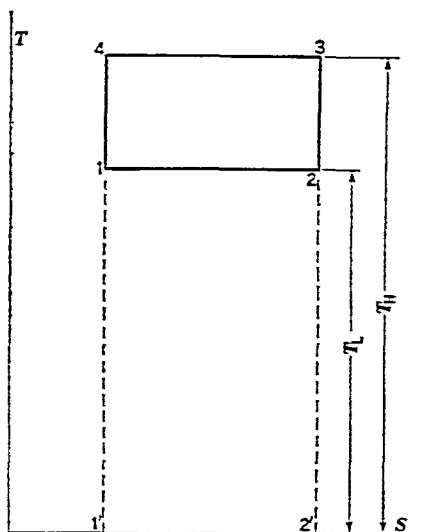


Fig. 12-2. Performance of Reversed Carnot Cycle

The reversed Carnot Cycle is shown in Fig. 12-1. Process 1-2 is a constant-temperature expansion, with heat picked up from the cold body. Between points 2 and 3 the working substance is compressed isentropically, until its temperature reaches that of the hot body. Heat is rejected to the hot body during the constant-temperature compression, 3-4. The working substance now expands isentropically, along path 4-1, until the cold-body temperature is reached.

As the Carnot Cycle is rectangular on the  $T$ - $S$  plane, this plane is very useful in easily determining the energy quantities involved. In Fig. 12-2 the refrigerating effect  $Q_R$  is represented by area 1-2-2'-1', and the heat  $Q_H$  delivered to the hot body is represented by area 3-4-1'-2'. As the difference between the heat delivered to the hot body and the heat abstracted from the cold body is equivalent to the work input, area 1-2-3-4 must represent the heat equivalent of this work input.

From the  $T$ - $S$  diagram, the coefficient of performance of a refrigerating machine operating on the reversed Carnot Cycle is

$$c.p. = \frac{Q_R}{W_k} = \frac{T_L(S_2 - S_1)}{T_H(S_2 - S_1) - T_L(S_2 - S_1)} = \frac{T_L}{T_H - T_L} \quad (12-3)$$

For a Carnot-Cycle heat pump,

$$c.p. = \frac{T_H}{T_H - T_L} \quad (12-4)$$

In Chapter 5 it was pointed out that in order to have high thermal efficiency there must be a wide range in temperatures. In order to have a high  $c.p.$ , the reverse is true, the temperature *range* must be minimized, as may be seen by an analysis of equations 12-3 and 12-4. With a high temperature of 80 F, the  $c.p.$  of a Carnot-Cycle refrigerating machine is 12.5 when the low temperature is 40 F. This  $c.p.$  reduces to 5.75 with a low temperature of 0 F, and to 2.0 with a low temperature of -100 F. This fundamental principle holds not only for the Carnot Cycle but for any reversed heat engine.

For a small refrigerating machine, heat may be rejected to the atmosphere. For larger units, the heat is normally rejected to cooling water. Generally there is little choice available in the temperature of the cooling medium which is to pick up the heat. Although a choice of cooling water at the lowest temperature available will result in the highest  $c.p.$ , such factors as cleanliness and water costs must be considered. The low temperature  $T_L$  depends on the temperature at which refrigeration is desired. For air-conditioning work this temperature is relatively high, for the production of liquefied gases, much-lower temperatures must be attained. Inherently, the maximum  $c.p.$  of a refrigerating machine depends on the use to which it is put.

★12-4 The Air Refrigerating System — Air as a refrigerant has two outstanding advantages: it is free and leakage will not cause trouble. Thermodynamically, it is a poor refrigerant and was abandoned upon development of refrigerants with superior thermodynamic properties.\* An analysis will be made, however, to discover its thermodynamic limitations.

In Fig. 12-3 is shown a line diagram of the open air system. This name is applied because the air flows into the open in the cold room at atmospheric pressure. The compressor draws the air from the cold room, compresses it, and delivers it to an air cooler. As the air flows through the cooler, heat is removed from it, causing a great decrease in its volume.

\* Recently, air refrigeration has been adopted for aircraft-cabin cooling; air from the cabin superchargers being used in connection with expansion turbines.

In the ideal case, the pressure remains constant; in the actual case, there is a slight pressure drop. As the minimum temperature of the air leaving the air cooler must be higher than the temperature of the cooling water, the air at the air-cooler exit is at too high a temperature to produce any

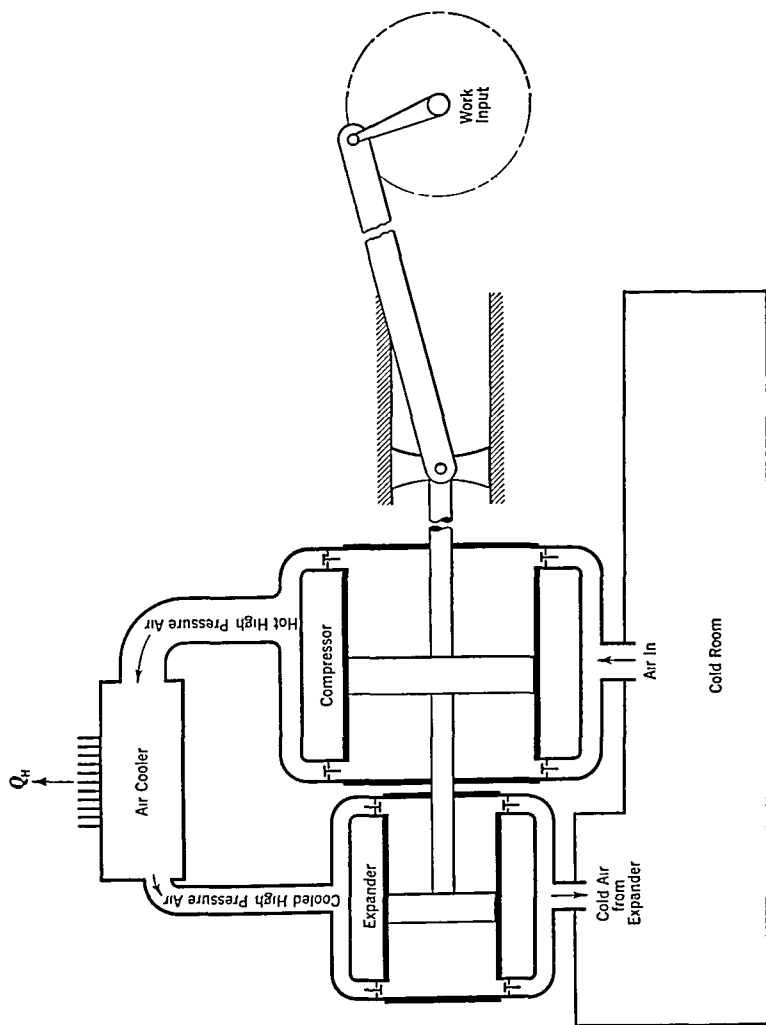


Fig. 12-3. Line Diagram of Open Air System

refrigeration. If the high-pressure air is throttled, its temperature will drop very slightly. If, however, the high-pressure air is made to do work, this work will be done at the expense of its enthalpy and there will be a large drop in temperature. This expansion is accomplished in the expansion

cylinder. The air is so low in temperature as it enters the cold room that it picks up much heat from the contents of the cold room.

The theoretical  $P$ - $V$  diagrams for both the compressor and the expander are shown in Fig. 12-4. In Fig. 12-5 (a) the  $P$ - $V$  diagram of the expander is superimposed on that of the compressor, giving the net diagram 1-2-3-4. The area of this net diagram represents the new work input to the system. The  $T$ - $S$  diagram in Fig. 12-5 (b) is a thermodynamic diagram for the system as a whole.

The characteristics of the open air refrigeration system are illustrated in the following example.

**Example 12-1.**—An open air machine is to produce 5 tons of refrigeration at a temperature of 20 F. Cooling water is available at 60 F. Assume standard atmospheric pressure in the cold room. The compressor discharges air at 147 psia. Calculate, for the theoretical case: (a) the net horsepower required; (b) the c.p.; (c) the piston displacement per minute required in the compressor; (d) the piston displacement per minute required in the expander.

**Solution**—(a) Refer to Fig. 12-5. The temperature of the air leaving the cold room, or  $t_1$ , is 20 F. In the theoretical case the air in the cooler may be cooled down to the temperature of the cooling water. Thus,  $t_4 = 60$  F. Hence,

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = (20 + 460) \left( \frac{147}{14.7} \right)^{\frac{0.4}{1.4}} = 927 \text{ R}$$

$$\text{and} \quad T_4 = \frac{T_2}{\left( \frac{P_2}{P_4} \right)^{\frac{k-1}{k}}} = \frac{60 + 460}{\left( \frac{147}{14.7} \right)^{\frac{0.4}{1.4}}} = 269 \text{ R}$$

As the process in the cold room is one of steady flow, then

$$Q_R = H_1 - H_4 = W c_p (T_1 - T_4)$$

or the weight of air is

$$W = \frac{5(200)}{0.24(480 - 269)} = 19.75 \text{ lb per min}$$

The heat rejected in the air cooler is

$$Q_H = H_2 - H_4 = W c_p (T_2 - T_4) = 19.75(0.24)(927 - 520) = 1929 \text{ Btu per min}$$

The net work input is the difference between the heat rejected in the air cooler and the refrigerating effect. Or

$$\frac{W_k}{J} = Q_H - Q_R = 1929 - 5(200) = 929 \text{ Btu per min}$$

The net horsepower is

$$\frac{929}{42.4} = 21.9 \text{ hp} \quad \text{Ans.}$$

$$(b) \quad \text{c.p.} = \frac{5(200)}{929} = 1.076 \quad \text{Ans.}$$

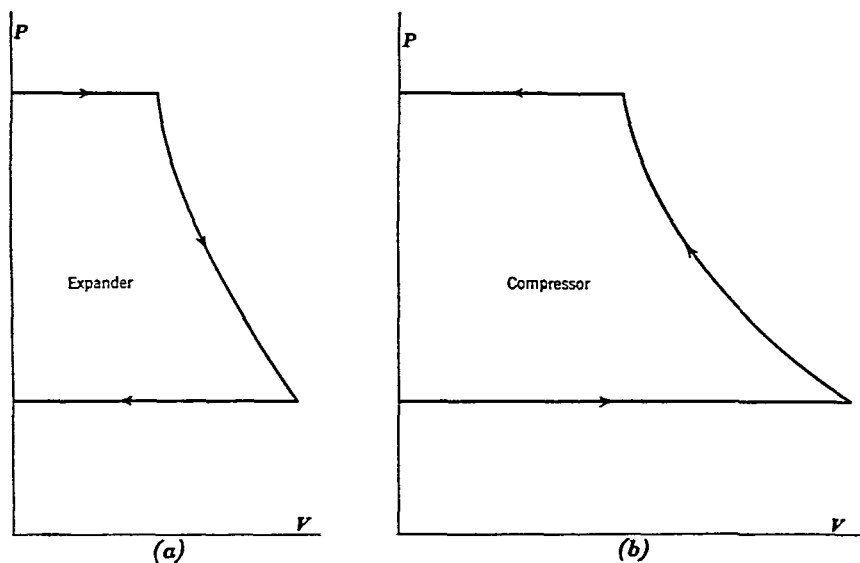
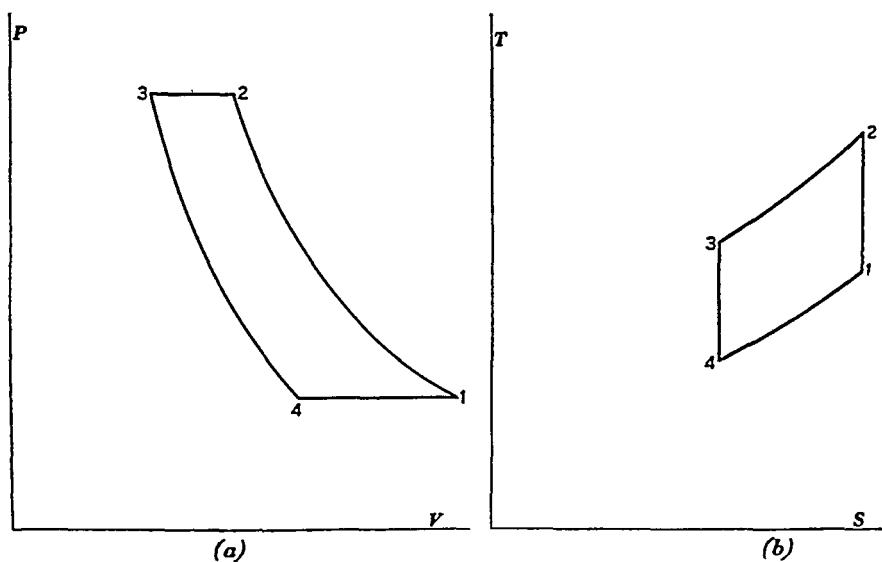
Fig. 12-4. Separate  $P$ - $V$  Diagrams for Open Air System

Fig. 12-5. Combined Diagrams for Open Air System

(c) The piston displacement of the compressor must be large enough to contain the air entering it. This volume is

$$V_1 = \frac{WRT}{P} = \frac{19.75(53.3)(480)}{144(14.7)} = 238.8 \text{ cu ft per min} \quad \text{Ans.}$$

The piston displacement of the expander must be large enough to contain the air after its expansion. This volume is

$$V_2 = \frac{WRT}{P} = \frac{19.75(53.3)(269)}{144(14.7)} = 133.8 \text{ cu ft per min} \quad \text{Ans.}$$

**Example 12-2**—Use the reversed Carnot Cycle and calculate the horsepower and the c p for the previous example.

**Solution**—The work per minute is

$$\frac{Wk}{J} = (5)(200) \left( \frac{520 - 480}{480} \right) = 83.3 \text{ Btu per min}$$

The net horsepower is

$$\frac{83.3}{42.4} = 1.964 \text{ hp} \quad \text{Ans.}$$

$$c.p. = \frac{1000}{83.3} = 12.0 \quad \text{Ans.}$$

By comparison with the reversed Carnot Cycle, the c p of the open air system is very low and the horsepower required for a given tonnage is excessive. Although the cylinder volumes required in the open air system are less than those in the Carnot Cycle, the open air system nevertheless is very bulky. The bulk of the system may be reduced by increasing the density of the air entering the compressor and leaving the expander. To do this, the air is kept in a coil in the cold room and is carried at a pressure of several atmospheres. Such a system is known as the dense air system. The dense air system was used for some time, particularly for marine refrigeration.

Although less bulky than the open air system, the dense air system is inherently too bulky. It also possesses the drawback of low c p, by the very nature of the manner in which it picks up and rejects heat. As it picks up heat at constant pressure, the temperature of the air entering the cold room must be far below the cold room temperature to keep the weight of air circulated and the cylinder volumes down. Likewise the temperature of the air entering the air cooler must be high. This means that the over-all temperature range in the cycle is very high in comparison to the temperature difference between the cold room and the cooling water.

**12-5 The Theoretical Vapor Compression System**—If the pressure of a vaporizing fluid is maintained at a constant value, the fluid picks up heat at constant temperature. Likewise, under constant pressure, a condensing vapor gives up heat at constant temperature. Thus, if a fluid is used which will vaporize under cold room conditions and can be liquefied

by the cooling water, the mean temperature range in the cycle will approach the temperature difference between the cold room and the cooling water.

The vaporizing temperature of a fluid depends on the pressure exerted on it (see Chapter 6). By adjusting the pressure on it, the fluid may be made to boil at a temperature close to the desired cold-room temperature. As the temperature of the cooling water exceeds that of the cold room, the vapor formed in the cold room must have its pressure built up in order that it may be condensed and reused. When the pressure on the condensed liquid is reduced, its temperature will also drop, part of the liquid flashing into vapor. Because of this there is no need for an expander.\*

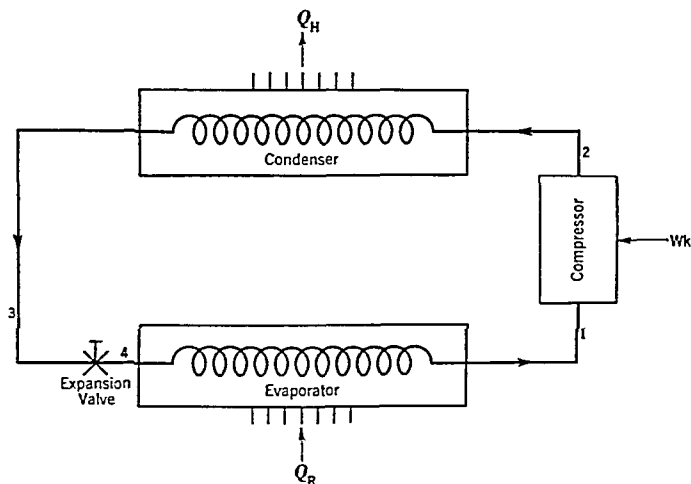


Fig. 12-6. Line Diagram of Vapor Compression System

A line diagram of the equipment used in a vapor compression system is shown in Fig. 12-6. The  $T$ - $S$  diagram for the theoretical case is shown in Fig. 12-7, which is numbered to agree with Fig. 12-6. In this case wet vapor leaves the evaporator and enters the compressor at point 1. Compression is isentropic and the vapor leaves dry saturated and enters the condenser at point 2. The condensate formed in the condenser leaves it and enters the expansion valve at point 3. The very wet vapor enters the evaporator at point 4. In the evaporator, the liquid part of the very wet vapor settles out and is changed into a vapor by boiling, producing refrigeration as it does so. Area 4-1-8-7 represents the refrigerating effect and area 2-3-5-8 represents the heat rejected.

\* If an expander is introduced in the vapor system, there will be a small improvement, theoretically, in the c.p.; but the operating difficulties will be increased because a liquid is fed to the expander. In addition, the bulk of the system will be increased.



**Example 12-3** — It is desired to furnish 5 tons of refrigeration at 20 F. The cooling water temperature is 60 F. The refrigerant, which is dichlorodifluoromethane (Freon, F-12), is dry saturated at the end of compression and is a saturated liquid entering the expansion valve. Calculate, for the theoretical case (a) the horsepower required (b) the c p, and (c) the piston displacement of the compressor.

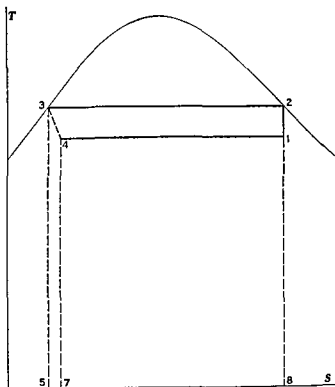


Fig 12-7 Theoretical Vapor Compression System With Wet Compression

**Solution** — (a) The properties of F-12 are taken from the Appendix. For the cycle represented in Fig 12-7,

$$S_1 = S_2 = 0.16741 = S_g - (1-x)S_{fg}$$

$$= 0.16949 - (1-x_1)(0.14166)$$

$$1 - x_1 = 0.01468$$

from which

$$\text{Hence, } h_1 = h_g - (1-x_1)h_{fg} = 80.49 - 0.01468(67.94) = 79.49 \text{ Btu per lb}$$

As process 3-4 is a throttling process, the enthalpy remains constant, and

$$h_4 = h_1 = 21.57 \text{ Btu per lb}$$

$$\text{Since } Q_R = H_1 - H_4 = W(h_1 - h_4),$$

$$W = \frac{5(200)}{79.49 - 21.57} = 17.27 \text{ lb per min}$$

Ans.

There being no heat losses the compressor work is

$$H_2 - H_1 = W(h_2 - h_1) = 17.27(84.82 - 79.49) = 92.0 \text{ Btu per min}$$

Then the horsepower is

$$\frac{92.0}{42.4} = 2.17 \text{ hp}$$

Ans

$$(b) \quad c.p. = \frac{5(200)}{92.0} = 10.87$$

Ans.

$$(c) \text{ The specific volume entering the compressor is } v_1 = v_g - (1 - x_1)v_{fg} = 1.121 - 0.01468(1.1097) = 1.105 \text{ cu ft per lb}$$

The piston displacement of the compressor is

$$(1.105)(17.27) = 19.08 \text{ cu ft per min}$$

Ans.

A comparison of the results of Example 12-3 with those obtained in Example 12-1 on the open air system shows that the vapor compression system is vastly superior to the open air system. Because of the throttling action in the expansion valve, the results for the vapor compression system are not quite so good as those for the reversed Carnot cycle.

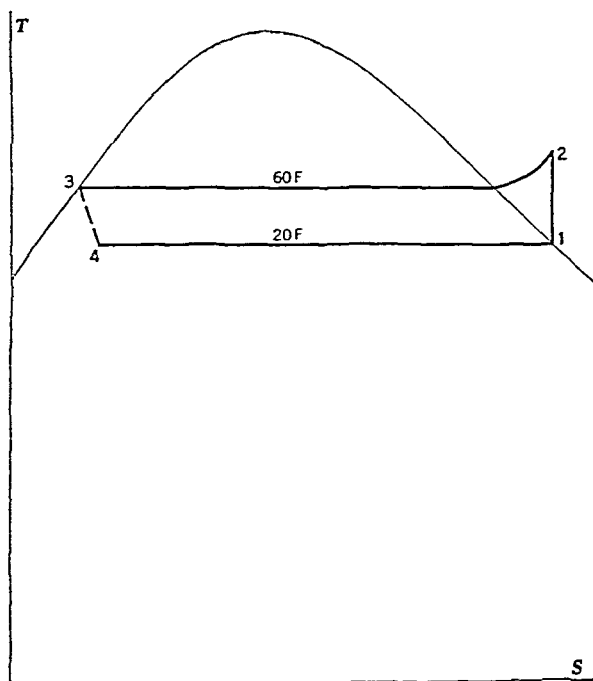


Fig. 12-8. Diagram for Example 12-4

In the vapor system just discussed, the compression was wholly in the saturated region; hence, this is known as wet compression. Operating difficulties may be encountered in taking such a wet vapor from the evaporator, as the operation there may not be steady. In this case, priming will take place (*i.e.*, slugs of liquid will be carried out of the evaporator into the compressor). Although the compressor is designed with a safety head, which will be raised in case too much liquid is present during com-

pression, and cylinder damage will be prevented, such operation is not desirable. For this reason, the liquid level in the evaporator is kept low, in order that there will be much liquid-separating space. Furthermore, as the vapor is in contact with more surface, it will receive more heat than when a high liquid level is maintained. To make certain that no appreciable amount of liquid can enter the compressor, the liquid level in the evaporator is carried so low that the vapor is often superheated upon leaving the evaporator. If the vapor is dry or superheated when entering the compressor, the compression is said to be dry.

*Example 12-4* —Repeat the previous example, assuming that the vapor is dry saturated upon leaving the evaporator

*Solution* —(a) As indicated in Fig. 12-8, in which the points are numbered to agree with Fig. 12-6, the enthalpy at points 3 and 4 is the same as that with the wet compression, namely, 21.57 Btu per lb. From the table in the Appendix, the enthalpy at point 1 is 80.49 Btu per lb and the entropy is 0.16949. At entrance to the condenser, or at point 2, the pressure is the saturation pressure for the condensing temperature, namely, 72.41 psia, and the entropy is 0.16949. By interpolation, the enthalpy at point 2 is 85.92 Btu per lb.

$$\text{As } Q_R = W(h_1 - h_4),$$

$$W = \frac{5(200)}{80.49 - 21.57} = 16.97 \text{ lb per min}$$

The compressor work is

$$H_2 - H_1 = W(h_2 - h_1) = 16.97(85.92 - 80.49) = 92.15 \text{ Btu per min}$$

and the horsepower is

$$\frac{92.15}{42.4} = 2.173 \text{ hp} \quad \text{Ans}$$

$$(b) \quad c.p. = \frac{5(200)}{92.15} = 10.85 \quad \text{Ans}$$

(c) As the specific volume entering the compressor is 1.121, the piston displacement of the compressor is

$$16.97(1.121) = 19.02 \text{ cu ft per min} \quad \text{Ans}$$

A comparison of the results of Examples 12-3 and 12-4 shows very little difference between wet and dry compression. If the compression were carried over a larger pressure range, the dry compression being thrown further into the superheat region, it would be found that the difference between the c.p. of the wet compression and that of the dry compression would be greater, in favor of the wet compression.

**12-6. The Actual Vapor Compression System.**—Some of the reasons why the actual system differs in performance from the theoretical are:

- (1) The condensing-vapor temperature must be higher than that of the condensing water in order to transfer heat to it.
- (2) The evaporating-liquid temperature must be lower than that of the refrigerated body in order to pick up heat from it.

- (3) The compression is not isentropic, and it may not be close to an adiabatic.
- (4) There will be appreciable pressure drops through both the suction valve and the discharge valve of the compressor.
- (5) Heat may be transferred into or out of the pipe lines and other parts of the system.
- (6) There may be slight pressure drops in other parts of the system, as well as through the compressor valves.
- (7) The liquid will be subcooled (*i.e.*, cooled below the saturation temperature for the given pressure) in the condenser.

The effects of some of the more important of these variables will be shown by the following examples.

*Example 12-5.*—To cause heat transfer in the condenser and the evaporator, assume a condensing vapor temperature of 70 F and an evaporating temperature of 10 F. Otherwise, use the same conditions as in Example 12-4. Calculate the horsepower required.

*Solution.*—From the table in the Appendix,  $h_3 = h_4 = 23.90$ ,  $h_1 = 79.36$ , and  $S_1 = 0.17015$ . The condensing pressure is  $P_2 = 84.82$  psia. By interpolation,  $h_2 = 87.87$  Btu.

As  $Q_R = W(h_1 - h_4)$ ,

$$W = \frac{5(200)}{79.36 - 23.90} = 18.03 \text{ lb per m in}$$

Then the work is

$$W(h_2 - h_1) = 18.03(87.87 - 79.36) = 153.4 \text{ Btu per min}$$

and the horsepower is

$$\frac{153.4}{42.4} = 3.62 \text{ hp} \quad \text{Ans.}$$

It is to be noted that the temperature differences in the evaporator and compressor cause a large increase in the horsepower required. If the original temperature range had been larger, the temperature differences in the evaporator and condenser would not show such a large percentage increase in power required.

*Example 12-6.*—Repeat Example 12-5, but take into account throttling through the compressor valves by assuming that the pressure at the end of suction is 27.0 psia and the pressure at the end of discharge is 90 psia.

*Solution.*—As shown in Fig. 12-9, points 1 and 2 are outside the compressor, and points 1' and 2' are inside the compressor. In the theoretical case, there is no net change in enthalpy during either suction or discharge. The enthalpy at points 3 and 4 remains at its value of the previous example, or 23.90 Btu per lb, and the enthalpy at point 1 remains 79.36 Btu per lb. The enthalpy at point 1' is also 79.36 Btu, and the pressure is 27 psia. It is found by interpolation that, for an entropy of 0.1720 and a pressure of 90 psia, the enthalpy at point 2', and hence at point 2, is 89.02 Btu per lb. As the con-

ditions in the evaporator remain the same the weight circulated remains at 18.03 lb per min. The work equals

$$W(h_2 - h_1) = 18.03(89.02 - 79.39) = 173.6 \text{ Btu per min}$$

and the horsepower is

$$\frac{173.6}{42.4} = 4.094 \text{ hp}$$

Ans.

By comparison with the previous example, it may be seen that in this case the pressure drops through the valves cause an increase in the power required of about 13 per cent.

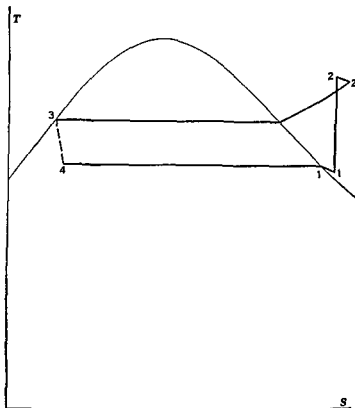


Fig 12-9 Diagram for Example 12-6

In case the compression is such that there is a high degree of super heat at the end of compression, an appreciable saving in the horsepower required may be made by water-jacketing the compressor. (In Chapter 11 this subject was discussed in relation to air compressors.) On the other hand, if the refrigerant is only slightly superheated at the end of compression, little is to be gained by the use of a water jacket. When no water jackets are used, there is an increase in entropy during the compression process by virtue of its irreversibility. Even when water jackets are used, there may be an increase in entropy during compression, as the

energy added to the fluid by virtue of piston friction may exceed that removed by the jacket.

The following example is one in which most of the deviations of the actual from the theoretical occur.

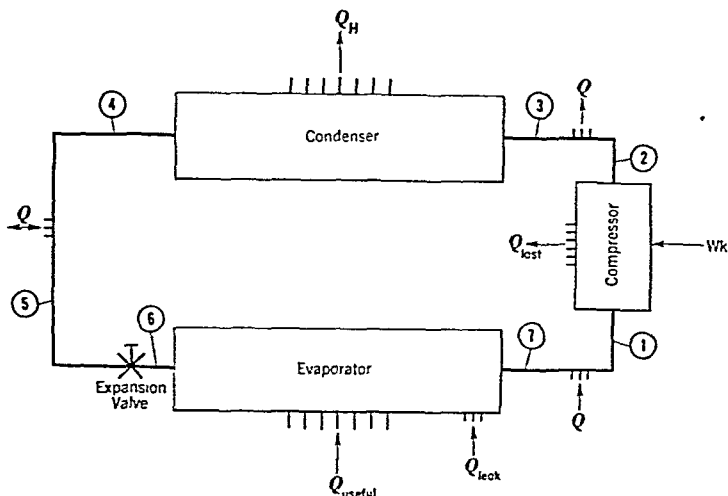


Fig. 12-10. Line Diagram for Example 12-7

**Example 12-7.**—In a compression system using Freon, F-12, the evaporator pressure is 30 psia and the condenser pressure is 90 psia. Freon leaves the condenser at 62 F and enters the expansion valve at 68 F. It leaves the evaporator at 20 F and enters the compressor at 30 F. It leaves the compressor at 140 F and enters the condenser at 130 F. The gross tonnage produced in the evaporator (i.e., the useful refrigeration plus the heat leakage into the evaporator) is 42.8 tons. The heat lost from the compressor by convection and radiation is 250 Btu per min. Calculate the horsepower and the c.p., and make an energy balance\* on the system. The connecting pipe lines are short, and pressure drops in them may be neglected.

**Solution.**—Fig. 12-10 shows a line diagram of the set-up and the corresponding T-S diagram is shown in Fig. 12-11. Note that the compression line is shown dotted. As the pressure drops through the valves are unknown, the terminal points of the compression line are indeterminate. The dotted line simply connects the state points existing outside the compressor.

From the table in the Appendix, the following enthalpies are obtained:  $h_1 = 82.15$ ,  $h_2 = 96.69$ ,  $h_3 = 95.08$ ,  $h_4 = 22.03$ ,  $h_5 = h_6 = 23.42$ , and  $h_7 = 80.73$ . As  $Q_R = W(h_7 - h_2)$ ,

$$W = \frac{42.8(200)}{80.73 - 23.42} = 149.3 \text{ lb per min}$$

\* The energy balance on the system assumes that the system is surrounded by an imaginary envelope. The energy passing into the system through this envelope is grouped under the heading of "energy in" and the energy passing out from the system is grouped under the heading of "energy out." As the energy in must equal the energy out, if equilibrium is established, the energy balance permits determining an unknown quantity or checking the accuracy of determinations.

The equation of steady flow for the compressor reduces to

$$Wh_1 + \frac{Wk}{J} = Wh_2 + Q_{\text{lost}}$$

In this case

$$\frac{Wk}{J} = 149.3(96.69 - 82.15) + 250 = 2421 \text{ Btu per min}$$

and the horsepower equals

$$\frac{2421}{42.4} = 57.10 \text{ hp} \quad \text{Ans.}$$

$$c.p. = \frac{42.8(200)}{2421} = 3.54 \quad \text{Ans.}$$

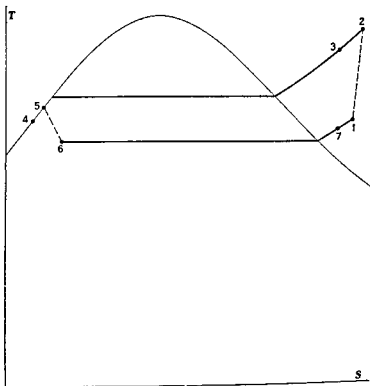


Fig 12-11 T-S Diagram for Example 12-7

### Energy Balance

Energy in (Btu per min)

Gross refrigeration

Work

Pipe line (condenser to valve)

Pipe line (evaporator to compressor)

$$(42.8)(200) = 8,560$$

$$= 2,421$$

$$149.3(23.42 - 22.03) = 207$$

$$149.3(82.15 - 80.73) = 212$$

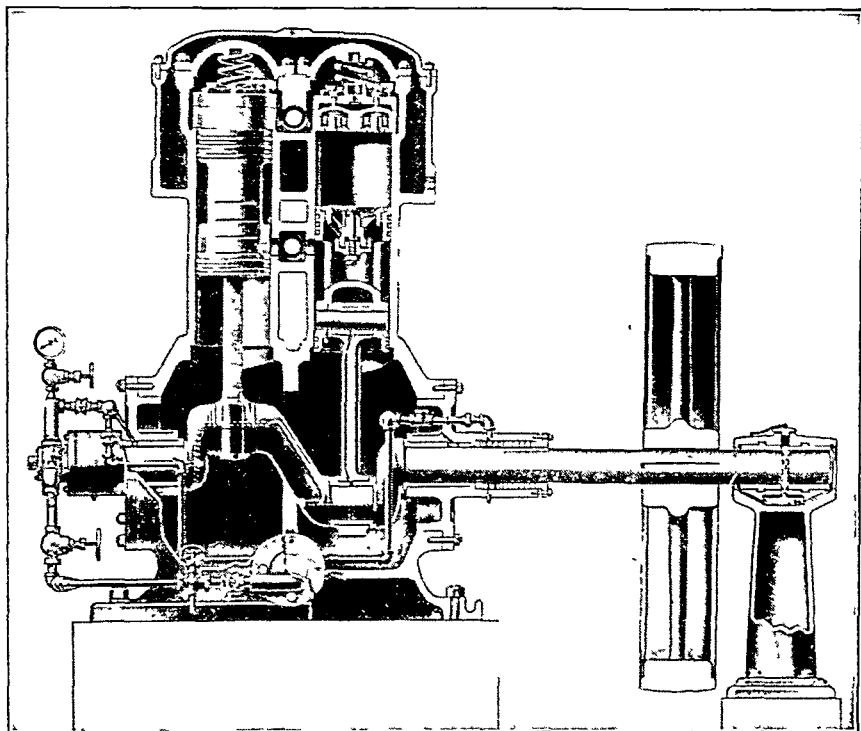
$$\text{Total} = 11,400$$

*Energy out (Btu per min)*

Condenser.....	149.3(95.08—22.03) = 10,906
Compressor heat lost.....	= 250
Pipe line (compressor to condenser).....	149.3(96.69—95.08) = 241

Total = 11,397

★12-7. **Units of the Actual System.**—The units in an actual vapor compression system are the compressor, the evaporator, and the condenser.



(Courtesy of Frick Company)

Fig. 12-12. Single-Acting, Jacketed Reciprocating Compressor

(A) **Compressors:** Refrigerating compressors may be classified as (a) reciprocating, (b) rotary, and (c) centrifugal. In the smaller sizes, particularly in the household field, the rotary type is used to some extent. Otherwise, the compressors are either of the reciprocating type or of the centrifugal type, with the reciprocating type predominating.

Except in the largest sizes, reciprocating compressors are single acting. In Fig. 12-12 is shown a single-acting jacketed compressor. This compressor, typical of the larger machines, differs from a gas compressor in



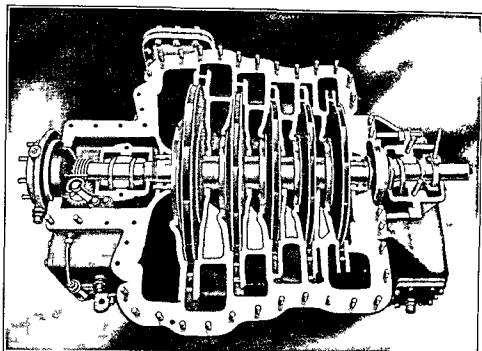


Fig 12-13 Centrifugal Compressor *(Courtesy of Carrier Co p)*

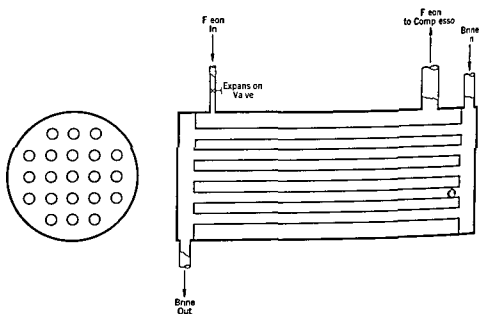


Fig 12-14 Shell and Tube Evaporator

that the suction valves are located in the piston. This construction makes it possible to have a larger discharge-valve area in the cylinder head.

As the reciprocating compressor cannot handle large volumes, it is necessary, when dealing with those vapors which have large specific volumes at compressor entrance, to use a centrifugal compressor. Fig. 12-13 shows the lower half of a centrifugal refrigerating compressor having five stages.

(B) *Evaporators*: Evaporators may be used to cool gases, particularly air in air-conditioning work. Although bare-tube evaporators may be used for this work, it is customary to use evaporators with tubes of the finned type. For refrigeration of liquids, the common type of evaporator employed is the shell-and-tube type. In Fig. 12-14 is shown a shell-and-tube evaporator used as a brine cooler. A double-pipe evaporator is sometimes used. As the name implies, this type consists of an inner pipe containing the refrigerant and a surrounding pipe through which flows the fluid to be refrigerated. Still another type of evaporator is the shell-and-coil type, which is illustrated in Fig. 12-15.

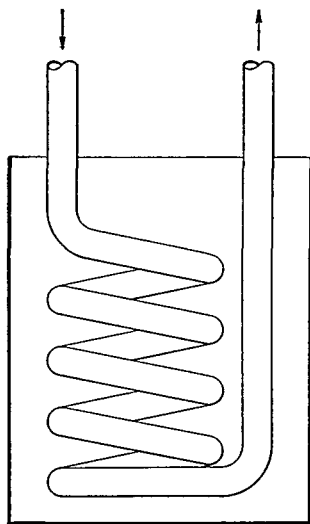


Fig. 12-15. Shell-and-Coil Evaporator

(C) *Condensers*: In the smaller sizes, condensers are usually air-cooled. The air-cooled condenser generally consists of finned tubes. In the larger sizes, shell-and-tube types of condensers are used to a large extent. Quite often, the shell-and-tube condenser is placed in a vertical position, the bottom of the shell acting as a receiver for the liquid refrigerant. In this way there is no necessity of having a separate receiver to store the liquid refrigerant. Double-pipe condensers and shell-and-coil condensers are also used to some extent. Another type of condenser used is the evaporative condenser. In this type, the vapor to be condensed passes through a coil, on which is sprayed water. Air is drawn through the spray to remove the water vapor formed as the condensing refrigerant gives up heat.

★12-8. *Refrigerants.\**—Many fluids are, or have been, used as refrigerants. Several of these had certain properties which led to their discontinuance, in spite of some desirable features. No refrigerant is

\* A more thorough discussion of refrigerants than is possible here is given in Chapter 7, *Refrigerating Data Book*, 1951, A.S.R.E.

perfect in all respects. In some fields of service some properties are extremely important, whereas in other fields of service these properties are much less important. For example, in the "air conditioning for comfort" field of service, a toxic refrigerant must be avoided. On the other hand the toxic properties of a refrigerant are not particularly objectionable in the manufacture of ice. Hence, before one can determine whether a refrigerant is suitable or not by comparing its good and bad properties, it becomes necessary to decide first on the use to which the refrigerant is to be put. After the use to which the refrigerant is to be put is known a decision should be made as to the importance of each of the various properties in connection with the given field of service.

Some of the properties to be considered are

- (1) Toxicity
- (2) Stability
- (3) Evaporating and condensing pressures
- (4) Vapor and liquid densities
- (5) Vapor and liquid viscosities
- (6) Critical and freezing points
- (7) Conductivity and film coefficient
- (8) Corrosiveness
- (9) Miscibility with oil
- (10) Latent heat
- (11) Electrical resistance
- (12) Explosiveness
- (13) Leak tendency
- (14) Action on refrigerated products
- (15) Cost

One of the earliest refrigerants used was carbon dioxide. Although carbon dioxide is a safe refrigerant and is relatively inexpensive, the c p obtained with it is so low that it is rarely used now as a refrigerant. The reason for such a low c p is that the operating range is close to the critical condition, as indicated in Fig. 12-16. If any other refrigerant were to be operated as close to its critical condition as is carbon dioxide, it too would show a low c p. Solid carbon dioxide (dry ice) is used to some extent when a small amount of refrigeration is desired intermittently. It has the advantage over water ice of producing lower temperatures, of sublimating rather than melting to wet the product, and of blanketing the product with a gas that is heavier than air.

Another early refrigerant was ammonia. Ammonia is relatively cheap, has a high c p, has a high chemical stability, does not corrode steel, and has a high latent heat. The over all operating cost is lower than that for

other refrigerants used in industrial compression systems. However, ammonia is highly toxic, and its use is therefore restricted to those fields where the toxic properties of the refrigerant are not a serious drawback (*i.e.*, refrigeration where many people are not present).

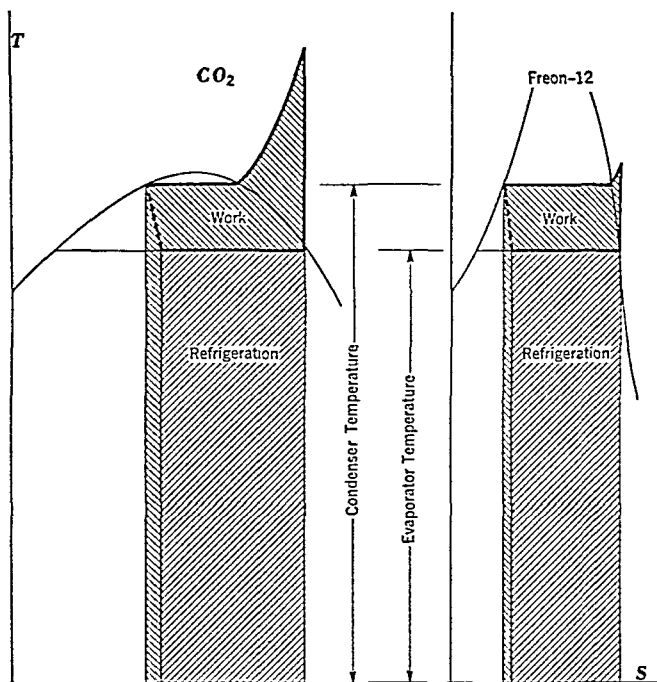


Fig. 12-16. Operating Ranges of Carbon Dioxide and Freon-12

A third early refrigerant was sulfur dioxide. It also is a cheap refrigerant and has a low operating cost, has a high c.p., and is very stable chemically. But, sulfur dioxide is highly toxic. Today its use is restricted almost entirely to the household field. The amount of the refrigerant used in a household refrigerator is so small that a leak should not be serious if the house is well ventilated. This is particularly true because the strong odor of sulfur dioxide quickly indicates the presence of even a very small leak.

A relatively new group of refrigerants is the Freon group. These are fluorine compounds. Although fluorine is a very active element, the Freon refrigerants are quite stable and are the least toxic of the refrigerants that have been discovered as yet. Therefore, in spite of the fact that they are relatively expensive, these refrigerants are very widely used, particularly in those fields where the toxic properties of the refrigerant have

to be considered. There are at least five substances in the Freon group F-11, F-12, F-21, F-113, and F-114. The thermodynamic properties of these refrigerants differ greatly. For instance, the specific volume of F-12 is low at compressor entrance and, hence, this substance is well suited for reciprocating compressors. On the other hand, the specific volume of F-113 vapor is large and, hence, this product is a good refrigerant for centrifugal compressors.

Many other refrigerants are used to a limited extent, particularly for special fields of service.

**★12-9 Operating Characteristics**—The performance of a refrigerating machine, in regard to both the  $cp$  and the tonnage produced by a machine of a given size, depends on the operating conditions. Therefore to rate a machine, the operating conditions must be specified. The standard conditions are defined as pressures corresponding to saturated vapor temperatures of 5 F at the compressor intake and 86 F at the compressor discharge, 9 deg F of subcooling and 9 deg F of superheating fix the final refrigerant temperatures used, respectively, at intake to the expansion valve and at intake to the compressor.

A change in any of the operating conditions, in addition to its individual effect, may cause secondary effects by changing other operating conditions. For instance, if the opening of the expansion valve is increased, more liquid will flow into the evaporator, and the evaporator pressure will be increased until the increased vapor density entering the compressor will be sufficient to allow the compressor to withdraw refrigerant from the evaporator at the rate at which it enters. The primary effect of the increased evaporator pressure, and hence temperature, will be to increase the  $cp$ , as has been discussed previously in this chapter. Along with the increased  $cp$ , there will be an increase in the tonnage of the machine, which is almost proportional to the percentage increase in the evaporator pressure. This is true because the tonnage is roughly proportional to the weight circulated. This weight, in turn, is approximately proportional to the density entering the compressor, which varies with the evaporator pressure.

The secondary effect of opening the expansion valve is some increase in the condenser pressure. This occurs because, with increase in flow, much more heat must be transferred in the condenser. To transfer the additional amount of heat, there must be a greater temperature difference between the condensing vapor and the condensing water.

With a decrease in the condensing-water temperature, there will be a decrease in condensing temperature and hence an increase in the  $cp$  of the system. However, it may not be economically sound to choose a condensing water with a low temperature if the cost of the water is high,

if the pumping charges are high, or if the water is so impure that it may foul the condenser tubes.

In case the heat-transfer surface of the condenser is of ample size, there will be some subcooling of the condensate. The amount of this subcooling will depend on the surface area, the rate of vapor flow to the condenser, and the velocity and temperature of the condensing water. There will be some improvement in the c.p. as the amount of subcooling increases, because more refrigeration will be obtained for the same amount of work.

The amount of superheat normally encountered at the compressor entrance has only a slight effect on the c.p., which decreases with an increase in the amount of superheat.

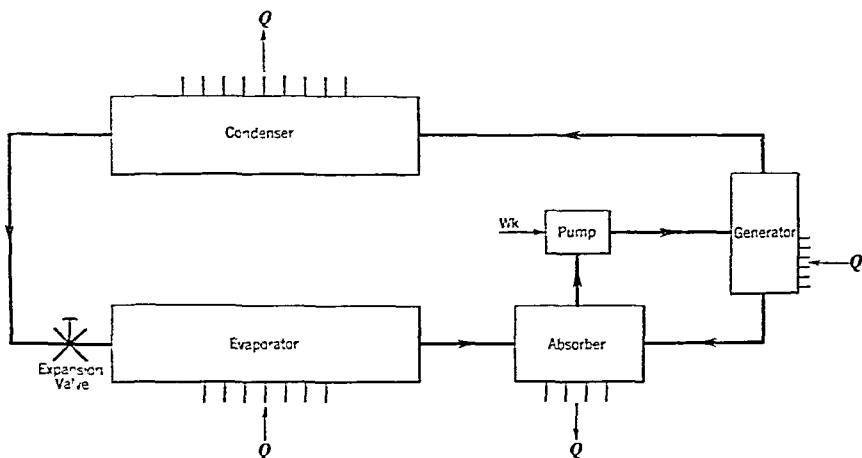


Fig. 12-17. Simple Absorption System

★12-10. The Absorption System.—One drawback of the vapor compression system is that it requires a compressor, which, as it must handle a vapor, is large and requires much power for its operation. If means were available for greatly reducing the vapor volume before it was pushed up to high pressure, then there would be a reduction in both the bulk of the system and the power required to operate it. Some liquids have the ability of absorbing large quantities of certain vapors, thus greatly reducing the total volume. A system that operates on this principle is called an absorption system. Water, in particular, has the ability to absorb very large quantities of ammonia vapor. In this case a pump may be used to build up the pressure on the resultant liquor, as it is known. Under the high pressure, the ammonia vapor must be taken out of solution in order that it may condense and continue to produce refrigeration. This is accomplished by adding heat to the high-pressure liquor, causing it to boil.

As the boiling temperature of ammonia is much lower than that of water the major portion of the vapors given off upon boiling is ammonia, which may be fed to the condenser

The essential parts of an absorption system are shown in Fig 12 17 By comparison with Fig 12-6, it is to be noted that systems of both types contain a condenser, an expansion valve, and an evaporator In place of the compressor of the compression system, there are the absorber, the pump, and the generator (where the high pressure liquor is boiled)

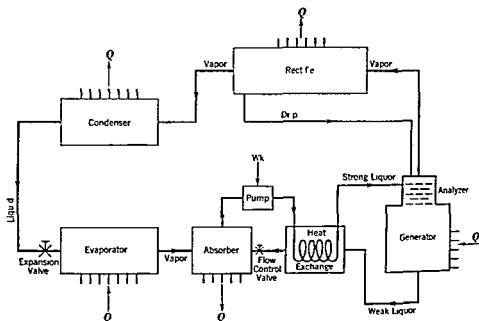


Fig 12 18. Absorption System With Heat Exchanger and Rectifier

At high temperatures the capacity of water for absorbing ammonia vapor is low. Hence, the weak liquor (water containing little ammonia) taken from the generator must be cooled before it can absorb appreciable quantities of vapor in the absorber. The heat removed from the weak liquor may be used to raise the temperature of the strong liquor coming from the absorber and going to the generator, and thus minimize the amount of heat to be supplied to the generator. This heat transfer is accomplished by adding a counter flow heat exchanger, as shown in Fig 12-18. The vapors leaving the generator are partly water vapor. Unless the major portion of this water vapor is removed before the vapors enter the condenser, so much water may enter the evaporator that it may freeze there. This water vapor is partly removed in an analyzer by passing the vapors up through a spray of the strong liquid coming from the heat

exchanger. A large part of the water vapor which leaves the analyzer is removed by cooling the vapors in a rectifier, which is a partial condenser.

It is difficult to compare the thermodynamic performance of the absorption system with that of the vapor compression system. In the vapor compression system, the energy supplied as work is entirely available energy. In the absorption system, the work input is extremely small. On the other hand, the heat which must be added to the generator is much larger in amount than the work required in the vapor system. However,

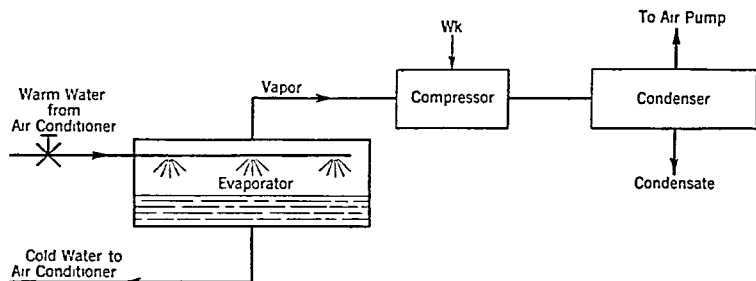


Fig. 12-19. Water Vapor Refrigeration System With Compressor

only a part of the heat added is available energy. The only real basis of comparison of the two systems is the relative economy. Where the energy required to drive the compressor is expensive, or where the generator is supplied largely with heat that otherwise would be wasted (say exhaust steam), the absorption system may be the cheaper one to operate.

A modification of the absorption system just described is the Servel-Electrolux System. In this system, which is widely used in household gas refrigerators, the pump is eliminated. The whole system is under substantially the same pressure. The evaporator of the system is charged with enough hydrogen to render the resulting ammonia pressure so low that the liquid ammonia there will boil and produce refrigeration. As the hydrogen is only slightly soluble in water and as liquid seals prevent backflow to the condenser, the hydrogen remains in the evaporator. The units of the system must be so arranged that flow will be obtained by thermal action.

★12-11. **Water Vapor Refrigeration.**—When water is to be used for the refrigeration of air in air-conditioning work, the water itself may be refrigerated by another refrigerant. However, the c.p. of the system may be increased if the secondary refrigerant is eliminated and the water itself is used as the only refrigerant. This may be done by spraying the warm return water into a tank (which acts as an evaporator), in which a very low pressure is maintained. If no air is present, the water will reach



the saturation temperature for the given pressure in the evaporator. Thus system is illustrated in Fig. 12-19.

If low temperature water is desired, the vapor volumes will be very large. For example, if water is desired at 40 F, the specific volume of the dry saturated vapor will be 2.444 cu ft per lb. This means that the compressor must handle about 475 cu ft of vapor per min per ton of refrigeration. This is far beyond the capacity of a reciprocating compressor. To handle the very large volumes, a centrifugal water-vapor compressor is often used.

In case steam at moderate pressure is available, a steam-jet ejector may be used as the compressor. Such an ejector is shown schematically in Fig. 12-20. Steam expands in the nozzle, coming out at high velocity, and enters the combining chamber. In the combining tube, the high velocity steam entrains the vapor from the evaporator and imparts kinetic energy to it. In the diffuser, the velocity of the mixture is decreased in an ordinary manner, the pressure being built up by this process.

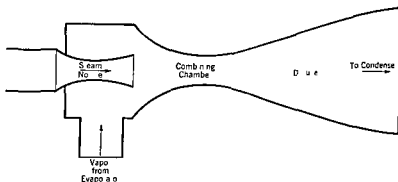


Fig. 12-20 Steam Jet Ejector Used as Compressor

**12-12 The Heat-Pump Method of Heating**—As mentioned in Art. 12-2, a reversed heat engine may be used for its heating effect as well as for its refrigerating effect. This idea is not a new one, having been proposed by Lord Kelvin many years ago for use in India. Such a machine may pump heat from outside a building to heat the building in the winter, and may pump heat out of the building in summer. In such a system the unit which is located outside the building acts as an evaporator in winter but becomes a condenser in summer, the unit located inside the building, which acts as a condenser in winter, becomes an evaporator in summer. The thermodynamics of such a system for winter operation is illustrated in the following example.

**Example 12-8**—When the outside temperature is 35 F, the heat lost from a building is 40 000 Btu per hr. Room temperature is 70 F. The building is to be heated by using a heat pump having Freon, F-12 as the refrigerant. To allow for heat transfer to the

evaporator, assume an evaporator pressure of 40 psia. To allow the building to be heated by air blown over the condenser without excessive velocities, assume a condenser pressure of 200 psia. Taking the isentropic compressor efficiency as 80%, calculate the horsepower required.

*Solution.*—The  $T$ - $S$  diagram is shown in Fig. 12-21. Here,

$$h_3 = h_4 = 39.21, h_1 = 81.16, S_1 = 0.16914, \text{ and } h_2 = 93.79$$

The isentropic work per pound is

$$h_2 - h_1 = 93.79 - 81.16 = 12.63 \text{ Btu}$$

The actual work per pound is

$$\frac{\text{Isentropic Work}}{\text{Compressor Eff.}} = \frac{12.63}{0.8} = 15.79 \text{ Btu}$$

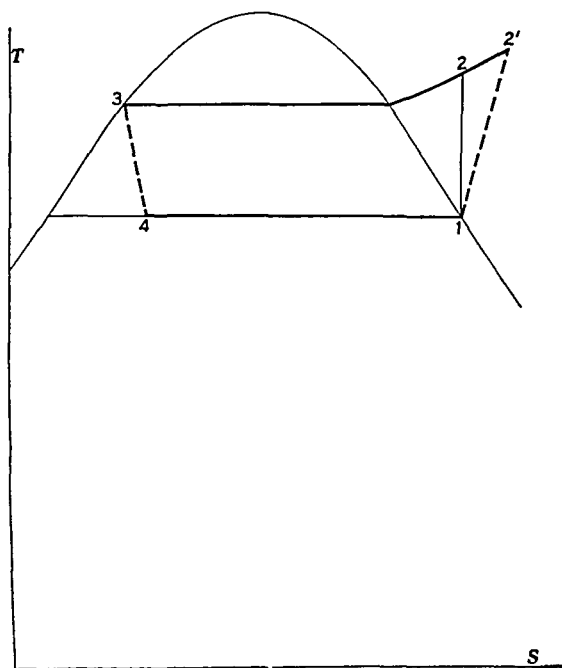


Fig. 12-21. Diagram for Example 12-8

The heat picked up outdoors is

$$h_1 - h_4 = 81.16 - 39.21 = 41.95 \text{ Btu per lb}$$

As all energy put into the compressor must be delivered to the building, the total energy to the building is

$$15.79 + 41.95 = 57.74 \text{ Btu per lb}$$

The work input is

$$40,000 \left( \frac{15.79}{57.74} \right) = 10,940 \text{ Btu per hr}$$

The horsepower input is

$$\frac{10\,940}{2\,544} = 4.30 \text{ hp} \quad \text{Ans}$$

*Example 12-9* —If in the preceding example electrical energy costs  $1\frac{1}{2}$  cents per kw hr and fuel oil costs 10 cents a gal compare the cost of operation of the heat pump with that of an oil burner. Assume that the oil has a heating value of 140 000 Btu per gal and the efficiency of the burner is 75%.

*Solution* —The cost of operating the heat pump is

$$(4.3)(0.746)(1.25) = 4.01 \text{ cents per hr}$$

For the oil burner the cost is

$$\frac{40\,000}{140\,000(0.75)}(10) = 3.81 \text{ cents per hr}$$

These results show that under the given conditions there will be a slight saving in burning oil. In case the outside temperature drops the advantage in favor of oil will be greater. With higher outside temperatures the advantage will shift to favor the heat pump.

If anything can be done to minimize the temperature range in the heat pump system for given outside conditions its operating costs will be reduced. A saving can be effected by using the condenser as a direct heater and by using much heat transfer surface and operating with a condenser temperature which is not greatly in excess of the atmospheric temperature.

Some work has been done with abstraction of heat from ground water which has a temperature in excess of that of the outside air. In some cases pipes are embedded in the earth with water circulating through them to pick up heat from the earth. These systems show promise because of their low operating costs. Although the initial costs of such heat pump systems are high at present it should be realized that such systems are not yet in quantity production. Furthermore they replace both the conventional heating system and an air conditioning system for summer use.

## CHAPTER 13

### GASEOUS MIXTURES AND COMBUSTION

13-1. **Dalton's and Avogadro's Laws.**—When a perfect gas A is introduced into a tank which contains another perfect gas B, nothing happens to gas B as a result of the addition of gas A, it being assumed that the two gases are at the *same temperature*. This is true because the size of the molecules of gas A may be neglected, and the total free volume through which the molecules of gas B may move is still the entire tank volume. Furthermore, there can be no intermolecular attractions between the molecules of the two gases; and the molecules of gas B will therefore travel in the same straight lines as they did originally. In addition, as the mean kinetic energies of the molecules of the two gases are equal, there can be no net exchange of energies between the molecules of the two gases. Hence, the pressure exerted by gas B will be the same as it was originally. Following the same line of reasoning, the pressure exerted by gas A is not influenced by gas B.

This fact was enunciated by Dalton and is known as Dalton's Law of partial pressures. It is stated as follows: *In a mixture of perfect gases, each gas occupies the entire volume at its own partial pressure; the total pressure equals the summation of the individual partial pressures.* The term "partial pressure" refers to that part of the total pressure which is exerted by a given gas. For a perfect gas this equals the pressure which that gas would exert if it alone occupied the same container at the same temperature.

In Art. 2-6 it was shown that the pressure exerted by a perfect gas is  $PV = \frac{1}{3} N m \overline{\text{Vel}}^2$ . Solving for the number of molecules  $N$ , we get:

$$N = \frac{3PV}{m\overline{\text{Vel}}^2}$$

Since temperature is a measure of the mean kinetic energy of the molecules, or  $\frac{m\overline{\text{Vel}}^2}{2}$ , it may be seen that the number of molecules of a perfect

gas in a given volume is independent of the nature of the gas but depends solely on the pressure and temperature of the gas. Avogadro phrased this fact as follows: *Equal volumes of perfect gases, under the same conditions of temperature and pressure, contain the same number of molecules.*

13-2. **The Mol.**—According to the line of reasoning in Art. 13-1, the total pressure exerted by a mixture of perfect gases, occupying a given

volume at a given temperature, is independent of the nature of the gases but depends solely on the total number of molecules present. The molecule is such a small quantity of matter that we are confronted with an extremely large number when dealing with the numbers of molecules present in the quantities of substances normally encountered. Thus, there is a need for a unit of quantity that is similar to the dozen, the gross, and the ream but involves a very large number per unit. There is also need for such a unit of quantity when dealing with combustion, because combustion is fundamentally action between individual molecules. Such a unit is the *mol*.

Because of inability to count the number of molecules and because of the very large number of molecules involved, the number of molecules per mol is not pertinent. The weight method is used to determine a very large number of similar objects. For example, a billion pins would be weighed rather than counted individually. Likewise, the mol is weighed. When the English system is used, a mol is that number of molecules of a substance whose weight in pounds is equivalent to its molecular weight. This mol is often termed the "pound mol." For example, a mol of oxygen weighs 32 lb.

As the weight of a mol of any substance is directly proportional to its molecular weight, it follows that the number of molecules in a mol\* is independent of the nature of the substance and is a constant, whether the substance is a solid, a liquid, or a gas.

The total pressure exerted by a mixture of perfect gases may be evaluated by determining the partial pressure of each of the constituent gases from the equation  $PV = WRT$  and then summing up the partial pressures. Another method would be to find a composite gas constant, designated as  $R$ , for the mixture and then to determine the total pressure directly. These laborious methods may be avoided by use of the mol. To use this procedure, start with the equation  $PV = \frac{1}{3} N m \overline{Vel}^2$ .

Let  $n$  = the number of molecules per mol = a constant,  $c'$ ,  
and  $M$  = the number of mols,

$N$  = the total number of molecules

Then  $nM = N$

But  $T = c \frac{m \overline{Vel}^2}{2}$  or  $m \overline{Vel}^2 = \frac{2T}{c}$

and  $PV = \frac{1}{3} nM \frac{2T}{c} = \frac{1}{3} c' \frac{2MT}{c}$

---

\* There are approximately  $2.7 \times 10^{23}$  molecules in a pound mol.

Substituting the constant  $B$  for all the constants in the foregoing equation, we obtain:

$$PV = MBT \quad (13-1)$$

As nothing has been said about the nature of the gas, other than that it was a perfect gas, equation 13-1 must hold for all perfect gases, and the constant  $B$  must have the same value for all perfect gases. For this reason,  $B$  is known as the *universal gas constant*. When pressures are expressed in pounds per square foot absolute, volumes are in cubic feet, and temperatures are in degrees Rankine, the value of  $B$  is 1545.

The number of mols of a substance may be determined by dividing the total weight by the weight per mol, or  $w'$ . Thus,

$$M = \frac{W}{w'} \quad (13-2)$$

If the equations  $PV = WRT$  and  $PV = MBT$  are compared,  $WR$  must equal  $MB$ . Substituting  $Mw'$  for  $W$ , we get  $Mw'R = MB$  or

$$R = \frac{B}{w'} \quad (13-3)$$

*Example 13-1.*—A gaseous mixture consisting of 32 lb of oxygen, 14 lb of nitrogen, 56 lb of carbon monoxide, and 8 lb of hydrogen is to be stored in a tank at a gage pressure of 200 psi and at a temperature of 130 F. Calculate the tank volume required.

*Solution.*—The number of mols of each gas is found as follows:

$$\text{mols of } O_2 = \frac{32}{32} = 1.00$$

$$\text{mols of } N_2 = \frac{14}{28} = 0.50$$

$$\text{mols of } CO = \frac{56}{28} = 2.00$$

$$\text{mols of } H_2 = \frac{8}{2} = 4.00$$

$$\text{Total mols} = 7.50$$

$$\text{Then, } V = \frac{MBT}{P} = \frac{7.5(1545)(130 + 460)}{144(14.7 + 200)}$$

or

$$V = 221 \text{ cu ft}$$

Ans.

**13-3. Volumetric and Weight Analyses.**—In some gases a volumetric analysis of a gaseous mixture is desired; in other cases it is the weight analysis which is desired. The transformation from one type of analysis to another is facilitated by the use of the mol. Since the number of molecules per mol is a constant and is independent of the nature of the substance, the following statement can be made by reversing Avogadro's Law: *Under given conditions of temperature and pressure, the volume per mol of any perfect gas equals that of another perfect gas.* In other words, the number

of mols of any gas in a mixture of perfect gases is proportional to the volume this gas would occupy if it were separated from the mixture and existed at the total pressure and at the same temperature. A molal analysis of a mixture of perfect gases must equal numerically the volumetric analysis. To convert a volumetric analysis to a weight analysis, use is made of this equality, as illustrated in the sample problem which follows.

*Example 13-2*—A gaseous mixture has the following volumetric analysis:  $CO_2=12\%$ ,  $O_2=5\%$ ,  $CO=1\%$ ,  $N_2=82\%$ . Determine the weight analysis. (Note: Carbon dioxide may not be treated as a perfect gas at high pressures, even at room temperatures, but, when it is a part of a gaseous mixture existing at atmospheric pressure, the partial pressure of carbon dioxide is so low that it may be treated as a perfect gas.)

*Solution*—The values may be tabulated as follows:

Substance	Per Cent by Volume	Mols per mol of Mixture	Weight lb per mol	Weight lb per mol of Mixture		Per Cent by Weight
$CO_2$	12	0.12	$\times 44$	$= 5.28$	5.28	$\div 30.12 = 17.53$
$O_2$	5	0.05	$\times 32$	$= 1.60$	1.60	$\div 30.12 = 5.31$
$CO$	1	0.01	$\times 28$	$= 0.28$	0.28	$\div 30.12 = 0.93$
$N_2$	82	0.82	$\times 28$	$= 22.96$	22.96	$\div 30.12 = 76.23$
				<u>30.12</u>		<u>100.00</u>

In this solution use is made of the fact that the molal analysis of a mixture of perfect gases is numerically equal to the volumetric analysis of the mixture. It should be noted that in the solution of this problem the weight of a mol of mixture, 30.12 lb, is found.

The conversion of a weight analysis to a volumetric one is illustrated by the following sample problem.

*Example 13-3*—A gas has the following weight analysis:  $CO_2=18\%$ ,  $H_2=2\%$ ,  $CH_4=1\%$ ,  $CO=4\%$ ,  $O_2=0.5\%$ ,  $N_2=74.5\%$ . Determine the volumetric analysis.

*Solution*—The results may be tabulated as follows:

Substance	Per Cent by Weight	Weight lb per lb of Mixture	Weight lb per mol	Mols per lb of Mixture		Molal or Volumetric Analysis Per Cent
$CO_2$	18.0	0.180	$\div 44$	$= 0.00409$	0.409	$\div 0.04291 = 9.53$
$H_2$	2.0	0.020	$\div 2$	$= 0.01000$	1.000	$\div 0.04291 = 23.30$
$CH_4$	1.0	0.010	$\div 16$	$= 0.00062$	0.062	$\div 0.04291 = 1.46$
$CO$	4.0	0.040	$\div 28$	$= 0.00143$	0.143	$\div 0.04291 = 3.33$
$O_2$	0.5	0.005	$\div 32$	$= 0.00016$	0.016	$\div 0.04291 = 0.37$
$N_2$	74.5	0.745	$\div 28$	$= 0.02661$	2.661	$\div 0.04291 = 62.01$
				<u>0.04291</u>		<u>100.00</u>

Care must be taken not to multiply the value found in the analysis by the weight per mol of each individual substance.

If the weight of a mol of this mixture is desired, it may be found readily, as the above calculations show that there is 0.04291 mol per pound of mixture. The weight of a mol then is  $\frac{1.000}{0.04291} = 23.30$  lb.

★13-4. **Properties of Gaseous Mixtures.**—The important properties of gaseous mixtures are pressures, volumes, intrinsic energies, enthalpies, entropies, and specific heats.

(a) *Pressures:* It has been shown that for perfect gases the total pressure is the summation of the individual partial pressures. The individual partial pressures may be determined if the number of mols of each substance is known.

*Example 13-4.*—A quantity of oxygen having a volume of 20 cu ft at 200 psi gage and 500 F and 30 cu ft of nitrogen at 120 psi gage and 300 F are placed in a 12 cu ft tank at a temperature of 200 F. Calculate the partial pressures of both the oxygen and the nitrogen and also the total pressure.

*Solution.*—The number of mols of each gas is found as follows:

$$\text{Oxygen} = \frac{PV}{BT} = \frac{144(14.7+200)20}{1545(500+460)} = 0.4169 \text{ mol}$$

$$\text{Nitrogen} = \frac{PV}{BT} = \frac{144(14.7+120)30}{1545(300+460)} = 0.4956 \text{ mol}$$

The total pressure after mixing is

$$\begin{aligned} \frac{MBT}{V} &= \frac{(0.4169+0.4956)1545(200+460)}{12} \\ &= 77,500 \text{ lb per sq ft abs} \\ &= 538 \text{ psia} \end{aligned}$$

Ans.

The partial pressures are:

$$\text{For oxygen,} \quad 538 \times \frac{0.4169}{0.4169+0.4956} = 246 \text{ psia}$$

Ans.

$$\text{For nitrogen,} \quad 538 \times \frac{0.4956}{0.4169+0.4956} = 292 \text{ psia}$$

Ans.

(b) *Volumes:* In a gaseous mixture, each gas occupies the entire volume. However, in some cases it is desirable to determine the partial volumes, i.e., the volumes that would be occupied by the individual gases if each existed at the total pressure and temperature of the mixture. The perfect gas law,  $PV = MBT$ , shows that each partial volume is dependent on the numbers of mols of the individual substances.

*Example 13-5.*—A mixture consisting of carbon monoxide and nitrogen occupies 42 cu ft at a pressure of 80 psi gage and 120 F. The weight of nitrogen present is 7.2 lb. Calculate the partial volumes of both the nitrogen and the carbon monoxide.

*Solution.*—The total number of mols equals

$$\frac{PV}{BT} = \frac{144(80+14.7)42}{1545(120+460)} = 0.639 \text{ mol}$$



and the number of mols of nitrogen is

$$\frac{7.2}{28} = 0.257 \text{ mol}$$

The partial volume of the nitrogen equals

$$42 \times \frac{0.257}{0.639} = 16.9 \text{ cu ft} \quad \text{Ans}$$

The partial volume of the carbon monoxide equals

$$42 \times \frac{0.639 - 0.257}{0.639} = 25.1 \text{ cu ft} \quad \text{Ans}$$

(c) *Intrinsic Energies* When gases are mixed adiabatically in a non flow manner, the total intrinsic energy after mixing equals the summation of the individual intrinsic energies before mixing provided that no energy in any other form enters or leaves the mixing chamber. This statement follows from the law of conservation of energy.

When flowing gases are mixed, there is little need for a knowledge of the intrinsic energy after mixing. However, if desired this quantity may be found by the application of the equation of steady flow.

(d) *Enthalpies* When *flowing* fluids are mixed adiabatically the total enthalpy of the mixture equals the sum of the individual enthalpies before mixing provided that energies in other forms are not involved in the mixing process. In case other forms of energies are involved the equation of steady flow must be resorted to in order to determine the enthalpy of the mixture.

The enthalpy of a fluid *at rest* has little significance, except in constant pressure processes. Keeping this in mind one must not infer that when fluids are mixed adiabatically in a non flow manner, the enthalpy of the mixture equals the summation of the enthalpies of the constituent fluids. In the case of perfect gases, however, there is no change in temperature during such a mixing and there is no change in the values of  $PV$  because of mixing\*. Hence the enthalpy of a mixture of perfect gases equals the summation of the enthalpies of the individual gases.

(e) *Entropies* Mixing is an *irreversible* process, as such it is accompanied by an *increase* in entropy. The amount of this increase in entropy depends on the drop in pressure suffered by the individual gases as a result of the mixing process to a common total pressure and volume.

*Example 13-6*—A 6 cu ft tank contains oxygen and nitrogen. The gases are initially in separate compartments and each is at a pressure of 20 psia and at a temperature of 140 F. The oxygen occupies a volume of 1 cu ft and the nitrogen a volume of 5 cu ft. Calculate the total increase in entropy occurring when the two gases are mixed adiabatically.

\* It must be kept in mind that this discussion concerns non flow mixing of perfect gases that are initially at the same temperature.

*Solution.*—As entropy is a point function, the increase in the entropy of each of the gases may be evaluated by replacing the irreversible mixing process by a reversible process having the same terminal states. Such a process is the reversible constant-temperature expansion process.

The increase in entropy for the oxygen is

$$S_2 - S_1 = \frac{Q_2}{T} = \frac{Wk_2}{JT} = \frac{PV \log_e \frac{V_2}{V_1}}{JT} = \frac{144 \times 20 \times 1 \log_e \frac{6}{1}}{778(140 + 460)} = 0.01105 \text{ unit}$$

For the nitrogen, the increase is

$$S_2 - S_1 = \frac{PV \log_e \frac{V_2}{V_1}}{JT} = \frac{144 \times 20 \times 5 \log_e \frac{6}{5}}{778(140 + 460)} = 0.00562 \text{ unit}$$

The total increase in entropy of the gases as a result of the mixing equals  $0.01105 + 0.00562 = 0.01667$  unit. Ans.

(f) *Specific Heats:* For perfect gases the specific heat of a gaseous mixture equals the weighted average of the specific heats of the individual gases. If the specific heats of the gases vary with temperature\* but the gases are perfect in all other respects, the specific heat of a gaseous mixture equals the weighted average of the individual specific heats of the constituent gases taken at the mixture temperature.

*Example 13-7.*—A gaseous mixture consists of 10 lb of CO, 8 lb of N<sub>2</sub>, and 6 lb of O<sub>2</sub>. Calculate the constant-pressure specific heat of the mixture.

*Solution.*—The values may be tabulated as follows:

Substance	Weight lb	Specific Heat Btu per lb per deg	Weight $\times$ Specific Heat Btu per deg
CO	10	0.245	2.450
N <sub>2</sub>	8	0.245	1.964
O <sub>2</sub>	6	0.219	1.314
	24		5.728

The specific heat of the mixture equals

$$\frac{5.728}{24} = 0.241 \text{ Btu per lb per deg} \quad \text{Ans.}$$

*Example 13-8.*—A gaseous mixture has the following composition by volume: CO<sub>2</sub>, 20%; CO, 30%; O<sub>2</sub>, 8%; and N<sub>2</sub>, 42%. The molal specific heats at constant volume, in Btu per mol per deg, are as follows: CO and N<sub>2</sub>, 4.97; CO<sub>2</sub>, 6.95; O<sub>2</sub>, 5.03. Calculate the constant-volume molal specific heat of the mixture.

\* See Chapter 14.

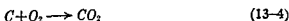
*Solution*—The values are shown in the following tabulation

Substance	Per Cent by Volume	Mols per mol of Mixture	Spec fic Heat Btu per mol per deg	Mols X Spec fic Heat per mol Btu per mol per deg
$CO_2$	20	0.20	6.95	1.390
$CO$	30	0.30	4.97	1.491
$O_2$	8	0.08	5.03	0.403
$N_2$	42	0.42	4.97	2.087
				<hr/> 5.371

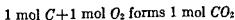
As the specific heats of the gases were given to only two decimal places, the value of the specific heat of the mixture should be rounded off to 5.37 Btu per mol per deg. Ans

**13-5. Combustion Equations**—Combustion is the combining of the atoms of fuel molecules with those of oxygen molecules. Fuels may contain carbon or hydrogen, or both, which take part in the reaction. To a limited extent, the fuel may contain sulfur, which will also react. Combustion equations express the reactions. Combustion may be complete or may be incomplete because of a deficiency of oxygen.

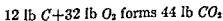
(a) *Complete Combustion* The complete combustion of carbon is expressed as follows



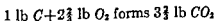
This indicates that a molecule of carbon combines with a molecule of oxygen to form a molecule of carbon dioxide. As indicated in Art. 13-2, the number of molecules, in most finite quantities of material, is extremely large. For this reason, it is desirable to deal with mols of substances rather than with molecules. As the number of molecules per mol is a constant, the combustion equation of carbon may be read as follows



The weights of the substances involved may be found by using the weights per mol of the various substances. Thus,



or

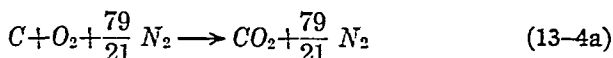


It is to be noted that, because of chemical contraction or expansion, the number of mols after combustion may not equal the original number of mols. However, the total weight cannot change during a combustion process.

The oxygen for combustion normally is supplied by the air. Air consists of oxygen, nitrogen, argon, and several inert gases which are present

in very small amounts. It is common practice to lump these inert gases and argon\* together with nitrogen and call the mixture *atmospheric nitrogen*. Atmospheric nitrogen weighs 28.16 lb per mol. On the volume basis, air contains 21 per cent of oxygen and 79 per cent of atmospheric nitrogen. On the weight basis, air consists of 23.2 per cent of oxygen and 76.8 per cent of atmospheric nitrogen. The weight of air is 28.97 lb per mol.

Although the nitrogen does not react during the combustion process, it is commonly included in the combustion equation to indicate its presence. The amount of atmospheric nitrogen equals the amount of oxygen multiplied by the nitrogen-oxygen ratio for air. Thus, the combustion equation for carbon becomes:



In the theoretical case it is assumed that the air supplied contains exactly the right amount of oxygen so that if it were completely utilized neither fuel nor oxygen would be left after combustion. This is termed "perfect" combustion and the air thus supplied is called the "theoretical" air.†

In the case of combustion of a hydrocarbon fuel,  $CO_2$  and  $H_2O$  are formed. When setting up combustion equations it is suggested that the following steps be taken:

- (1) Assume that 1 mol of fuel is present. (It is recognized that this may call for a fraction of a mol of oxygen, but fractions of mols are realities whereas fractions of molecules are not. The advantage of starting with 1 mol of fuel is that all other quantities are then expressed per mol of fuel.)
- (2) Determine the amounts of  $CO_2$  and  $H_2O$ .
- (3) Determine the amount of  $O_2$  required.
- (4) Determine the amount of  $N_2$ .

*Example 13-9.*—Write the combustion equation for octane,  $C_8H_{18}$ .

*Solution.*—The mol of  $C_8H_{18}$  is composed of carbon (C) in the amount of 8 mols and hydrogen ( $H_2$ ) in the amount of 9 mols. The 8 mols of carbon will form 8 mols of  $CO_2$  and the 9 mols of hydrogen will form 9 mols of  $H_2O$ .

The 8 mols of  $CO_2$  contain 8 mols of  $O_2$ , and the 9 mols of  $H_2O$  contain  $4\frac{1}{2}$  mols of  $O_2$ .

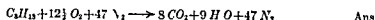
Thus, the number of mols of  $O_2$  required per mol of fuel is  $8 + 4\frac{1}{2} = 12\frac{1}{2}$  mols.

The number of mols of  $N_2$  accompanying the  $O_2$  is  $12\frac{1}{2} \times \frac{79}{21} = 47$  mols.

\* Oxygen and nitrogen together constitute about 99 per cent of air.

† The theoretical air is also commonly called the "stoichiometric" air.

Thus, the combustion equation becomes



Even though the nitrogen present after a combustion process has not resulted from the reaction, it is intimately mixed with substances formed by the reaction, hence, it is considered one of the "products of combustion". In a like manner, when excess oxygen (i.e., more oxygen than is theoretically required) is used, this oxygen will augment the final volume. In this case it, too, will be considered to be a product of combustion.

*Example 13-10*—Calculate the volume of the products of combustion per pound of  $C_4H_{10}$  at 500 F and 15 psia.

*Solution*—Although  $CO_2$  and  $H_2O$  normally are considered to be vapors, and not gases, their pressures in this case are so low that they may be treated as perfect gases with ordinary engineering accuracy. The total number of mols of products per mol of fuel equals  $9 + 8 + 47 = 64$ . A mol of fuel weighs  $8 \times 12 + 10 = 114$  lb. Hence, a pound of fuel equals  $1 \div 114 = 0.00877$  mol. The volume of products equals

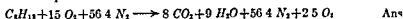
$$V = \frac{MBT}{P} = \frac{0.00877(64)(1545)(500 + 460)}{144(15)} = 38.5 \text{ cu ft} \quad \text{Ans.}$$

*Example 13-11*—Write the combustion equation for octane,  $C_8H_{18}$ , when supplied with 20 per cent of excess air (i.e., air supplied in excess of the theoretical and expressed as a per cent of the theoretical air). Calculate also the number of pounds of air supplied per pound of fuel.

*Solution*—As determined in Example 13-9, a mol of octane requires, theoretically,  $12\frac{1}{2}$  mols of oxygen. The excess oxygen equals  $(0.20)(12.5) = 2.5$  mols and the total oxygen supplied equals 15.0 mols.

$$\text{The total nitrogen equals } 15.0 \left( \frac{79}{21} \right) = 56.4 \text{ mols}$$

The combustion equation then becomes



The number of mols of air per mol of fuel equals

$$\frac{\text{mols } O_2 \text{ per mol fuel}}{\text{mols } O_2 \text{ per mol air}} = \frac{15.0}{0.21} = 71.4$$

The weight of air per mol of fuel equals

$$71.4 \times 28.97 = 2070 \text{ lb}$$

As a mol of fuel weighs 114 lb, the weight of air per pound of fuel equals

$$\frac{2070}{114} = 18.15 \text{ lb} \quad \text{Ans}$$

It should be emphasized that the excess  $O_2$  appears as  $O_2$  on the right-hand side of the equation, and the total  $N_2$  appears on both sides.

★ (b) *Combustion With Deficiency of Oxygen*. When there is a deficiency in the amount of air supplied, incomplete combustion will result. Hydrogen has a greater affinity for oxygen than has carbon. In case there is not much deficiency of oxygen, it may be assumed that the hydrogen takes what

oxygen it requires for complete combustion\* and the carbon has available the remainder of the oxygen supplied. The amounts of carbon monoxide and carbon dioxide formed\* may be determined by setting up two equations, one a carbon balance and the other an oxygen balance, and solving them simultaneously. The application of this method is illustrated by the following example.

★*Example 13-12.*—Benzene,  $C_6H_6$ , is burned with 95 per cent of the theoretical amount of air. Determine, theoretically, the combustion equation.

*Solution.*—For the theoretical amount of air, the combustion equation may be written as follows:



When 95 per cent of the theoretical air is supplied, the amount of  $O_2$  equals  $0.95(7.5) = 7.12$  mols, and the amount of nitrogen equals  $0.95(28.2) = 26.8$  mols.

The carbon dioxide and the carbon monoxide formed, together, must contain all of the carbon in the fuel, namely, 6 mols. Each mol of carbon dioxide contains an amount of carbon equal to 1 mol. A mol of carbon monoxide also contains carbon in the amount of 1 mol. Hence, the number of mols of carbon monoxide and carbon dioxide must total 6, to contain the 6 mols of carbon in the fuel. Or the equation for carbon balance is

$$\text{mols } CO_2 + \text{mols } CO = 6.00 \quad (a)$$

A mol of carbon dioxide contains oxygen ( $O_2$ ) in the amount of 1 mol. A mol of carbon monoxide contains oxygen in the amount of  $\frac{1}{2}$  mol. Together the carbon dioxide and carbon monoxide must contain the oxygen left over when the hydrogen takes its  $1\frac{1}{2}$  mols out of the 7.12 mols of oxygen supplied. Thus, the oxygen left to burn the carbon equals

$$7.12 - 1.5 = 5.62 \text{ mols}$$

The equation for oxygen balance then becomes:

$$\text{mols } CO_2 + \frac{1}{2} \text{ mols } CO = 5.62 \quad (b)$$

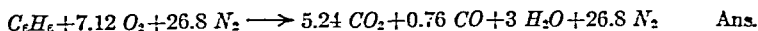
Subtracting equation (b) from equation (a), we obtain

$$\frac{1}{2} \text{ mols } CO = 0.38 \text{ and mols } CO = 0.76$$

Therefore,

$$\text{mols } CO_2 = 6.00 - 0.76 = 5.24$$

Then the combustion equation becomes:



13-6. *The Orsat.*—The completeness of combustion, as well as a knowledge of the amount of air supplied, can be determined if it is possible to obtain an analysis of the products of combustion. An apparatus to make such an analysis is known as the Orsat. By use of the Orsat, a volumetric analysis of the dry products (*i.e.*, the products with water vapor

\* In the actual case the amount of water vapor will be somewhat less than that shown by this assumption, as will the amount of carbon monoxide. There will be slightly more carbon dioxide.

removed) is obtained. In the Orsat, chemicals absorb carbon dioxide, oxygen, and carbon monoxide. The remainder of the dry gases is assumed to be nitrogen. In case there is a large deficiency of air, the dry products may contain free hydrogen and possibly methane, which will not be detected by the simple Orsat.

The Orsat analysis is used to determine the per cent of theoretical air\* supplied. The carbon content of a given amount of dry products and also that of a given amount of fuel are determined. Then a relation may be established between the amounts of fuel and dry products†. Likewise by use of nitrogen, a relationship between the amounts of air and dry products‡ may be established. The ratio of fuel to dry products and the ratio of air to dry products being known, the air-fuel ratio may be calculated. This nitrogen-carbon method is illustrated by the following examples.

*Example 13-13*—A fuel consists of 14.9 per cent of hydrogen and 85.1 per cent of carbon by weight. The Orsat analysis shows 12.5%  $CO_2$ , 2.5%  $O_2$ , and 0.7%  $CO$ . Determine the actual air-fuel ratio and the per cent of theoretical air.

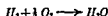
*Solution*—A mol of the dry products contains  $0.125 + 0.007 = 0.132$  mol of carbon. The weight of this carbon per mol of dry products is  $0.132 \times 12 = 1.584$  lb. A pound of fuel contains 0.851 lb of carbon. Hence the number of pounds of fuel per mol of dry products equals  $1.584 / 0.851 = 1.86$  lb.

A mol of dry products contains  $1.000 - (0.125 + 0.025 + 0.007) = 0.843$  mol of nitrogen. The number of mols of air required to supply this nitrogen is  $0.843 / 0.79 = 1.067$  mols. The weight of air per mol of dry products equals  $1.067 \times 28.97 = 30.91$  lb.

As the weight of air and the weight of fuel per mol of dry products are both known, the air-fuel ratio may be obtained by dividing one of these quantities by the other. Thus the actual air-fuel ratio equals

$$\frac{30.91}{1.86} = 16.6 \text{ lb air per lb fuel} \quad \text{Ans}$$

To determine the per cent of theoretical air, the theoretical air-fuel ratio must be found. As shown in the first part of Art. 13-5, a pound of carbon requires  $2\frac{1}{2}$  lb of oxygen. To find the oxygen required per pound of hydrogen, the combustion equation for hydrogen will be used. This is



Thus a mol of hydrogen requires  $\frac{1}{2}$  mol of oxygen. 2 lb of hydrogen requires 16 lb of oxygen, and 1 lb of hydrogen requires 8 lb of oxygen.

\* Actual air supplied divided by the theoretical air, expressed on the percentage basis.

† If some of the carbon is unburned and separates out, say in an ash pit, this amount must be subtracted from the carbon in the fuel to find the amount of carbon in the dry products.

‡ Some fuels contain small amounts of nitrogen. This nitrogen content of the fuel must be subtracted from that of the dry products to obtain the amount from the air.

The values may be tabulated as follows:

Substance	Pounds per lb of Fuel	Pounds $O_2$ per lb Substance	Pounds $O_2$ per lb Fuel
$H_2$	0.149	8	1.192
C	0.851	$2\frac{1}{2}$	2.269
			<hr/> 3.461

The theoretical air-fuel ratio equals

$$\frac{\text{lb } O_2 \text{ per lb fuel}}{\text{lb } O_2 \text{ per lb air}} = \frac{3.461}{0.232} = 14.9 \text{ lb air per lb fuel}$$

Then the per cent of theoretical air equals

$$\frac{16.60}{14.9} \times 100 = 111.4\% \quad \text{Ans.}$$

This may be expressed as 11.4% excess air.

*Example 13-14.*—A gaseous fuel has the following volumetric analysis:  $CO = 25.5\%$ ,  $CO_2 = 6.58\%$ ,  $H_2 = 13.20\%$ ,  $H_2O = 6.23\%$ , and  $N_2 = 48.49\%$ . The Orsat analysis shows:  $CO_2 = 17.70\%$ ,  $CO = 0.17\%$ , and  $O_2 = 2.68\%$ . Calculate the volume of air actually supplied per cu ft of fuel and the per cent of theoretical air used.

*Solution.*—The number of mols of carbon contained in a mol of dry products equals  $0.1770 + 0.0017 = 0.1787$  mol. The number of mols of carbon contained in a mol of fuel equals  $0.2550 + 0.0658 = 0.3208$  mol. Hence, the number of mols of fuel per mol of dry products equals

$$\frac{0.1787}{0.3208} = 0.557 \text{ mol}$$

The number of mols of nitrogen, from the air supplied, per mol of dry products equals the mols of nitrogen in the dry products minus the mols of nitrogen in the fuel. The number of mols of nitrogen per mol of dry products equals  $1.000 - (0.1770 + 0.0017 + 0.0268) = 0.7945$  mol. The number of mols of nitrogen in the fuel per mol of dry products is found by multiplying the number of mols of nitrogen per mol of fuel by the number of mols of fuel per mol of dry products, and equals  $0.4849 \times 0.557 = 0.2701$  mol. Then the number of mols of nitrogen, in the air supplied, per mol of dry products equals  $0.7945 - 0.2701 = 0.5244$  mol.

The air supplied per mol of dry products equals

$$\frac{\text{mols } N_2 \text{ per mol dry products}}{\text{mols } N_2 \text{ per mol air}} = \frac{0.5244}{0.79} = 0.664 \text{ mol}$$

The number of mols of air per mol of fuel equals

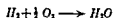
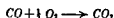
$$\frac{\text{mols air per mol dry products}}{\text{mols fuel per mol dry products}} = \frac{0.664}{0.557} = 1.19 \text{ mols}$$

As both the air and the fuel may be treated as perfect gases, the volume per mol of each is the same for the same conditions. Therefore,

$$\text{Actual air supplied} = 1.19 \text{ cu ft air per cu ft fuel} \quad \text{Ans.}$$

The theoretical air-fuel ratio will be found in order to find the per cent of theoretical air used. Only the carbon monoxide and the hydrogen in the fuel will burn. The combustion equations for these are:





Thus, a mol of each of these combustibles requires  $\frac{1}{2}$  mol of oxygen. The computations may be arranged as follows

Substance	Mols per mol Fuel	Mols $O_2$ per mol Substance	Mols $O_2$ per mol Fuel
CO	0.255	0.5	0.1275
$H_2$	0.132	0.5	0.0660
			0.1935

The theoretical air fuel ratio equals

$$\frac{\text{mols } O_2 \text{ per mol fuel}}{\text{mols } O_2 \text{ per mol air}} = \frac{0.1935}{0.21} = 0.921 \text{ mols air per mol fuel}$$

$$= 0.921 \text{ cu ft air per cu ft fuel}$$

The per cent of theoretical air equals

$$\frac{1.190}{0.921} \times 100 = 129\% \quad \text{Ans.}^*$$

**13-7 Heating Values**—The heating value of a fuel is the amount of heat that must be removed from the products of complete combustion of a unit amount of fuel to cool the products down to the temperature of the original air-fuel mixture.

Most common fuels contain combustible hydrogen, which, upon burning, forms water vapor. Normally, as the products of combustion are being cooled, part of the water vapor condenses, giving up its latent heat. The heating value, *i.e.*, the amount of heat removed from the products of combustion, thus depends on the amount of water vapor condensed. For a given fuel, the amount of water vapor that condenses is a function of (a) the temperature of the original mixture, (b) the barometric pressure, (c) the relative humidity of the air, (d) the excess air used.

It may be seen that, according to the definition given in the first paragraph, there are an infinite number of heating values for a fuel. When this definition is used, however, there are a minimum heating value and a

\* The authors realize that problems of these types may be solved in a shorter manner by combining several steps and expressing relations in equation form. However, they feel that when the student starts on the subject of combustion, he should go through the problems step by step. After he gains an understanding of the subject, either he or his instructor may then introduce the commonly used equations to reduce the time required for solution of problems.

maximum heating value for any fuel. The minimum heating value will be obtained when sufficient excess air is used to prevent any condensation of water vapor of combustion. This heating value is the "lower" or "net" heating value, and is designated  $Q_L$ . The maximum heating value of a fuel is obtained when the air contains sufficient moisture to saturate the products of combustion. In this case, *all* the water vapor of combustion will condense. This is the "higher" or "gross" heating value, and is designated  $Q_H$ .

The difference between the higher heating value and the lower heating value is a constant for any particular fuel.\* In the case of constant-pressure combustion, this difference is the enthalpy of vaporization  $h_{fg}$  of the water vapor of combustion, taken at the original temperature of the air-fuel mixture. In accordance with common practice when dealing with heating values, the term "latent heat" will be used in place of the term enthalpy of vaporization. The heating values of fuels commonly are reported at 25 C or 77 F. The latent heat of water vapor at this temperature is 1050.4 Btu per pound. In case the combustion is a constant-volume one, the difference between the higher and lower heating values equals the internal energy of vaporization  $u_{fg}$  of the water vapor. At 77 F this is 991.3 Btu per pound.

Most of our fuels are solids or liquids. Their heating values generally are determined in a bomb calorimeter. This is a constant-volume device into which is placed first a known quantity of fuel and then a charge of oxygen. The bomb is immersed in water, which absorbs energy from the products after combustion takes place. If a few drops of water are added to the inside of the bomb before it is sealed, this will saturate the space within the bomb; hence, all of the water vapor of combustion must condense. Because the temperature of the water surrounding the bomb increases, it cannot quite cool the products down to the temperature of the original air-fuel mixture. However, a correction is made for this condition in "standardizing" the bomb. Hence, the heating value cal-

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\* Before combustion, the air-fuel mixture possesses a definite amount of both chemical and sensible molecular energy. During combustion, new substances are formed and chemical energy is transformed into sensible molecular energy. Generally the sensible molecular energy of the products is somewhat different from that of the air-fuel mixture, even at the same temperature. The heat that is removed is the algebraic sum of the chemical energy of the fuel and the net change in sensible molecular energy. The difference between the sensible molecular energies of an air-fuel mixture and its products of combustion at the same temperature varies as the initial temperature is varied. Thus, the heating value of a fuel will vary slightly with temperature. However, this variation with temperature is so slight that it may be neglected except where extreme accuracy is desired. In this case the sensible molecular energies may be calculated by the aid of equations showing variations in specific heats with temperatures or by tables of intrinsic energies or enthalpies.

culated from the results obtained with a bomb calorimeter is the higher heating value at constant volume, which is designated  $Q_{HV}$

The heating values of the gaseous fuels, and sometimes of the lighter of the liquid fuels, are determined in the gas calorimeter. In this type of calorimeter the air-fuel mixture is burned in a bunsen type of burner. Water, circulating through the calorimeter, cools the products of combustion down to the temperature of the entering air-fuel mixture. Under normal conditions, a portion of the water vapor of combustion is condensed. The heating value thus obtained is intermediate in amount between the higher and lower values. The heating value is that of a flow process, which may be shown to equal the constant-pressure non flow heating value. The energy equation for the non-flow constant-pressure combustion is

$$U_{\text{air and fuel}} + \text{Chem Energy}_{\text{fuel}} = U_{\text{products}} + \frac{{}_1Wk_2}{J} + Q_p$$

Since the pressure is constant,

$${}_1Wk_2 = P(V_{\text{prod}} - V_{\text{air and fuel}}) = PV_{\text{prod}} - PV_{\text{air and fuel}}$$

Substituting for  ${}_1Wk_2$  and rearranging, we obtain

$$U_{\text{air and fuel}} + \frac{P V_{\text{air and fuel}}}{J} + \text{Chem Energy}_{\text{fuel}} = U_{\text{prod}} + \frac{P V_{\text{prod}}}{J} + Q_p$$

or 
$$H_{\text{air and fuel}} + \text{Chem Energy}_{\text{fuel}} = H_{\text{prod}} + Q_p$$

Applying the equation of steady flow to the gas calorimeter, we get

$$H_{\text{air and fuel}} + \text{Chem Energy}_{\text{fuel}} = H_{\text{prod}} + Q_{\text{flow}}$$

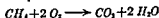
If the last equation is subtracted from the one immediately preceding it, the result is

$$Q_{\text{flow}} = Q_p$$

Thus, the heating value determined from the gas calorimeter equals the constant-pressure heating value. To determine the higher heating value, the latent heat of the part of the water vapor of combustion that was *not condensed* must be added to the heating value obtained with the gas calorimeter. When the latent heat of the part of the water vapor of combustion that was *condensed* is subtracted from the calorimeter heating value, the lower heating value will be obtained.

*Example 13-15*—Methane,  $\text{CH}_4$ , has a heating value of 934 Btu per cu ft at 77 F when 0.05 lb of water vapor per cu ft of fuel is condensed. Calculate the higher and lower heating values of methane at constant pressure.

*Solution*—The combustion equation for methane is



The volume per mol of gas under the given conditions equals

$$\frac{MBT}{P} = \frac{1(1545)(77+460)}{144(14.7)} = 392 \text{ cu ft}$$

The number of mols in the 2 cu ft of water vapor formed equals

$$\frac{2}{392} = 0.0051 \text{ mol}$$

and the weight of water vapor is 0.0918 lb.

The higher heating value at constant pressure is

$$Q_{HP} = 934 + (0.0918 - 0.05)1050.4 = 978 \text{ Btu per cu ft} \quad \text{Ans.}$$

The lower heating value at constant pressure is

$$Q_{LP} = 934 - (0.05)(1050.4) = 881.5 \text{ Btu per cu ft} \quad \text{Ans.}$$

The difference between the heating values at constant pressure and constant volume is very small and is generally neglected.\*

It should be noted that liquid fuels are supplied to the calorimeter in the liquid state. The fuels are vaporized at the expense of the sensible heat of the products of combustion. If the fuel is vaporized (as it is in the intake manifold of an engine), the heating value exceeds the heating value of the liquid by an amount equal to the latent heat of the fuel. However, the energy supplied that is chargeable to an engine is the heating value of the liquid fuel (unless the engine uses a gaseous fuel). Because the air and fuel flow into the engine and the products flow away from the engine, the heating value to be used is the heating value of flow, which equals the constant-pressure heating value.

There is considerable controversy as to whether the higher heating value or the lower heating value is to be used in determining the energy supplied to an engine. Strictly speaking, neither of these heating values is the correct one. An engine having a very high thermal efficiency would cool the products down to the temperature of the entering air-fuel mixture. In doing this, there would be partial condensation of the water vapor of combustion, and the energy released would be intermediate in amount between the higher and lower heating values. This must be the correct energy chargeable to the engine.† However, as was pointed out in the first part of this article, many factors affect the amount of condensation; hence, the actual heating value varies from day to day. In order to have the

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\* If the final volume of the products of a constant-pressure combustion exceeds the original volume, part of the fuel energy must be used to push the atmosphere out of the way. In this case, the heating value at constant pressure is less than that at constant volume by a quantity equal to the work done in pushing the atmosphere out of the way. If the final volume of the products is less than the original volume, the constant-pressure heating value exceeds the constant-volume one by a quantity equal to the work done on the gas by the atmosphere during the volume contraction.

† It is true that no actual engine can cool the products down to the temperature of the entering air-fuel mixture. This, however, does not in any way fix or limit the amount of energy chargeable to the engine.

energy chargeable to the engine a fixed quantity, it is desirable to make an approximation and select either the higher heating value or the lower heating value. In the United States, many engineers charge an engine with the higher heating value, although there is some tendency, particularly among the gas-turbine engineers, to use the lower heating value. Of course, thermal efficiencies calculated by use of the lower heating values are materially higher.

Approximate higher heating values of gaseous fuels at constant pressure are given in Table 13-1.

TABLE 13-1

APPROXIMATE HIGHER HEATING VALUES OF GASEOUS FUELS AT CONSTANT PRESSURE  
(Btu per mol)

Fuel	Formula	Heat ing Value	Approx mate Latent Heat
Hydrogen	$H_2$	123 000	
Carbon monoxide	$CO$	121,700	
Methane	$CH_4$	383 000	
Ethyl Alcohol	$C_2H_5O$	606 200	
Methyl Alcohol	$CH_3O$	328 600	
Benzene	$C_6H_6$	1,417 000	15 000
Octane	$C_8H_{18}$	2 368 000	18 000
Dodecane	$C_{12}H_{26}$	3,499,000	23 000

## CHAPTER 14

### IMPERFECT GASES

**14-1. Factors Causing Gases to Be Imperfect.**—Thus far, gases have been considered to behave in either of two ways. Some were so nearly like perfect gases that they were treated as such for ordinary engineering calculations; the properties of others differed so radically from those of perfect gases that there was no hope of dealing with their changes of state in any simple manner and it was therefore necessary to use tables of their properties. However, although under certain conditions any gas approximates a perfect gas, all known gases may be liquefied. As they approach their point of liquefaction, their deviation from perfect-gas laws is marked. There are tables of properties for a few substances over a wide range of conditions, but tables of thermodynamic properties of most substances are sketchy and, in many cases, of questionable accuracy. Hence, it is desirable to see the factors which cause a gas to deviate in its behavior from a perfect gas and to ascertain, if possible, methods of allowing for these deviations.

In Art. 2-5, it was pointed out that, for a gas to be perfect, there were no appreciable intermolecular attractions. As the force between molecules is inversely proportional to some power\* of the distance between the molecules, the number of molecules per unit volume is an important factor in predicting the amount of intermolecular attraction. This means that, for any given temperature, the higher the pressure, the closer together will be the molecules, the greater will be the intermolecular attractions and the greater will be the deviation from the perfect-gas laws.

The molecules on the average may be far apart; but, because of continuous molecular motion, many molecules will come close to each other and some will collide with one another. Under these conditions, intermolecular attractions will tend to prevent straight-line molecular travel. Resisting this interference with their straight-line travel is the inertia of the molecules. By virtue of their velocities, they tend to persist in a straight-line path. The higher the kinetic energy of the molecules, the greater will be their inertia. As temperature is a measure of the mean kinetic energy of the molecules, it follows that the higher the temperature, the more the molecular inertia tends to counteract intermolecular attractions.

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\* There is some reason to believe that the intermolecular force is inversely proportional to about the seventh to ninth power of the distance between the molecules.

Although the part played by temperature and pressure is recognized in predicting the tendencies of actual gases to deviate from the perfect gas law the amounts of the deviations cannot be determined if only the temperature and pressure are known. The deciding factor seems to be the relations of these temperatures and pressures to the critical temperature and pressure of the substance in question. In Fig 14-1 is shown the

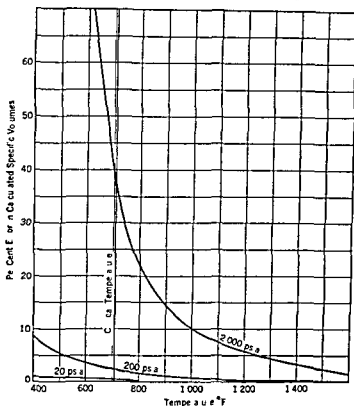


Fig 14-1 Error in Calculated Specific Volume

effect of pressures and temperatures on the error in the calculation of specific volumes of steam by use of the perfect gas law. It should be kept in mind that the critical pressure of steam is 3206.2 psia and the critical temperature is 705.4 °F. A curve of errors in calculated volumes for other substances would show roughly the same percentage errors if the conditions at which the other substances existed bore the same relations to their respective critical conditions as does the steam (i.e. a substance existing at 30 per cent of its critical pressure and 80 per cent of its critical temperature should have about the same error in its calculated specific volume as would steam when the steam was at 30 per cent of the critical steam pressure and 80 per cent of the critical steam temperature).

★14-2. **Variable Specific Heats.**—The specific heat of a perfect gas is a constant. Over a small range in temperature, there is little variation in the specific heat of the so-called permanent gases. However, over a wide range of temperatures the specific heat cannot be taken as a constant. In general, specific heats are affected by temperatures, pressures, and the number of atoms per molecule. The specific heats of even perfect gases depend on the number of atoms per molecule. In the case of an actual gas, the behavior of the atoms in the molecule causes variations in the specific heat. Hence, it is desirable to analyze the effects of the atoms on specific heat, first in the ideal case and then in the actual case.

Gas molecules, as was indicated in Chapter 2, have motion in three planes. As the pressures exerted in the three planes must be equal, the means of the squares of the velocities in the three planes must also be equal. Thus, the mean kinetic energy of the molecules traveling in one plane must equal the mean kinetic energy of the molecules traveling in each of the other planes. The total energy of translation of the molecules equals three times the energy of the translation in one plane.

In Art. 2-6, it was shown that  $PV = \frac{1}{3} N m \overline{Vel}^2$ . From Art. 13-2,  $PV = MBT$ . Therefore,  $\frac{1}{3} N m \overline{Vel}^2 = MBT$ . But the energy of translation equals  $\frac{N m \overline{Vel}^2}{2}$ . Or

$$N m \overline{Vel}^2 = 2 U_{\text{translation}}$$

$$\text{Hence,} \quad \frac{2}{3} U_{\text{translation}} = MBT$$

$$\text{or} \quad U_{\text{translation}} = \frac{3}{2} MBT$$

The total molecular energy is  $U = MC_v T$ , where  $C_v$  is the molal specific heat at constant volume. In case the molecule can possess only energy of translation, then  $U = U_{\text{translation}}$  and

$$U_{\text{translation}} = MC_v T$$

Equating the two equations for  $U_{\text{translation}}$ , we obtain:

$$\frac{3}{2} MBT = MC_v T$$

$$\text{or} \quad C_v = \frac{3}{2} B$$

As the energy of translation expressed in terms of velocities will be in foot-pounds,  $C_v$  will be in foot-pounds per mol per deg F. On the Btu basis,  $C_v = \frac{3}{2} \frac{B}{J} = 2.978$ .



If, in deriving the relations between  $C_v$  and  $C_p$  in Art 3-6, quantities were evaluated on the mol basis, rather than on the pound basis, then it would be found that  $C_p = C_v + \frac{B}{J}$ . Substituting for  $C_v$ , we have

$$C_p = \frac{3}{2} \frac{B}{J} + \frac{B}{J} = \frac{5}{2} \frac{B}{J} = 4.963$$

The adiabatic exponent is

$$k = \frac{C_p}{C_v} = \frac{\frac{5}{2} \frac{B}{J}}{\frac{3}{2} \frac{B}{J}} = \frac{5}{3}$$

The molal specific heats  $C_p$  and  $C_v$  as well as the value of  $k$ , of monatomic gases agree with the foregoing values very closely. However, the specific heats for other gases differ greatly from these values. This fact leads to the conclusions that gases other than monatomic must possess energy in excess of their energy of translation. Consider the diatomic gas. Assume that its two atoms are held at a fixed distance from each other and that the two together behave as a rigid entity. Conceivably, this molecule will have three degrees of rotation around its three principal axes. However, as the mass of the atom is concentrated in its nucleus, which is located at its center, the molecule can possess no appreciable energy of rotation about the axis running through the two atoms. It does, however, possess energy of rotation about its other two axes.

Boltzmann demonstrated that the energy of rotation about each axis must equal the energy of translation in one plane. This result will be accepted as the demonstration is beyond the scope of this text. Earlier in this article it was shown that the total energy of translation was  $\frac{3}{2} MBT$  and that the energies of translation in the three planes were equal. Thus the energy of translation in each plane equals  $\frac{1}{3} \left( \frac{3}{2} MBT \right) = \frac{1}{2} MBT$ . Or the energy of rotation about each axis equals  $\frac{1}{2} MBT$ . The total molecular energy of a diatomic gas should be

$$U_{\text{total}} = \frac{3}{2} MBT + 2 \left( \frac{1}{2} MBT \right) = \frac{5}{2} MBT$$

Since  $U_{\text{total}} = MC_v T$ ,

$$\frac{5}{2} MBT = MC_v T$$

or 
$$C_v = \frac{5}{2}B \text{ ft-lb per mol per deg}$$

Also, 
$$C_v = \frac{5}{2} \frac{B}{J} = 4.963 \text{ Btu per mol per deg}$$

As  $C_p = C_v + \frac{B}{J}$ ,

$$C_p = \frac{7}{2} \frac{B}{J} = 6.948 \text{ Btu per mol per deg}$$

and 
$$k = \frac{C_p}{C_v} = \frac{\frac{7}{2}}{\frac{5}{2}} = 1.40$$

The specific heats of such diatomic gases as oxygen, nitrogen, and carbon monoxide agree with these values within 1 per cent at standard atmospheric pressure and at 32 F. The specific heats of many diatomic gases exceed the foregoing values even under such conditions. At high temperatures the specific heats of the so-called permanent diatomic gases also exceed the values obtained by this simple theory.

Gases possessing more than two atoms per molecule have molecular energy, in addition to the energy of the diatomic gases, equal to the energy of rotation about the third axis, or  $\frac{1}{2} MBT$ . The total molecular energy of such a gas equals  $\frac{6}{2} MBT$ . Equating this to the molecular energy expressed in terms of  $C_v$ , we obtain  $\frac{6}{2} MBT = MC_v T$  or

$$C_v = 3B \text{ ft-lb per mol per deg}$$

Also, 
$$C_v = 3 \frac{B}{J} = 5.955 \text{ Btu per mol per deg}$$

Then 
$$C_p = 4 \frac{B}{J} = 7.940 \text{ Btu per mol per deg}$$

and 
$$k = \frac{4}{3}$$

In general, the specific heats of polyatomic gases are higher than the values obtained by this simple analysis. This fact leads to the conclusion that gases other than monatomics possess additional energies. The atoms, particularly at high temperatures, must move in relation to each other and hence possess this additional energy. The evaluation of this energy is beyond the scope of this text, but it may be mentioned that the specific heat depends on the frequency of vibration of the atoms and on the temperature, and that it increases as the temperature increases.

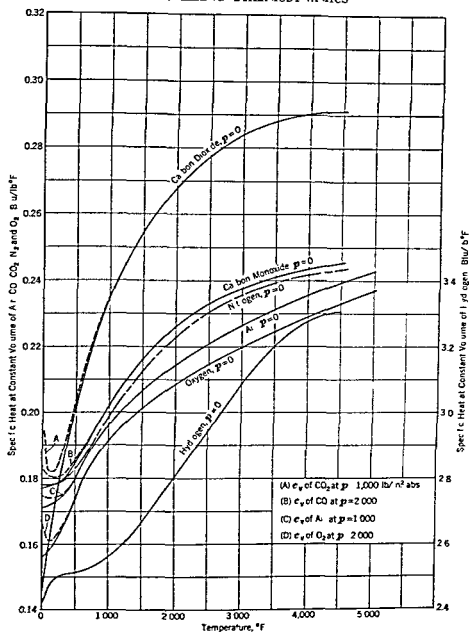


Fig 14-2. The Effect of Temperature and Pressure on  $C_p$  of Various Gases\*

As the specific heats of gases do not vary in a straight line with temperature when there is a large change in temperature, it is necessary to perform an integration to determine the heat added. The procedure is illustrated in the following example.

\*F. O. Ellenwood, N. Aulk, and N. R. Gay, "The Specific Heat of Certain Gases over Wide Ranges of Pressures and Temperatures," Cornell University Engineering Experiment Station Bulletin 20. Used by permission of the authors.

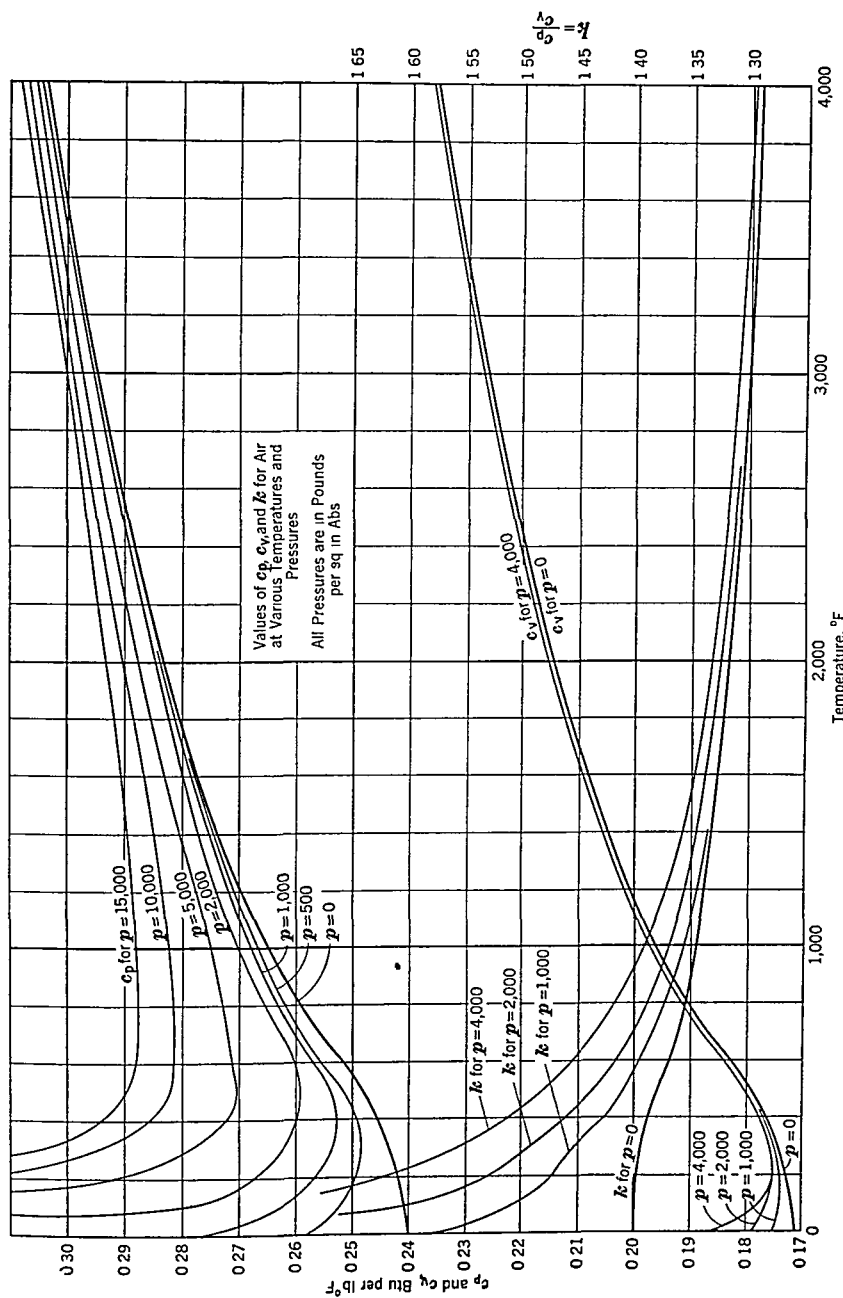


Fig. 14-3. The Effect of Temperature on  $C_p$ ,  $C_v$ , and  $k$  of Dry Air at Various Pressures (see Footnote, p. 276)

★*Example 14-1*—Nitrogen is heated at constant pressure from 240 F to 2540 F. Calculate (a) the heat added per mol, (b) the work done per mol (non flow), (c) the mean molal specific heat at constant pressure over this temperature range.

*Solution*—(a) It will be assumed that the pressure on the nitrogen is not extremely high and that the equations of Sweigert and Beardsley will give sufficiently accurate results. By the definition of  $C_p$ , the product of  $C_p$  and  $dT$  equals  $dQ$  per mol. This is shown as the cross-hatched area in Fig. 14-4. The total area underneath the curve 1-2

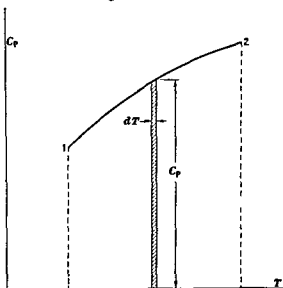


Fig. 14-4 Diagram for Example 14-1

represents the heat added per mol. This area, or  ${}_1Q_2$ , is  $\int_1^2 C_p dT$  per mol. Writing a general expression for heat added, we get

$${}_1Q_2 = M \int_1^2 C_p dT$$

Substituting for  $C_p$  the value given by Sweigert and Beardsley, we obtain

$$\begin{aligned} {}_1Q_2 &= M \int_1^2 \left( 9.47 - \frac{3.47 \times 10^3}{T} + \frac{1.16 \times 10^6}{T^2} \right) dT \\ &= M \left[ 9.47(T_2 - T_1) - 3.47 \times 10^3 \log_e \frac{T_2}{T_1} + 1.16 \times 10^6 \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \right] \end{aligned}$$

But, in this problem,  $M=1$ ,  $T_2=2540+460=3000$  R, and  $T_1=240+460=700$  R. For these values the heat added is

$$\begin{aligned} {}_1Q_2 &= 1 \left[ 9.47(3000-700) - 3.47 \times 10^3 \log_e \frac{3000}{700} + 1.16 \times 10^6 \left( \frac{1}{700} - \frac{1}{3000} \right) \right] \\ &= 21,781 - 5050 + 1270 = 18,001 \text{ Btu per mol} \end{aligned} \quad \text{Ans.}$$

(b) The work during addition of heat may be found by use of the general energy equation, when the change in intrinsic energy is known. Thus,

$$U_2 - U_1 = M \int_1^2 C_v dT$$

Since  $C_v = C_p - \frac{B}{J}$ ,

$$U_2 - U_1 = M \int_1^2 \left( C_p - \frac{B}{J} \right) dT = M \int_1^2 C_p dT - M \frac{B}{J} \int_1^2 dT = 18,001 - M \frac{B}{J} \int_1^2 dT$$

But 
$$\frac{MB}{J} \int_1^2 dT = \frac{1 \times 1545}{778} (3000 - 700) = 4567 \text{ Btu}$$

Therefore, 
$$U_2 - U_1 = 18,001 - 4,567 = 13,434 \text{ Btu}$$

and 
$$\frac{{}_1Wk_2}{J} = 18,001 - 13,434 = 4,567 \text{ Btu}$$

from which 
$${}_1Wk_2 = 3,553,500 \text{ ft-lb} \quad \text{Ans.}$$

As the process was a constant-pressure one, the work done could have been found as follows:

$${}_1Wk_2 = P(V_2 - V_1) = MB(T_2 - T_1) = 1 \times 1545(3000 - 700) = 3,553,500 \text{ ft-lb} \quad \text{Ans.}$$

(c) The mean molal specific heat is that specific heat by which the product of the temperature change and the number of mols must be multiplied to give the heat added. Thus, the mean molal specific heat equals

$$\frac{{}_1Q_2}{M(T_2 - T_1)} = \frac{18,001}{1 \times (3000 - 700)} = 7.827 \text{ Btu per mol per deg} \quad \text{Ans.}$$

Comparison of this value of 7.827 with the value of 5.948 for the ideal diatomic gas shows the effect of temperature on the specific heat.

Because of the work involved in integration, it is desirable to find other means of calculating heat added in case the specific heat varies. Such means are provided in Table 14-2, based on the work of Hershey, Eberhardt, and Hottel.\* Table 14-2 gives values of intrinsic energies of various gases at various temperatures, as well as values of the term  $\frac{PV}{J}$ .

The effect of temperature on intrinsic energy is taken into account. The datum plane for this table is 60 F; the intrinsic energy of the gas is assumed to be zero at this temperature.

★ *Example 14-2.*—Using Table 14-2, compute the heat added in Example 14-1.

*Solution.*—As the process is a constant-pressure one, the heat added equals the difference in enthalpies, which is

$$H_2 - H_1 = (U_2 - U_1) + \left( \frac{P_2 V_2}{J} - \frac{P_1 V_1}{J} \right)$$

Substituting for these quantities their values from Table 14-2, and using straight-line interpolation for values at 700 R, we obtain:

$${}_1Q_2 = H_2 - H_1 = (14,236 - 897) + (5957 - 1390) = 17,906 \text{ Btu} \quad \text{Ans.}$$

In addition to its value when determining temperature rises due to addition of heat, Table 14-2 is of value in computing chemical energies

\* R. L. Hershey, J. E. Eberhardt, and H. C. Hottel, "Thermodynamic Properties of the Working Fluid in Internal Combustion Engines." *S.A.E. Transactions*, Vol. 31, p. 409 (1936).

TABLE 14.2\*  
INTERNAL ENERGY OF GASES  
(*Htu per mol the internal energy of the gas is taken as 0 at 500 R*)

Temp. °Rankine	O <sub>2</sub>	N <sub>2</sub>	Ar	CO <sub>2</sub>	H <sub>2</sub> O	CO	H <sub>2</sub>	CH <sub>4</sub>	C <sub>2</sub> H <sub>6</sub>	C <sub>3</sub> H <sub>8</sub>	$\frac{PV}{J}$
500	0	0	0	0	0	0	0	0	0	0	1.033
527	85	82	83	118	101	83	82	100	610	603	1.066
550	200	193	197	290	244	196	193	261	1534	2159	1.112
600	402	393	396	570	430	390	390	533	3163	4516	1.191
800	1419	1379	1403	2120	1731	1492	1386	2072	12750	18303	1.588
1000	2520	2416	2431	3552	3009	2430	2387	3065	21717	25615	1.985
1500	5472	5091	5113	8764	6468	5115	4916	9781	65205	91526	2.979
2000	8629	7683	8087	14230	10327	8109	7517				4.070
2500	11916	11056	11185	20033	14578	11220	10263				5.083
3000	15209	14236	14591	26055	19160	14413	13133				6.057
3500	18570	17512	17692	32217	24057	17744	16143				6.918
4000	22314	20830	21037	38522	29178	21107	19217				7.612
4500	25658	24194	24437	44996	34470	24503	22497				8.191
5000	29610	27569	27872	51265	39885	27907	25819				9.927
5500	33350	31096	31353	57873	45402	31363	29298				10.920

\* Values for CH<sub>4</sub>, C<sub>2</sub>H<sub>6</sub>, and C<sub>3</sub>H<sub>8</sub> were computed from equations given in Table 14.1. Values for the other gases were based on data from the following sources: O<sub>2</sub>, N<sub>2</sub>, Ar, and CO from Giauque and Benedict; H<sub>2</sub>O from Benedict, Giauque, and Lippert; CH<sub>4</sub> from Benedict and Giauque; C<sub>2</sub>H<sub>6</sub> from Benedict and Giauque; C<sub>3</sub>H<sub>8</sub> from Benedict and Giauque. Values are in Btu per mol.

of fuels. Chemical energy is that energy contained in a fuel which will be released when the fuel is completely burned. It is approximately equal to the lower heating value of the fuel at constant volume. It is of particular value in computing the maximum possible temperature rise during combustion.

Unlike heating values, the chemical energy of a fuel is a fixed quantity and is *independent* of both the *nature* of the combustion process and the *initial temperature* of the air-fuel mixture. However, there is no direct way of measuring chemical energy. It must be calculated from a measured heating value. For a non-flow constant-volume combustion process, the energy equation is

$$U_{\text{air and fuel}} + \text{Chem. Energy}_{\text{fuel}} = U_{\text{prod.}} + Q_r$$

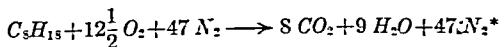
Because absolute values of intrinsic energies cannot be obtained, the energies must be calculated with respect to some datum plane. Hence, the calculated values of chemical energies are not absolute values, but depend on the datum plane used in calculating the intrinsic energies. Although calculated values of chemical energies are thus dependent on a datum plane, they are independent of both the nature of the combustion process and the initial temperature of the air-fuel mixture. Once a chemical energy is obtained, care must be taken to use the chemical energy with intrinsic energies having the *same datum plane*.

★*Example 14-8.*—Gaseous octane,  $C_8H_{18}$ , has a higher heating value at constant pressure of 2,368,000 Btu per mol at 77 F. Calculate the chemical energy per mol.

*Solution.*—The energy equation for constant-pressure combustion, as shown in Art. 13-7, is

$$H_{\text{air and fuel}} + \text{Chem. Energy} = H_{\text{products}} + Q_{HP}$$

The combustion equation for  $C_8H_{18}$  is



Then the energy equation becomes

$$\left(U + \frac{PV}{J}\right)_{1 \text{ mol } C_8H_{18}} + \left(U + \frac{PV}{J}\right)_{12\frac{1}{2} \text{ mols } O_2} + \text{Chem. Energy}_{1 \text{ mol } C_8H_{18}} = \left(U + \frac{PV}{J}\right)_{8 \text{ mols } CO_2} + \left(U + \frac{PV}{J}\right)_{9 \text{ mols } H_2O} + Q_{HP \text{ 1 mol } C_8H_{18}}$$

The water must be in the liquid form, as the heating value given is the higher heating value. As the enthalpy of water equals the enthalpy of the vapor minus the enthalpy of vaporization,

$$\left(U + \frac{PV}{J}\right)_{9 \text{ mols water}} = \left(U + \frac{PV}{J}\right)_{9 \text{ mols water vapor}} - (H_{fg})_{9 \text{ mols water vapor}}$$

\* Nitrogen does not have to be considered here, as the nitrogen is cooled down to its original temperature, possessing its original energy.



At 77 F the latent heat of water vapor may be taken as 1050 Btu per lb. Substituting values in the energy equation, and combining the terms  $\frac{PV}{J}$  on each side of the equation, we obtain

$$1 \times 640 + 12\frac{1}{2} \times 85 + 13\frac{1}{2} \times 1060 + \text{Chem. Energy}_{1 \text{ mol } C_8H_{18}} = 8 \times 118 + 9 \times 104 + 17 \times 1060 - 18 \times 9 \times 1050 + 2,368,000$$

$$\text{Chem. Energy} = 2,202,000 \text{ Btu per mol} \quad \text{Ans}$$

This result has been rounded off to the nearest thousand because the heating value is given to the nearest thousand.

★*Example 14-4*—Gaseous octane is mixed with the theoretical amount of air at 77 F. Calculate the maximum possible temperature that can be reached if this mixture is burned at constant pressure.

*Solution*—To attain the maximum theoretical temperature, there can be no heat lost. Hence, the energy equation is

$$\left[ U_{1 \text{ mol } C_8H_{18}} + U_{12\frac{1}{2} \text{ mols } O_2} + U_{47 \text{ mols } N_2} + \frac{PV}{(6 + 12\frac{1}{2} + 47) \text{ mols}} \right]_{77 F} + \text{Chem. Energy}_{1 \text{ mol } C_8H_{18}} = \left[ U_{8 \text{ mols } CO_2} + U_{9 \text{ mols } H_2O} + U_{17 \text{ mols } N_2} + \frac{PV}{(8 + 9 + 17) \text{ mols}} \right]_{T_{\max}}$$

Evaluating the left-hand side of this equation, we get

$$640 + 12\frac{1}{2} \times 85 + 47 \times 82 + 60.5 \times 1066 + 2,202,000 = 2,272,000 \text{ Btu}$$

The enthalpy of the products must equal this quantity at the maximum product temperature. A trial-and-error solution will be resorted to in the determination of this maximum temperature. Values for two temperatures follow.

Substance	Mols	Assumed Temp = 4000 R		Assumed Temp = 4500 R	
		Btu per mol	Total Btu	Btu per mol	Total Btu
CO <sub>2</sub>	8	38,522	308,176	44,906	359,248
H <sub>2</sub> O	9	29,178	262,602	34,459	310,131
N <sub>2</sub>	47	20,830	979,010	24,194	1,137,118
$\frac{PV}{J}$	64	7,942	508,288	8,934	571,776
			<u>2,058,076</u>		<u>2,378,273</u>
			or 2,058,000		or 2,378,000

The value of the left-hand side of the equation, or 2,272,000 Btu, lies between the enthalpies of the products at 4000 R and 4500 R. By straight-line interpolation, the enthalpy of the products equals 2,272,000 Btu at 4330 R. Hence, the maximum temperature is 4330 R. Ans.

★14-3. *Equations of State.*—Because of the limitations of the perfect-gas equation of state,  $PV = WRT$ , many attempts have been made to modify it to obtain an equation of state which would apply to actual gases.

One of the earliest of such equations was that of Van der Waals in 1873. Van der Waals realized that the two outstanding faults of the perfect-gas equation of state were that it neglected the effect of intermolecular attractions and it also neglected molecular size. Thus, because of intermolecular attractions, the observable pressure exerted by an actual gas is reduced below that of a perfect gas. To obtain the true pressure, the observable pressure must be increased by a factor which accounts for the intermolecular attraction. Van der Waals reasoned that this attractive force is inversely proportional to the square of the distance between the molecules and, hence, inversely proportional to the square of the specific volume. The true free volume, in which the molecules may travel, equals the observable volume minus the total volume of the molecules themselves. When these corrections are made, Van der Waals' equation of state becomes:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \quad (14-1)$$

In this equation,  $a$  and  $b$  are constants, the values of which are given in Table 14-3. Common practice in the use of Van der Waals' equation

TABLE 14-3  
VALUES OF THE CONSTANTS\* IN VAN DER WAALS' EQUATION

Substance	$a$	$b$
Air	344	0.587
Argon	346	0.618
Carbon Dioxide	925	0.686
Carbon Monoxide	375	0.634
Hydrogen	62.8	0.427
Methane	577	0.683
Nitrogen	346	0.618
Oxygen	350	0.510
Sulfur Dioxide	1730	0.909
Water	1400	0.488

\* These constants are based on the use of the following units in Van der Waals' equation:  $P$  in atmospheres;  $V$  in cu ft per mol;  $T$  in degrees Rankine; and  $R = 0.729$ .

is to express the pressure  $P$  in atmospheres, the volume  $V$  in cu ft per mol, and the temperature  $T$  in degrees Rankine. The values of the constants in Table 14-3 are based on these units. When these units are used, the value of  $R$  for all substances is 0.729.

The discrepancy between the value obtained by Van der Waals' equation and the value obtained by the perfect-gas equation and the actual value is shown in the following example.

★*Example 14-5*—A pound of steam occupies 1 000 cu ft at a temperature of 820 F. Compute the pressure it exerts by use of Van der Waals' equation, and compare this pressure with the value calculated by use of the perfect-gas equation and also with the steam table value.

*Solution*—Solving Van der Waals' equation for  $P$ , we obtain

$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

The volume per mol is  $1 \times 18 = 18$  cu ft. Then,

$$P = \frac{0.729(820+460)}{18.00-0.488} - \frac{1400}{(18)^2} = 48.96 \text{ atmospheres or } 719.7 \text{ psia} \quad \text{Ans}$$

For the perfect gas,

$$P = \frac{MBT}{V} = \frac{\frac{1}{18}(1545)(820+460)}{1} = 109.860 \text{ lb per sq ft or } 762.9 \text{ psia} \quad \text{Ans}$$

For the given conditions the steam table gives a pressure of 720 psia.

If Example 14-5 were repeated for steam having a specific volume of 0.102 cu ft per lb at 720 F, Van der Waals' equation would give a pressure of 3422 psia, while the perfect-gas value would be 6883 psia and the steam table value is 3206.2 psia.

The foregoing examples show that Van der Waals' equation is a great improvement over the perfect-gas laws under some conditions. However, under certain conditions, the results calculated by use of Van der Waals' equation are too inaccurate for engineering purposes. For this reason several individuals have developed equations of state to obtain greater accuracy. These equations are more accurate but are more involved than that of Van der Waals. One such equation is the Beattie-Bridgman equation, which is

$$PV^2 = RT \left[ V + B_0 \left( 1 - \frac{b}{V} \right) \right] \left[ 1 - \frac{c}{VT^3} \right] - A_0 \left( 1 - \frac{a}{V} \right) \quad (14-1a)$$

In this equation,  $a$ ,  $b$ ,  $A_0$ ,  $B_0$ , and  $c$  are empirical constants. This equation produces accurate results over a wide range of conditions. However, because of the labor involved in its use, the average engineer prefers to try to find other means of obtaining gas and vapor properties.

★*14-4 Corresponding States*—It would seem reasonable that the properties of various gases would be similar to one another if the gases were at similar conditions. Van der Waals decided that gases would be under similar conditions if their various properties were the same proportion of their respective critical properties. Properties expressed in terms of critical properties are known as reduced properties. Thus

$$P_R = \frac{P}{P_c} \quad (14-2a)$$

$$T_R = \frac{T}{T_c} \quad (14-2b)$$

$$V_R = \frac{V}{V_c} \quad (14-2c)$$

The subscript  $R$  denotes "reduced," and the subscript  $c$  denotes "critical." When no subscript is used with the property, the actual value of the property is to be used.

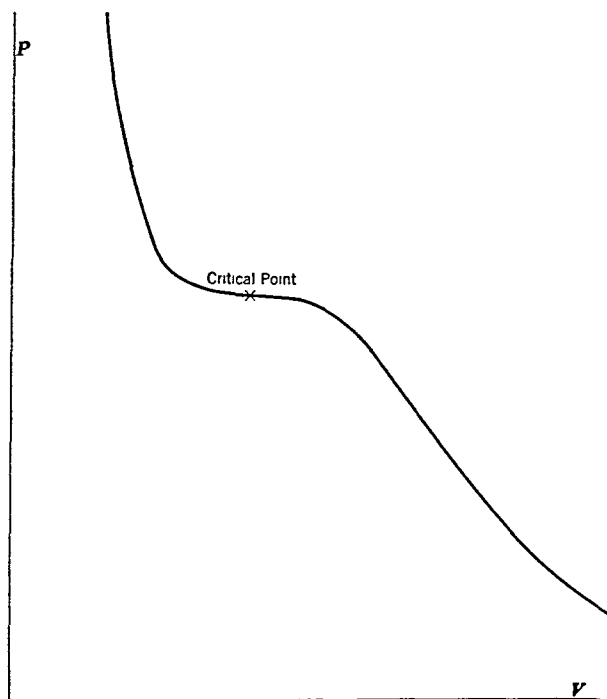


Fig. 14-5. Line of Constant Temperature

The relationship between properties of substances obeying Van der Waals' law may be obtained by a study of this law. The formula for the pressure is  $P = \frac{RT}{V-b} - \frac{a}{V^2}$ . A line of constant temperature equal to the critical temperature appears on the  $P$ - $V$  plane as shown in Fig. 14-5. At the critical point,  $\left(\frac{dP}{dV}\right)_T = 0$  and  $\left(\frac{d^2P}{dV^2}\right) = 0$ . But, for Van der Waals' equation,

$$\left(\frac{dP}{dV}\right)_T = \frac{-RT}{(V-b)^2} + \frac{2a}{V^3}$$

Therefore,

$$\frac{RT_c}{(V_c-b)^2} = \frac{2a}{V_c^3}$$

or

$$RT_c = \frac{2a}{V_c^3} (V_c - b)^2 \quad (14-3)$$

Now,

$$\left(\frac{d^2P}{dV^2}\right)_T = \frac{2RT}{(V-b)^3} - \frac{6a}{V^4}$$

At the critical point,

$$\frac{2RT_c}{(V_c-b)^3} = \frac{6a}{V_c^4}$$

or

$$RT_c = \frac{3a(V_c-b)^3}{V_c^4} \quad (14-4)$$

Combining equations 14-3 and 14-4, we obtain

$$V_c = 3b \quad (14-5)$$

When this value of  $V_c$  is substituted in equation 14-4,

$$T_c = \frac{8a}{27Rb} \quad (14-6)$$

Substitution of the values of  $V_c$  and  $T_c$  in the equation  $P_c = \frac{RT_c}{V_c - b} - \frac{a}{V_c^2}$  gives

$$P_c = \frac{a}{27b^2} \quad (14-7)$$

If equations 14-2a, 14-2b, and 14-2c be solved for values of  $P$ ,  $V$ , and  $T$ , respectively, and the values of  $P_c$ ,  $V_c$ , and  $T_c$  be substituted for those quantities, the following equations will be obtained

$$P = P_R \frac{a}{27b^2}$$

$$V = 3bV_R$$

$$T = T_R \frac{8a}{27Rb}$$

Substituting these values in Van der Waals' original equation, we obtain

$$\left(P_R \frac{a}{27b^2} + \frac{a}{9b^2V_R^2}\right)(3bV_R - b) = \frac{8aRT_R}{27Rb}$$

Multiplying each side of this equation by  $\frac{9b}{a}$  and simplifying, we get:

$$\left(P_R + \frac{3}{V_R^2}\right)\left(V_R - \frac{1}{3}\right) = \frac{8}{3}T_R \quad (14-8)$$

An examination of equation 14-8 shows that the gas constants have been eliminated and that the equation therefore holds for all gases obeying Van der Waals' equation. This equation indicates that all gases which obey Van der Waals' equation have similar properties when they exist at the same proportion of their respective critical conditions. Table 14-4 contains critical-point data on several commonly used substances.

TABLE 14-4  
CRITICAL PRESSURES AND TEMPERATURES  
(Based on Data International Critical Tables)

Substance	Formula	Critical Pressure psia	Critical Temperature °F
Air		547	-220.3
Alcohol, Wood	$CH_4O$	1156	464.0
Alcohol, Grain	$C_2H_6O$	927.5	469.6
*Ammonia	$NH_3$	1657	271.4
Argon	$A$	705	-188.3
Benzene	$C_6H_6$	701	551.3
Carbon dioxide	$CO_2$	1074	88.0
Carbon monoxide	$CO$	514	-218.2
Helium	$He$	33.2	-450.2
Hydrogen	$H_2$	188.2	-399.8
Methane	$CH_4$	673	-116.5
Nitrogen	$N_2$	492	-232.8
Oxygen	$O_2$	731	-181.8
Sulfur dioxide	$SO_2$	1143	315.0
†Water	$H_2O$	3206.2	705.4

\* Values for ammonia taken from Bureau of Standards Circular No. 142.

† Values for water taken from Keenan and Keyes Steam Tables

As has been discussed in Art. 14-3, Van der Waals' equation does not yield accurate results for all conditions. Hence, it would appear that the properties of various gases are not similar, even under the same relative conditions. However, if the properties of various gases deviate from Van der Waals' equation in the same manner, the properties would be similar. Considerable investigational work has been done to discover whether or not a similarity in properties actually exists. It appears that there is some correlation between the properties of various substances, but that the correlation is not perfect. However, in general the correla-

tion is good, and, even in the worst cases, such a correlation—while not of sufficient accuracy for refined engineering work—is of sufficient accuracy for rough calculations

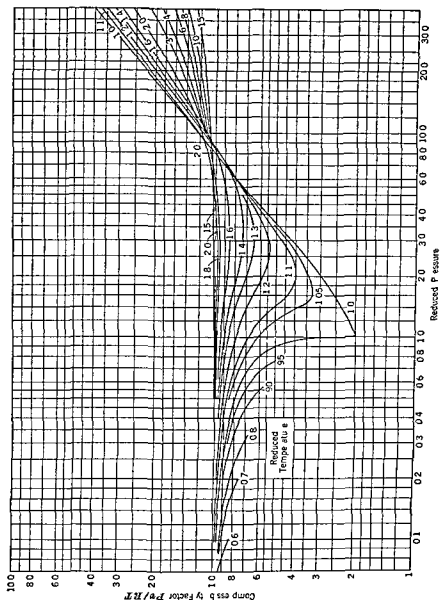


Fig 14-6 Compressibility Factors of Gases and Vapors

(Reproduced by permission of the authors from *Industrial Chemical Calculations* by Hougden and Watson, John Wiley & Sons, Inc.)

One way in which use is made of the similarity of various substances is in the adjustment of the perfect-gas equation of state by means of a compressibility factor. The compressibility factor is a factor which must be introduced into the perfect gas equation of state to make it valid

The value of the compressibility factor varies with both temperature and pressure. This variation is shown in Fig. 14-6, which is taken from Hougen and Watson.\* The adjusted equation of state becomes

$$PV = C' MBT \quad (14-9)$$

where  $C'$  is the compressibility factor.

★*Example 14-6.*—Calculate, by use of Fig. 14-6, the specific volume of steam under a pressure of 5000 psia at 1200 F, and compare the result with the steam-table value.

*Solution.*—The critical pressure of water is 3206.2 psia, and its critical temperature is 705.4 F. Then

$$P_R = \frac{5000}{3206.2} = 1.56 \text{ and } T_R = \frac{1200 + 460}{705.4 + 460} = 1.424$$

For these values, the compressibility factor from Fig. 14-6 is 0.84. If this factor is used, the specific volume equals

$$v = \frac{C' MBT}{P} = \frac{0.84 \left( \frac{1}{18} \right) (1545) (1200 + 460)}{144(5000)} = 0.166 \text{ cu ft} \quad \text{Ans.}$$

The steam-table volume is 0.1696 cu ft.

Ans.

Considerable work has been done in establishing the similarity between some properties of various substances. Much more work may be done along these lines. Engineers should utilize the work that has been done and should encourage additional work.

★14-5. **General Equations of Thermodynamics.**—Up to now, substances have been handled by being divided into three classes: (1) perfect gases, for which perfect-gas laws may be used; (2) vapors, for which tables are available; and (3) imperfect gases and vapors, for which means such as those discussed in Art. 14-4 are employed in property determinations. It is possible to establish property relations that apply equally well to all classes of substances. These equations are known as the general equations of thermodynamics. The number of such equations is very large and the derivation of many of them is involved. Only a few of these equations will be derived here to show the method applied and to point out their usefulness.

As many of these equations are differential equations, it is desirable to review very briefly some facts concerning equations of this type.

Suppose  $Z = f(X, Y)$ . Let there be an infinitesimal change in  $Z$  when  $Y$  is held constant. Then let there be an infinitesimal change in  $Z$  when  $X$  is held constant. The total change in  $Z$  is

$$dZ = \left( \frac{dZ}{dX} \right)_Y dX + \left( \frac{dZ}{dY} \right)_X dY \quad (14-10)$$

---

\* *Industrial Chemical Calculations*, by O. A. Hougen and K. M. Watson, John Wiley and Sons.



Equation 14-10 is said to be a perfect differential equation, as it is based on quantities that are functions of one another. The test for determining whether an equation is a perfect differential equation is as follows. In a perfect differential equation the derivative of the first coefficient in respect to the second variable must equal the derivative of the second coefficient in respect to the first variable. This may be shown as follows

$$\frac{d\left(\frac{dZ}{dX}\right)}{dY} = \frac{d^2Z}{dX dY} \text{ and } \frac{d\left(\frac{dZ}{dY}\right)}{dX} = \frac{d^2Z}{dX dY}$$

Therefore,

$$\frac{d\left(\frac{dZ}{dX}\right)}{dY} = \frac{d\left(\frac{dZ}{dY}\right)}{dX} \quad (14-11)$$

Such quantities as work may be evaluated in terms of pressure and volume changes once the path is known. It is therefore possible to write a differential equation involving these variables. Thus,

$$d(Wk) = \left(\frac{d(Wk)}{dP}\right)_V dP + \left(\frac{d(Wk)}{dV}\right)_P dV$$

Such an equation is an imperfect differential, as the work is not a point function dependent on  $P$  and  $V$ , but is a path function. While the coefficient  $\left(\frac{dZ}{dX}\right)_Y$  is fixed when the relationship between  $X$ ,  $Y$ , and  $Z$  is fixed, the coefficient  $\left(\frac{d(Wk)}{dP}\right)_V$  is not fixed until the path is fixed.

Suppose that general equations are desired to evaluate changes in intrinsic energy, enthalpy, and entropy. These may be obtained as follows.

In the derivations of these equations, all quantities will be put on the pound basis, and to avoid carrying along the term  $J$ , both  $Q$  and  $U$  will be expressed in foot pounds. Start with the general energy equation

$$dq = du + p dv \quad (14-12)$$

or

$$du = dq - p dv \quad (14-13)$$

But, as  $q$  varies with  $T$  and  $v$ ,

$$dq = \left(\frac{dq}{dT}\right)_v dT + \left(\frac{dq}{dv}\right)_T dv \quad (14-14)$$

The coefficient  $\left(\frac{dq}{dT}\right)_v$  will be recognized as  $c_v$ . Let  $\left(\frac{dq}{dv}\right)_T = l_v$ , the coefficient of volume expansion. Substituting these values in equation 14-14 and then substituting for  $dq$  in equation 14-13, we obtain

$$du = c_v dT + (l_v - P)dv \quad (14-15)$$

Much data exist on the value of  $c_v$ . This is not so regarding  $l_v$ . Hence, it is desirable to find a substitute for  $l_v$ . For a reversible process,

$$dq = T ds$$

or

$$ds = \frac{dq}{T}$$

Substituting for  $dq$  the value obtained from equation 14-14 by use of the coefficients  $c_v$  and  $l_v$ , we obtain

$$ds = \frac{c_v}{T} dT + l_v \frac{dv}{T} \quad (14-16)$$

But  $s$  is a point function, and  $s = f(T, v)$ . Therefore,

$$ds = \left( \frac{ds}{dT} \right)_v dT + \left( \frac{ds}{dv} \right)_T dv \quad (14-17)$$

Comparison of equations 14-16 and 14-17 shows that

$$\frac{c_v}{T} = \left( \frac{ds}{dT} \right)_v$$

and

$$\frac{l_v}{T} = \left( \frac{ds}{dv} \right)_T \quad (14-18)$$

Equation 14-18 gives a means of determining  $l_v$ , but the quantity  $\left( \frac{ds}{dv} \right)_T$  is not readily obtainable and hence should be evaluated in terms of other quantities.

Because we have need of one of the four relations of Maxwell at this point and have need of others later on, all four of these relations will be derived. Maxwell derived these relations by taking the quantities  $u$  and Gibbs' three functions denoted by  $\chi$  (or  $h$ ),  $\zeta$ , and  $\psi$ .

From equation 14-13,

$$du = dq - P dv$$

Substituting for  $dq$  its value  $T ds$  for a reversible process, we get

$$du = T ds - P dv$$

This is a perfect differential equation, as  $u = f(s, v)$ . Hence,

$$\left( \frac{dT}{dv} \right)_s = - \left( \frac{dP}{ds} \right)_v \quad (14-19)$$

This is the first of Maxwell's relations. To find the second, start with the relation

$$dh = du + P dv + v dP$$

Since  $du + P dv = dq = T ds$ ,

$$dh = T ds + v dP$$

Also, as  $h = f(s, P)$ ,

$$\left(\frac{dT}{dP}\right)_s = \left(\frac{dv}{ds}\right)_P \quad (14-20)$$

To find the third of Maxwell's relations, use the Gibbs function  $\zeta$ . As  $\zeta = h - Ts$ ,

$$d\zeta = dh - T ds - s dT$$

Substituting  $T ds + v dP$  for  $dh$ ,

$$d\zeta = T ds + v dP - T ds - s dT = v dP - s dT$$

As  $\zeta = f(P, T)$ ,

$$\left(\frac{dv}{dT}\right)_P = -\left(\frac{ds}{dP}\right)_T \quad (14-21)$$

To find Maxwell's fourth relation, use the Gibbs function  $\psi$ . As  $\psi = u - Ts$ ,

$$d\psi = du - T ds - s dT$$

But,  $du = dq - P dv$ , and, for a reversible process,  $dq = T ds$ . Hence,

$$d\psi = T ds - P dv - T ds - s dT$$

$$= -P dv - s dT$$

As  $\psi = f(v, T)$ ,

$$\left(\frac{dP}{dT}\right)_v = \left(\frac{ds}{dv}\right)_T \quad (14-22)$$

We will now proceed with the derivation of an expression for  $du$ . From equation 14-18,  $l_v = T\left(\frac{ds}{dv}\right)_T$ . Substituting for  $\left(\frac{ds}{dv}\right)_T$  its value from equation 14-22, we get

$$l_v = T\left(\frac{dP}{dT}\right)_v \quad (14-23)$$

Substituting the value of  $l_v$  from equation 14-23 in equation 14-15, we obtain

$$du = c_v dT + \left[ T\left(\frac{dP}{dT}\right)_v - P \right] dv \quad (14-24)$$

Equation 14-24 requires, for its use, a knowledge of the variations of  $c_v$  with  $T$  in the region where the change in intrinsic energy is desired and also requires an equation of state which is accurate in this region.

To establish an equation for the change in enthalpy,  $dh$ , start with the general expression for  $dh$ , which is

$$dh = du + P dv + v dP = dq + v dP \quad (14-25)$$

Since  $dq$  varies with  $P$  and  $T$ ,

$$dq = \left( \frac{dq}{dP} \right)_T dP + \left( \frac{dq}{dT} \right)_P dT \quad (14-26)$$

The coefficient  $\left( \frac{dq}{dT} \right)_P$  is recognized as  $c_p$  and the coefficient  $\left( \frac{dq}{dP} \right)_T$  is the coefficient of pressure expansion, or  $l_p$ . Substituting these values in equation 14-26 and substituting the resulting value of  $dq$  from equation 14-26 in equation 14-25, we obtain

$$dh = c_p dT + l_p dP + v dP \quad (14-27)$$

As  $ds = \frac{dq}{T}$  in a reversible process, it follows from equation 14-26 that

$$ds = \frac{l_p dP}{T} + \frac{c_p dT}{T} \quad (14-28)$$

Since  $s = f(T, P)$ ,

$$ds = \left( \frac{ds}{dT} \right)_P dT + \left( \frac{ds}{dP} \right)_T dP \quad (14-29)$$

Comparison of equations 14-28 and 14-29 shows that

$$\frac{c_p}{T} = \left( \frac{ds}{dT} \right)_P$$

and

$$\frac{l_p}{T} = \left( \frac{ds}{dP} \right)_T \quad (14-30)$$

From equation 14-30,

$$l_p = T \left( \frac{ds}{dP} \right)_T \quad (14-31)$$

Substituting the value of  $\left( \frac{ds}{dP} \right)_T$  from equation 14-21 in equation 14-31, we obtain

$$l_p = - \left( \frac{dv}{dT} \right)_P T \quad (14-32)$$

Substitution of the value of  $l_v$  from equation 14-32 in equation 14-27 gives

$$dh = c_p dT + \left[ v - \left( \frac{dv}{dT} \right)_P T \right] dP \quad (14-33)$$

Equation 14-33 requires, for its use, a knowledge of the variations of  $c_p$  with  $T$  and also an accurate equation of state for the regions in which the change in enthalpy is desired

An expression for entropy change that involves  $c_v$  and relations of  $T$ ,  $P$ , and  $v$  for the substance in question may be derived by substituting the value of  $l_v$  from equation 14-23 in equation 14-16. Thus,

$$ds = c_v \frac{dT}{T} + \left( \frac{dP}{dT} \right)_v dv \quad (14-34)$$

The general equations of thermodynamics are extremely useful in tying together various thermodynamic properties which have been determined experimentally. This use may be illustrated by setting up a relationship between the change in enthalpy and the change in volume during vaporization of a liquid

$$\begin{aligned} \text{Equation 14-23 is } l_v &= T \left( \frac{dP}{dT} \right)_v. \quad \text{Since, by definition, } l_v = \left( \frac{dq}{dv} \right)_T \\ T \left( \frac{dP}{dT} \right)_v &= \left( \frac{dq}{dv} \right)_T \end{aligned}$$

For a saturated vapor,

$$\left( \frac{dP}{dT} \right)_v = \left( \frac{dP}{dT} \right)_{\text{saturation}} \quad \text{and} \quad \left( \frac{dq}{dv} \right)_T = \frac{h_{fg}}{v_{fg}}$$

Therefore,

$$T \left( \frac{dP}{dT} \right)_{\text{saturation}} = \frac{h_{fg}}{v_{fg}}$$

$$\text{or} \quad h_{fg} = v_{fg} T \left( \frac{dP}{dT} \right)_{\text{saturation}} \quad (14-35)$$

Equation 14-35, which is known as the Clapeyron equation, points out the relationship that must exist between the enthalpy change during vaporization, the volume change during vaporization, and the rate of change of saturation pressure with temperature at any given temperature. If experimental results do not fit into equation 14-35, then one or more of the experimental results must be in error.

## CHAPTER 15

### AIR CONDITIONING

15-1. **Air Conditioning; Its Meaning and Use.**—Fundamentally, the term air conditioning means the act of putting the air in the proper condition for the use desired. This may mean an adjustment of the cleanliness, the temperature, and the water-vapor content (humidity) of the air.

The problem of cleanliness is a mechanical one. It is concerned with the removal of dirt and dust, smoke, odors, and other impurities. Dirt and dust may be removed by mechanical filters. Passing the air through liquid baths or sprays (oil or water) reduces not only the dirt and dust content but also the smoke and odor. Further consideration of the problem of cleanliness is beyond the scope of this text.

The problem of adjusting the temperature and humidity of the air is a thermodynamic one and is the subject of this chapter.

The uses of air conditioning can be divided into two general classes: (a) industrial air conditioning and (b) air conditioning for comfort. In some cases the two classes may overlap. For instance, an industrial operation may require reducing both the temperature and humidity of the air in summer. This will also increase the comfort of the personnel.

Air conditioning has made it possible to manufacture a better product at a higher rate in such diversified fields as book and newspaper printing, bakeries, textile manufacturing, and candy making. Air conditioning has been applied to the air supplied to blast furnaces. Because of the very large quantities of air required by blast furnaces, such air-conditioning installations are large and expensive. It has been stated that the increased production of pig iron justifies the high installation and operating costs, although some operators question this. In addition to the fields just mentioned, high-temperature air is used to a large extent for drying, *i.e.*, removing various types of liquids from products which became wet in the process of their manufacture.

Air conditioning for comfort\* is becoming almost a prerequisite for success in the merchandising field, particularly in the southern part of the United States. In addition to stores of all sorts, other businesses that are being air-conditioned to a large extent are hotels, theaters, and

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\* The specifications for air that is comfortable depend on many factors, such as air velocity, kind of activity of the individual, outside temperature, and individual preferences. The discussion of these items is beyond the scope of this text. See the "Guide" of The American Society of Heating and Ventilating Engineers for a good treatment of this subject.

restaurants The gain in customer good will is reflected by such an increase in business that air conditioning more than pays for itself In the home, where there is no such financial gain possible, summer air conditioning has lagged Another factor causing the lag is that in the normal home the number of persons together at once is small, hence, the increase in air temperature as a result of body heat is not so great as it is in places of public gatherings One factor which is keeping down the demand for household air conditioning is the high initial cost In turn, the cost is kept high by the low rate of production Certainly, any great increase in demand should be accompanied by a lowering of the initial cost and vice versa

**15-2 Fundamentals**—(a) *Absolute and Relative Humidities* When the atmosphere contains no water vapor, the mixture of gases is known as "dry air" If any water vapor is present, the gaseous mixture is called "wet air," i.e., a mixture of dry air and water vapor In most humidity problems, the total pressure of the wet air is seldom very much greater than atmospheric and each of the individual partial pressures is only a fraction of this amount Furthermore as the temperatures encountered are rarely very high (i.e., above 150 F), the specific heats of the constituents of the air may be considered constant, and all the gases, including water vapor, may be treated as perfect gases for ordinary engineering calculations Thus, Dalton's law of partial pressures applies, and it may be assumed that each gas occupies the entire volume at its own partial pressure None of the gases depends on the others for its existence The dry air does not "hold" the water vapor any more than the water vapor "holds" the air

The amount of water vapor in a given space is specified by the *absolute humidity* of the air The amount of space involved may be the total volume encountered or it may be a unit volume—say the cubic foot The amount of the vapor is specified by its weight This may be expressed in pounds or in grains There are 7,000 grains per pound

The maximum amount of water vapor that can be present in a given space depends on its temperature If the pressure of water vapor in a given space is less than the saturation pressure for the given temperature, but some water is present, vaporization will continue until the pressure reaches the saturation pressure for the given temperature This is true whether or not air is present, although the presence of air prevents boiling *Relative humidity* is used to express the degree of saturation of a given space It is defined as the ratio of the amount of water vapor present in a given space to the amount that would be present in the same space if that space were saturated at the same temperature Because a given space normally

is not completely saturated, the vapor pressure is generally less than the saturation pressure for the given temperature. Or, the actual vapor temperature exceeds the saturation temperature for the actual vapor pressure. Under these conditions, the water vapor must be superheated.

This may be shown by reference to Fig. 15-1. Here is illustrated water vapor existing at point 1, at a temperature of 79.58 F and a pressure of 0.2 psia. Steam tables show that the saturation pressure for 79.58 F is 0.5 psia and that the saturation temperature for a pressure of 0.2 psia is 53.14 F. Because the vapor at point 1 exists at a temperature in excess of 53.14 F, it is superheated vapor.

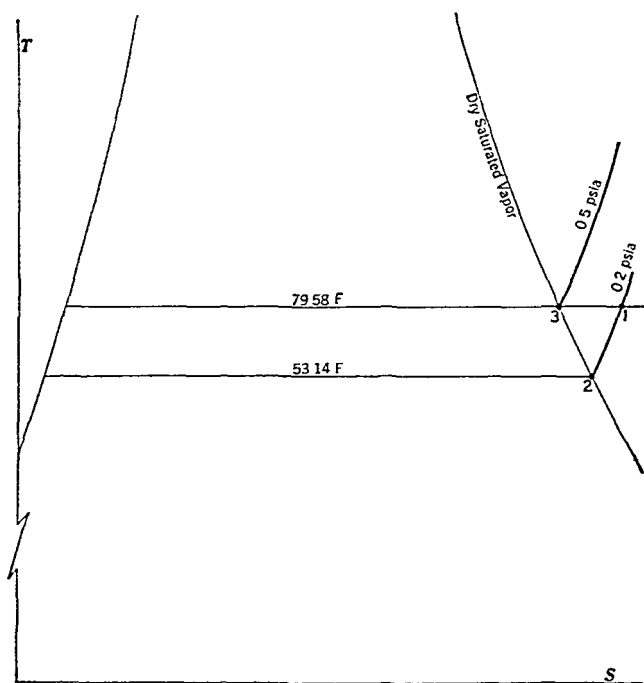


Fig. 15-1. T-S Diagram Showing Superheated Water Vapor at Point 1

It is difficult to determine directly the weight of vapor present in a given space. If the water vapor might be removed from the air, then the amount of vapor could be determined by finding the difference between the original weight and the final weight. However, although water vapor may be removed from air by cooling it or by physical-chemical means, complete removal of water vapor from the air is extremely difficult. Thus, other means must be used to determine the humidity of the air. If the



partial pressure of the vapor is available, the relative humidity of the air may be calculated by treating the vapor as a perfect gas. Writing the perfect-gas laws for the actual vapor (indicated by the subscript act), we obtain

$$P_{act}V_{act} = W_{act}RT_{act} \quad (15-1)$$

For vapor in the saturated state (indicated by the subscript sat),

$$P_{sat}V_{sat} = W_{sat}RT_{sat} \quad (15-2)$$

When one of these two equations is divided by the other and the relative humidity (R H) is substituted for  $\frac{W_{act}}{W_{sat}}$ , where the weights are for the same volumes at the same temperatures, the result is

$$R H = \frac{P_{act}}{P_{sat}} \quad (15-3)$$

(b) *Dew Point* The actual pressure of the vapor may be determined if the dew-point temperature of the air is known. As may be inferred from its name, dew-point temperature is the temperature at which dew starts to form. The natural process of forming dew is a constant pressure cooling one. Hence, "dew-point temperature" is that temperature at which condensation starts when wet air is cooled at constant pressure. If the total pressure is kept constant as the air is cooled to the dew point temperature, the pressure of the individual gases must also remain constant. Thus, the pressure of the water vapor at the point where condensation starts equals the original water-vapor pressure. But, when condensation starts, the water vapor is saturated. The original vapor pressure therefore equals the saturation pressure for the dew point temperature. Point 2 in Fig. 15-1 is the dew point. Process 1-2 represents a constant pressure cooling process to the dew point.

Although commercial means are available for dew-point determinations, such equipment seldom can be used conveniently. Furthermore, much care must be taken with most dew-point apparatus to obtain accurate results.

(c) *Wet Bulb and Dry Bulb Thermometers* A convenient means of obtaining the partial pressure of the water vapor in the air is the psychrometer. This consists of two thermometers, a dry bulb and a wet bulb, mounted side by side. The dry bulb is an ordinary thermometer and measures the actual air temperature. The bulb of the wet bulb thermometer is surrounded by a wick, which is kept moistened when in use. The action of the wet bulb depends on continuous vaporization from the wick. There must therefore be positive motion between the air and the

wet bulb. This may be provided by attaching a handle to the psychrometer and rotating it. As the air passes over the wet bulb, some water vaporizes, tending to saturate the air in immediate contact with the wet bulb. The vaporization requires heat, which comes from the air and its vapor. This heat reduces both the air temperature and the wet-bulb temperature.

Over a third of a century ago, Willis Carrier concluded that, in the ideal case, the action taking place at the wet-bulb thermometer was one of adiabatic saturation. The term "adiabatic saturation" refers to the process of air becoming saturated in an adiabatic manner. In other words, Carrier assumed that room air, in the theoretical case, passed over the wet bulb, leaving the bulb saturated, with no heat coming in from the outside. As the water on the wick was in intimate contact with the air, the water quickly assumed the temperature of the leaving air. Thus, Carrier reasoned that the observable wet-bulb temperature was that of adiabatic saturation. The adiabatic-saturation equation is one of steady flow and is as follows:

$$(W_{\text{air}} h_{\text{air}} + W_{\text{vap}} h_{\text{vap}})_{\text{to bulb}} + (W_{\text{water}} h_{\text{water}}^*)_{\text{vaporized}} = (W_{\text{air}} h_{\text{air}} + W_{\text{vap}} h_{\text{vap}})_{\text{from bulb}} \quad (15-4)$$

Carrier rearranged equation 15-4 and evaluated enthalpies in terms of temperatures and evaluated weights in terms of pressures, volumes, and temperatures. The reduced equation became

$$P_v = P_{v'} - \frac{(P_B - P_{v'})(t - t')}{2800 - 1.3t'} \quad (15-5)$$

where  $P_v$  = actual vapor pressure, psia;

$P_{v'}$  = saturated vapor pressure for the wet-bulb temperature, psia;

$P_B$  = atmospheric pressure, psia;

$t$  = dry-bulb temperature, °F;

$t'$  = wet-bulb temperature, °F.

The following factors cause the actual wet-bulb temperature to differ from the theoretical temperature:

- (1) Radiation to the wet bulb from the surroundings.
- (2) Conduction along the stem of the wet-bulb thermometer.
- (3) Impact of the air on the wet bulb.

Carrier made adjustments to equation 15-5 to account for these effects.

However, a careful analysis of the action on the theoretical wet bulb has shown that the action there is not a truly adiabatic saturation process.

\* Strictly speaking, as the water on the bulb is at rest, its energy is of the intrinsic form. However, the intrinsic energy of the water is practically equal to its enthalpy.

Surrounding the wet bulb is a very thin stagnant film of air and vapor. The heat required for vaporization must be transferred through this film. Hence, the air temperature leaving the wet bulb must exceed that of the wet bulb. Water vapor must pass outward through the thin film to the moving air by a process of diffusion. Hence, the partial pressure of the vapor in the air is less than that on the wet bulb. In the case of certain liquids vaporizing into some gases, the combined effect of diffusivity and heat transfer through the stagnant film causes the observable vaporiza-

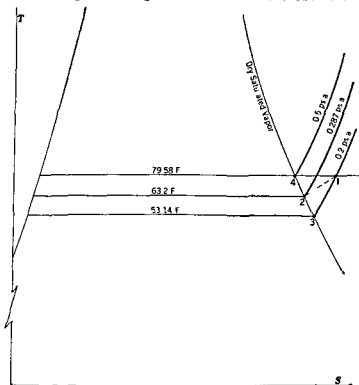


Fig 15-2 Action on Wet Bulb

tion temperature to differ materially from the adiabatic saturation temperature. (The wet-bulb temperature is a vaporization temperature.) In the case of water vaporizing into air, however, when the air velocity over the wet bulb approximates 1,000 ft per min, the observable wet bulb temperature equals the adiabatic saturation temperature within the range of ordinary engineering accuracy, provided that no attempt is made to shield the wet bulb from radiation. Hence, Carrier's uncorrected equation, or equation 15-5, is to be used unless extreme accuracy is desired. Furthermore, in dealing with drying problems, it will be assumed that the air is moving over the vaporizing water with considerable velocity, hence, it

will be assumed that the vaporization temperature equals the adiabatic saturation temperature.

The action on the wet bulb is illustrated in Fig. 15-2. Point 1 represents the condition of the vapor in the air as it comes up to the wet bulb, and point 2 represents the condition of the vapor in the air as it leaves the wet bulb. Because the air picks up vapor as it passes over the wet bulb, the amount—and hence the pressure—of the vapor increases. Thus, the vapor pressure leaving the wet bulb exceeds that of the room air; and the saturation temperature,  $t_2$ , of the vapor in the leaving air must exceed the dew-point temperature,  $t_3$ .

(d) *Specific Humidity*: When dealing with air flowing through devices which either humidify or dehumidify the air, there will be changes in the total volume flowing, as well as a change in the amount of vapor flowing. The weight of dry air flowing remains constant, however. Hence, it is desirable to express the various quantities involved, particularly the amount of vapor, per pound of dry air. "Specific humidity"\* is used to denote the number of pounds of water vapor per pound of dry air. If it is assumed that both the air and the vapor may be treated as perfect gases, the following equations of state may be written for each:

$$\begin{aligned}P_v V_v &= W_v R_v T_v \\P_a V_a &= W_a R_a T_a\end{aligned}$$

where the subscript  $v$  refers to the water vapor and  $a$  to the dry air.

By dividing one of these two equations by the other and utilizing the fact that the air and vapor both occupy the entire volume, we obtain:

$$\frac{W_v}{W_a} = \text{Specific Humidity (S.H.)} = \frac{P_v R_a}{P_a R_v} = \frac{P_v (53.3)}{P_a (85.8)} = 0.622 \frac{P_v}{P_a} \quad (15-6)$$

*Example 15-1.*—Air at 90 F and 14.7 psia has a wet-bulb temperature of 74 F. Determine: (a) the absolute humidity; (b) the relative humidity; (c) the specific humidity; (d) the dew-point temperature.

*Solution.*—(a) From equation 15-5,

$$P_v = P_v' - \frac{(P_B - P_v')(t - t')}{2800 - 1.3t'} = 0.4156 - \frac{(14.7 - 0.4156)(90 - 74)}{2800 - 1.3(74)} = 0.3311 \text{ psia}$$

To find the absolute humidity, use the equation of state,  $PV = WRT$ . Thus,

$$W = \frac{144 \times 0.3311 \times 1}{85.8(90 + 460)} = 0.001010 \text{ lb per cu ft}$$

or 7.070 grains per cu ft

Ans.

\* There is some objection to this definition of specific humidity. Instead of the term specific humidity, the Guide of the American Society of Heating and Ventilating Engineers uses the term "humidity ratio." For a more detailed discussion of the thermodynamic properties of air-vapor than can be given here, see Chapter 3 in the ASHVE Guide.

(b) To find the relative humidity, use equation 15-3. Hence,

$$R.H. = \frac{P_{act}}{P_{sat}} = \frac{0.3311}{0.6982} = 0.474 \text{ or } 47.4\% \quad \text{Ans.}$$

(c) To determine the specific humidity, use equation 15-6 which gives

$$S.H. = 0.622 \times \frac{0.3311}{14.7 - 0.3311} = 0.01433 \text{ lb vapor per lb dry air} \quad \text{Ans.}$$

(d) The saturation temperature corresponding to the pressure of 0.3311 is 67.3 F. Therefore the dew point temperature equals 67.3 F. Ans.

*Example 15-2*—Air has a relative humidity of 72% when its temperature is 80 F and its pressure is 14.9 psia. Determine its specific humidity and also its dew point temperature.

*Solution*—By equation 15-3,

$$P_{act} = R.H. (P_{sat}) = 0.72(0.5069) = 0.365 \text{ psia}$$

By equation 15-6,

$$S.H. = 0.622 \times \frac{0.365}{14.9 - 0.365} = 0.01562 \text{ lb vapor per lb dry air} \quad \text{Ans.}$$

From the vapor pressure of 0.365, the dew-point temperature is 70.2 F. Ans.

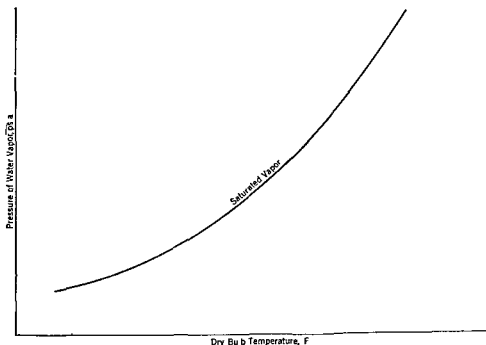


Fig. 15-3 Saturated Vapor Line on Psychrometric Chart

**15-3 Psychrometric Charts.**—Much work has been done in presenting properties of air-vapor mixtures in chart form, generally at standard atmospheric pressure. To illustrate the preparation of such a chart, take vapor pressures as ordinates and dry-bulb temperatures as abscissas.

The saturated-vapor line may be plotted as in Fig. 15-3, the vapor pressures being taken directly from the steam tables for the various dry-bulb temperatures.

Examination of equation 15-5 shows that for a given wet-bulb temperature the variation of the vapor pressure  $P_v$  with the dry-bulb temperature  $t$  is represented by a straight line. (The atmospheric pressure  $P_B$  is a constant, as is the saturation pressure  $P_{v'}$  corresponding to the fixed wet-bulb temperature  $t'$ .) Lines of constant wet-bulb temperature appear in Fig. 15-4.

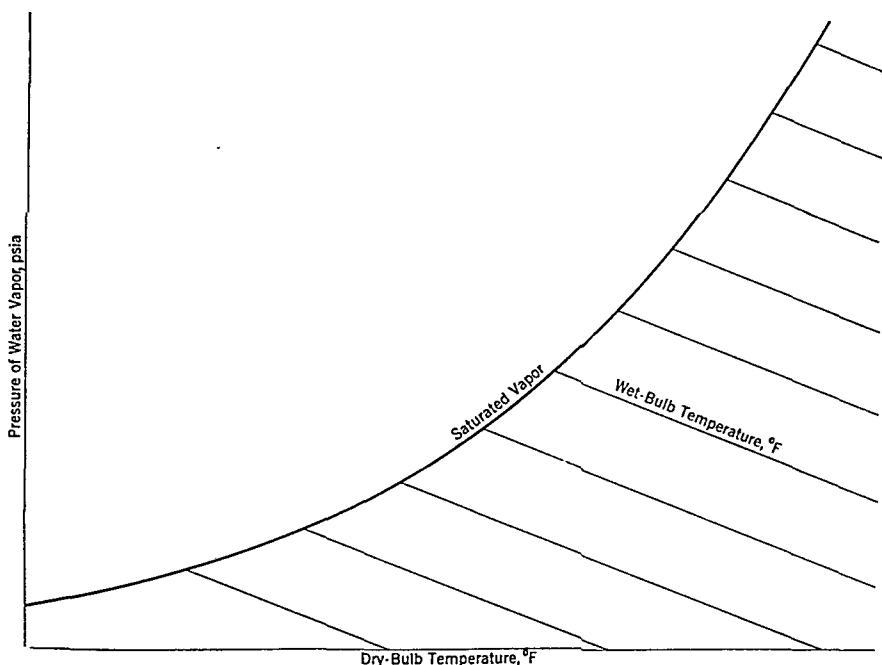


Fig. 15-4. Lines of Constant Wet-Bulb Temperature on Psychrometric Chart

The actual partial pressure of the vapor at any dry-bulb temperature and any fixed relative humidity may be calculated by multiplying the saturated pressure at the given temperature by the relative humidity, as indicated by equation 15-3. Lines of constant relative humidity are shown in Fig. 15-5.

The volume per pound of dry air can be calculated from the equation of state of a perfect gas. For a fixed volume per pound of dry air, the air

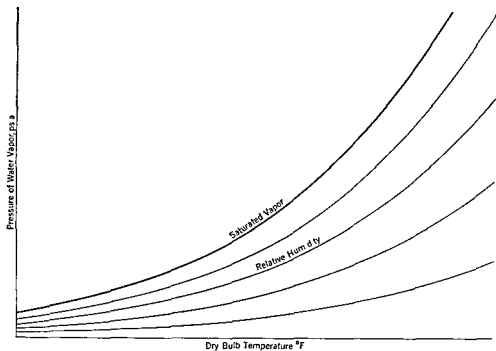


Fig 15-5 Lines of Constant Relative Humidity on Psychrometric Chart

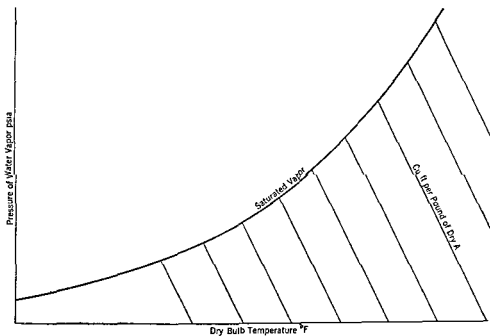
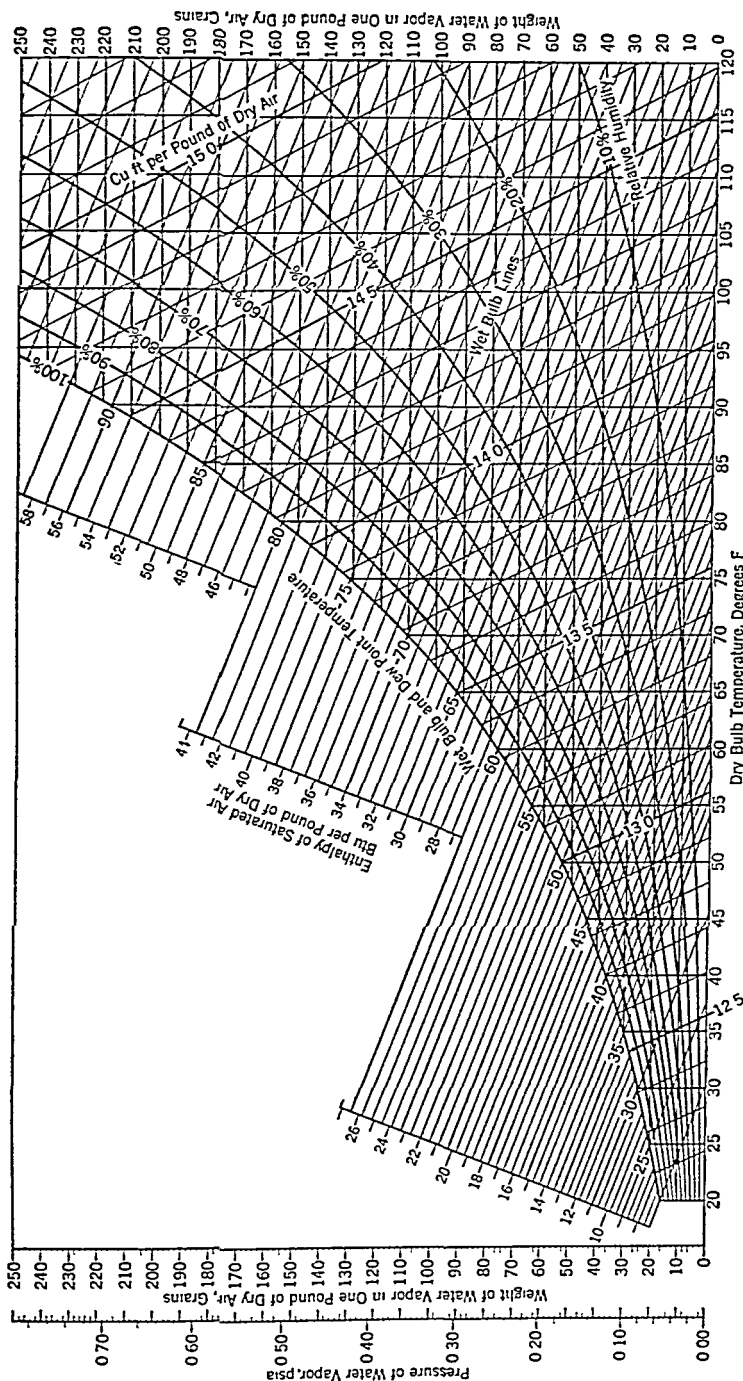


Fig 15-6 Lines of Constant Volume of Dry Air on Psychrometric Chart



(Courtesy of General Electric Company)

Fig. 15-7. Commercial Psychrometric Chart



pressure  $P_a$  equals the total pressure  $P_T$  minus the vapor pressure  $P_v$ . That is,

$$P_a = P_T - P_v = \frac{WRT}{V}$$

or

$$P_v = P_T - \frac{WRT}{V}$$

Thus, for a fixed volume of dry air and a given total pressure, the variation of the vapor pressure with the dry bulb temperature is represented by a straight line. Lines of constant volume are shown in Fig 15-6

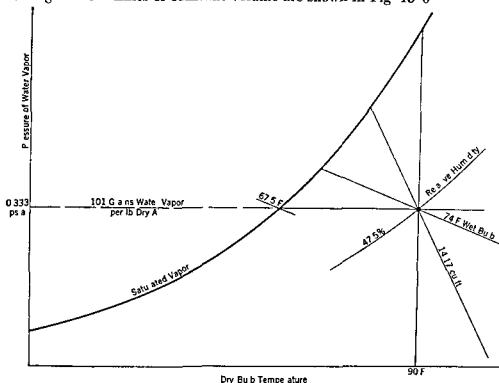


Fig 15-8 Solution of Example 15-3

Fig 15-7 shows a psychrometric chart. According to equation 15-6 specific humidity is almost in direct proportion to vapor pressure, particularly at low vapor pressures. In Fig 15-7 the specific humidity is taken as the ordinate. The vapor pressure also is shown as the ordinate, an adjusted scale being required to correlate it with specific humidity.

For any fixed wet-bulb temperature the enthalpy of the saturated air vapor leaving the wet bulb, expressed in Btu per pound of dry air, is a constant. Hence, lines of constant wet bulb temperature can point to a scale of enthalpy of saturated mixture per pound of dry air. This relation

is shown in Fig. 15-7. The enthalpy of water is very small and hence may be neglected for routine engineering calculations, except at very high wet-bulb temperatures. By reference to equation 15-4 it may be seen that the enthalpy of the air-vapor mixture passing to the wet bulb (*i.e.*, room air) equals the enthalpy of the mixture leaving the wet bulb, the enthalpy of the water vaporized at the wet bulb being neglected. But, as the enthalpy leaving the wet bulb is fixed by the wet-bulb temperature, *the enthalpy of an air-vapor mixture at any dry-bulb temperature equals the enthalpy of the saturated mixture at its wet-bulb temperature*, if the enthalpy of the water vaporized to adiabatically saturate the air is neglected. The use of this fact will be demonstrated later on in this chapter.

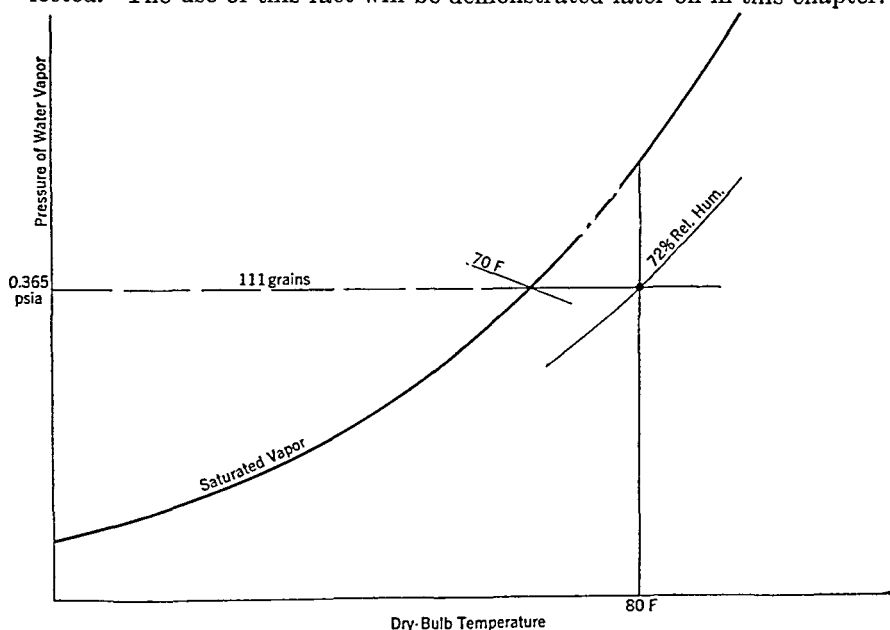


Fig. 15-9. Solution of Example 15-4

**Example 15-8.**—Solve Example 15-1 by use of the psychrometric chart in Fig. 15-7.

**Solution.**—The lines and points that are located on Fig. 15-7 are shown in Fig. 15-8. The intersection of the vertical line corresponding to a dry-bulb temperature of 90 F and the inclined line for a wet-bulb temperature of 74 F on the chart gives the following results:

101 grains moisture per pound of dry air  
 0.333 psia vapor pressure (0.3311 calculated)  
 47.5% relative humidity (47.4% calculated)  
 14.17 cu ft per pound dry air

From these values, the specific humidity equals

$$\frac{101}{7000} = 0.01443 \text{ lb per lb of dry air (0.01433 calculated)}$$

The absolute humidity equals

$$\frac{0.01443}{14.17} = 0.001018 \text{ lb per cu ft (0.001010 calculated)}$$

For the vapor pressure of 0.333 psia, the chart shows a dew point temperature of 67.5 F

*Example 15-4* — Use the chart in Fig. 15-7 to solve Example 15-2

*Solution* — The points and lines of importance in this case are shown in Fig. 15-9. From the chart in Fig. 15-7, the pressure of the vapor is 0.365 psia, which is also the calculated value. The dew point is 70.2 F. The chart shows a specific humidity of 111 grains or 0.01586 lb vapor per pound of dry air, as compared to the calculated value of 0.01562 lb. The reason for the difference between the two values of specific humidity is that the chart method does not take into account the fact that the total pressure is 14.9 psia instead of the standard value. The chart error is not large in this case, as the pressure is close to the standard value.\*

**15-4 Simple Heating and Cooling** — Most heating and cooling of air in air-conditioning work is done at substantially constant pressure. In case the water-vapor content of the air is neither increased nor decreased during the heating or cooling process, the pressure of vapor must also remain substantially constant. Although steam may be blown into the air, the normal way in which the vapor content of the air may be increased is by the introduction of water into the air. Unless removed by physical-chemical means, water vapor is removed only by cooling the air to a temperature below its dew point. Although the heat removed from or added to an air-vapor mixture may be evaluated by treating the air and vapor separately and adding the results together, it is common practice to treat the mixture as an entity. This will be done here.

For air, the datum is 0 F, the enthalpy of air being taken at zero at the datum plane. For water, the enthalpy of the liquid is taken as zero at 32 F, as in the steam tables. The enthalpy of the mixture per pound of dry air is

$$h_m = 0.24 t + S H (h_g) \quad (15-7)$$

where  $S H$  is the specific humidity, in pounds of water vapor per pound of dry air, and  $h_g$  is the enthalpy of the vapor, in Btu per pound.

Because at low partial pressures the vapor closely obeys the law  $PV = WRT$ , the enthalpy of the vapor is independent of pressure and is a function of temperature alone. An examination of the superheated steam tables justifies this statement, as there is very little difference between the enthalpies of steam at any given temperature when the pressure is 1 psia, 2 psia, or 3 psia. However, at low temperatures, a further increase in pressure materially reduces the enthalpy. Although the steam tables give accurate values of enthalpies for various pressures and temperatures,

\* Some psychrometric charts have been devised to take account of variations in barometric pressure. Allowance is generally made however by applying a correction to the value obtained from the standard atmospheric pressure chart.

most problems call for interpolation or extrapolation of steam-table values for their solution. To eliminate this, many equations have been proposed which are satisfactory over a certain range of conditions. The authors present the following equation:

$$h = 1061.0 + 0.445 t \quad (15-8)$$

where  $h$  is the enthalpy of the vapor, in Btu per pound; and  $t$  is the Fahrenheit temperature.

The maximum error of equation 15-8 is 1.4 Btu per lb of vapor over the range of  $-40^\circ\text{F}$  to  $+250^\circ\text{F}$ , provided that the vapor pressure does not exceed 3 psia. When the value of  $h$  is assumed to be given by equation 15-8, equation 15-7 becomes

$$h_m = 0.24 t + \text{S.H.}(1061.0 + 0.445 t) \quad (15-9)$$

The energy quantities involved in problems of simple heating or cooling are  $h_{m1}$ ,  $Q$ , and  $h_{m2}$ . As the velocities are relatively low in air-conditioning work, the kinetic energies may be neglected. The solution for the heat added or removed involves the use of the steady-flow equation.

*Example 15-5.*—A quantity of air having a volume of 5000 cu ft at  $85^\circ\text{F}$  and 40% R.H. is cooled to  $60^\circ\text{F}$ . The air pressure is 14.2 psi. Calculate the heat removed.

*Solution.*—The original partial pressure of the vapor, by equation 15-3, is

$$P_v = \text{R.H.} (P_{sat}) = (0.4)(0.5959) = 0.2384 \text{ psia}$$

At  $60^\circ\text{F}$  the saturated partial pressure is 0.2563. As the original vapor pressure is less than this value, there can be no condensation during the cooling process and hence there can be no change in the specific humidity. The original specific humidity, according to equation 15-6, is

$$\begin{aligned} \text{S.H.} &= 0.622 \frac{P_v}{P_a} \\ &= 0.622 \times \frac{0.2384}{14.2 - 0.2384} \\ &= 0.01062 \text{ lb vapor per lb dry air} \end{aligned}$$

By equation 15-9, the enthalpy into the cooler equals

$$\begin{aligned} h_{m1} &= 0.24 t + \text{S.H.}(1061.0 + 0.445 t) \\ &= 0.24(85) + 0.01062[1061.0 + 0.445(85)] \\ &= 32.07 \text{ Btu per lb dry air} \end{aligned}$$

In a like manner, the enthalpy leaving the cooler equals

$$\begin{aligned} h_{m2} &= 0.24(60) + 0.01062[1061.0 + 0.445(60)] \\ &= 25.95 \text{ Btu per lb dry air} \end{aligned}$$

Then the heat removed per pound of dry air is

$$Q = h_{m1} - h_{m2} = 32.07 - 25.95 = 6.12 \text{ Btu per lb dry air}$$

The weight of dry air, based on the original volume, is

$$W = \frac{PV}{RT} = \frac{(14.2 - 0.2384)(144)(5000)}{53.3(85 + 460)} = 346.3 \text{ lb}$$

Then the total heat removed equals

$$(346.3)(6.12) = 2119 \text{ Btu}$$

Ans.

*Example 15-6*—Saturated air at 35 F is heated to 110 F. Calculate the heat added per pound of dry air and the final relative humidity. Assume standard atmospheric pressure.

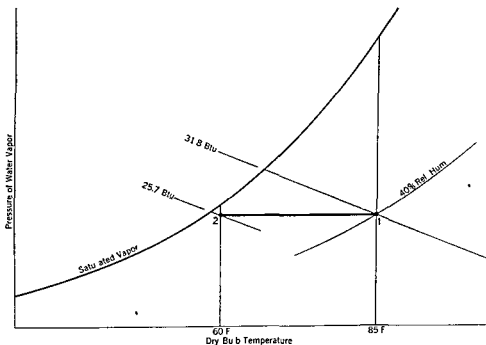


Fig. 15-10 Solution of Example 15-7

*Solution*—The original partial pressure of the vapor is 0.09995 psia, which is also the final vapor pressure. Then the final relative humidity is

$$R.H. = \frac{P_v}{P_{sat}} = \frac{0.09995}{1.2748} = 0.0784 \text{ or } 7.84\% \quad \text{Ans}$$

The specific humidity is

$$\begin{aligned} S.H. &= 0.622 \frac{P_v}{P_a} \\ &= 0.622 \times \frac{0.09995}{14.7 - 0.09995} \\ &= 0.00426 \text{ lb vapor per lb dry air} \end{aligned}$$

The enthalpy of the mixture entering the heater is

$$\begin{aligned} h_{m1} &= 0.24(35) + 0.00426[1061.0 + 0.445(35)] \\ &= 12.99 \text{ Btu per lb dry air} \end{aligned}$$

The enthalpy of the mixture leaving the heater is

$$\begin{aligned} h_{m_2} &= 0.24(110) + 0.00426[1061.0 + 0.445(110)] \\ &= 31.13 \text{ Btu per lb dry air} \end{aligned}$$

The heat added in the heater is

$$Q = h_{m_2} - h_{m_1} = 31.13 - 12.99 = 18.14 \text{ Btu per lb dry air} \quad \text{Ans.}$$

**Example 15-7.**—Determine the heat transferred in the two preceding examples by means of the psychrometric chart in Fig. 15-7.

**Solution.**—For the *cooling problem*, the enthalpy of saturated air at the original conditions is 31.8 Btu from the chart (see Fig. 15-10). As the cooling process is one at constant pressure, the final state point may be found by going horizontally to the left from the initial point until a temperature of 60 F is reached. At this point the enthalpy of the saturated air is 25.7 Btu. The difference between the original and final enthalpies of saturated air is  $31.8 - 25.7 = 6.1$  Btu per pound of dry air. This compares with the calculated heat removed of 6.12 Btu per pound of dry air.

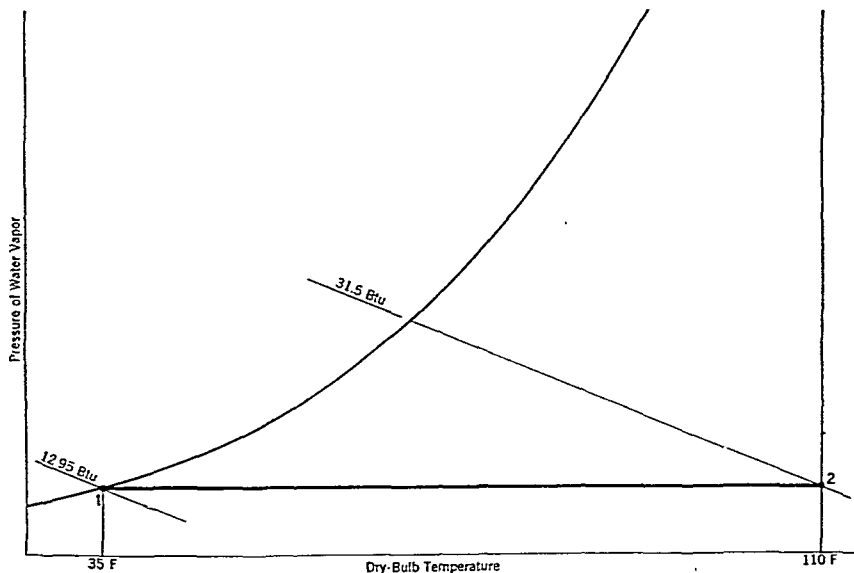


Fig. 15-11. Solution of Example 15-7

There are two errors involved in the use of the chart method. First, the pressure is not standard atmospheric, although it is close to it. Second, the enthalpy of the air is not exactly equal to the enthalpy of saturated air having the same wet-bulb temperature. These two errors evidently tend to compensate each other in this case but will not do so in all cases.

For the *heating problem*, the enthalpy of the original mixture is 12.95 Btu from the chart (see Fig. 15-11). Moving horizontally to the right from the original state point until a temperature of 110 F is reached locates the final state point. At the final state, the enthalpy of the saturated air is 31.5 Btu per pound of dry air. The difference between the two values of enthalpy is 18.55 Btu per pound of dry air. This compares with the calculated amount of heat added of 18.14 Btu per pound of dry air.

**15-5 Dehumidification**—Water vapor may be removed from air chemically, physically, or thermodynamically. Several chemicals are used for dehumidification, such as calcium oxide, calcium chloride, and anhydrous calcium sulfate. Physically, water vapor may be removed by absorption. Some substances used for this purpose are activated alumina and silica gel. Thermodynamically, water vapor may be removed from air by cooling the air below its dew point. This method is the subject of this article. It should be noted that the removal of water vapor from the air is relatively easy when the humidity is high, but becomes progressively more difficult as the humidity decreases. Although all the methods are used to reduce the humidity of the air to a low value, none of them produces air with zero humidity.

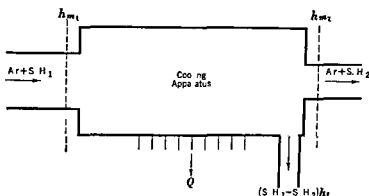


Fig 15-12 Dehumidification by Cooling and Condensation

When air is cooled below its dew point, the vapor remaining is saturated, provided it is not reheated after the cooling process. Because of condensation of the water vapor, both the specific humidity and the vapor pressure decrease. As the total pressure remains constant, the partial pressure of the dry air must increase as the vapor pressure decreases because of condensation. Fig 15-12 shows a schematic diagram under steady flow conditions.

The energy equation for a dehumidifier is

$$h_{m1} = h_{m2} + (S H_1 - S H_2)h_f + Q$$

where  $(S H_1 - S H_2)h_f$  is the energy in the condensate leaving. There are two types of dehumidifiers. The temperature of the condensate at exit depends on the type of dehumidifier used and on its method of operation.

In one type of dehumidifier, the cooling is accomplished by finned tubes which serve as the evaporator for a refrigeration system. When

moderate temperatures are maintained in the tubes, the condensate collects on the tubes and then runs off. With lower tube temperatures the condensate collects as frost on the tubes. This condition necessitates shutting down the evaporator and defrosting it.

In the other type of dehumidifier, dehumidification is accomplished by spraying cold water into the air. The spray water must have a temperature lower than that of the entering air; the lower the spray temperature, the greater the amount of dehumidification. The condensate mixes with the spray water and leaves with it. Even though the temperature of the leaving condensate is thus indefinite, very little error is introduced here as the enthalpy of the liquid is small in comparison with the other enthalpies involved, particularly if the temperature of the condensate is low. Lacking more specific information, the condensate will be assumed to leave the dehumidifier at the same temperature as does the saturated air.

*Example 15-8.*—Air enters a dehumidifier at 94 F and 70% R.H., and leaves at 50 F. Calculate the heat removed per pound of dry air. There is standard atmospheric pressure.

*Solution.*—The entering vapor pressure is

$$P_{v_1} = 0.70(0.7906) = 0.5534 \text{ psia}$$

The saturated-vapor pressure at the exit temperature of 50 F is 0.17811. As this is lower than the entering-vapor pressure, condensation takes place and the air is saturated at exit. The original specific humidity is

$$\text{S.H.}_1 = 0.622 \times \frac{0.5534}{14.7 - 0.5534} = 0.02429 \text{ lb vapor per lb dry air}$$

The final specific humidity is

$$\text{S.H.}_2 = 0.622 \times \frac{0.17811}{14.7 - 0.17811} = 0.00763 \text{ lb vapor per lb dry air}$$

The difference between these two specific humidities is the amount of condensate formed. This equals  $0.02429 - 0.00763 = 0.01666$  lb per lb of dry air.

The original enthalpy is

$$\begin{aligned} h_{m_1} &= 0.24 t + \text{S.H.}_1 (1061 + 0.445 t) \\ &= 0.24(94) + 0.02429[1061 + 0.445(94)] \\ &= 49.35 \text{ Btu per lb dry air} \end{aligned}$$

The enthalpy of the exit air is

$$\begin{aligned} h_{m_2} &= 0.24(50) + 0.00763[1061 + 0.445(50)] \\ &= 20.26 \text{ Btu per lb dry air} \end{aligned}$$

The enthalpy per pound of condensate may be taken from the steam tables or calculated from the approximate equation for the enthalpy of liquid at temperatures below 250 F; this equation is

$$h_f = t - 32$$

If it is assumed that the condensate leaves at 50 F, the enthalpy of the condensate per pound of dry air equals

$$0.01666(50 - 32) = 0.30 \text{ Btu}$$



From the equation of steady flow applied to the dehumidifier, the heat removed is  
 $Q = h_{m_1} - h_{m_2} - (S H_1 - S H_2)h_f = 49.35 - (20.26 + 0.30) = 28.79$  Btu per lb dry air    Ans

It is to be noted that an error of about 1 per cent will be caused by neglecting the enthalpy of the condensate. At higher exit temperatures the error will be larger.

**Example 15-9**—Work the preceding example by use of the psychrometric chart in Fig. 15-7 by taking the difference between the enthalpies of the saturated vapor at entrance and exit.

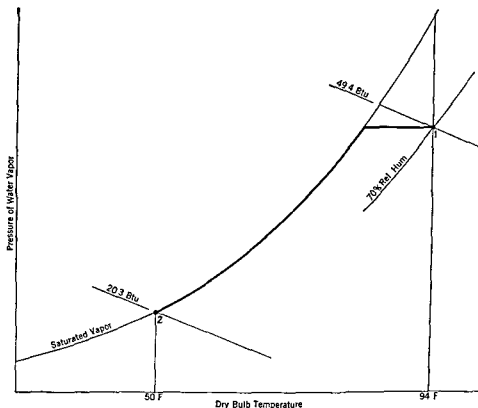


Fig. 15-13 Solution of Example 15-9

**Solution**—The enthalpy of the saturated vapor at the entering wet-bulb temperature from Fig. 15-7 is 49.4 Btu per lb of dry air (see Fig. 15-13). At the exit conditions the enthalpy of the saturated air is 20.3 Btu. The difference between these two values of enthalpy, or the heat removed is 29.1 Btu. This value agrees with the value of 29.09 Btu calculated when the enthalpy of the condensate is neglected.

**15-6. Humidification**—The problem of humidification of air is the reverse of that of dehumidification. Although humidification may be accomplished by addition of steam to the air, common practice is to supply the water in the liquid state. To aid in the ease of humidification, the water for humidification may be heated. Humidification may be accomplished by spraying the water into the air or by passing the air over the

surface of the water. Because humidification is desired in the winter at the same time that heat must be added to make up heat losses from buildings, it is common practice to heat and humidify the air at the same time. Note Fig. 15-14.

The energy equation in this case is

$$h_{m_1} + (S.H._2 - S.H._1)h_f + Q = h_{m_2}$$

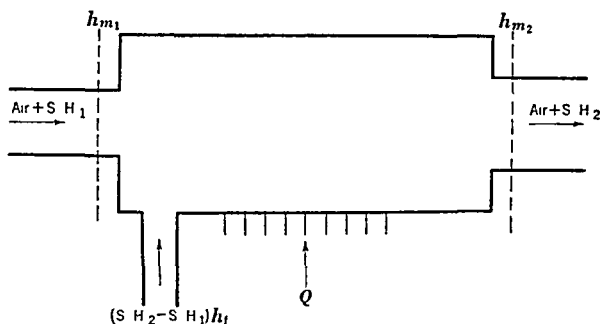


Fig. 15-14. Schematic Arrangement of Adding Heat and Humidity to Air

*Example 15-10.*—A quantity of air having a volume of 2,000 cu ft at 70% R.H., 60 F, and 14.0 psia is to be heated to 90 F. At this temperature the R.H. is to be 50%. Water is supplied at 100 F. Find the heat added.

*Solution.*—The vapor pressure at entrance is

$$P_{v_1} = 0.7(0.2563) = 0.1794 \text{ psia}$$

At this point the specific humidity is

$$S.H._1 = 0.622 \times \frac{0.1794}{14.0 - 0.1794} = 0.00808 \text{ lb per lb dry air}$$

At exit the vapor pressure is

$$P_{v_2} = 0.5(0.6982) = 0.3491 \text{ psia}$$

and the specific humidity is

$$S.H._2 = 0.622 \times \frac{0.3491}{14.0 - 0.3491} = 0.01591 \text{ lb per lb dry air}$$

The difference in the specific humidities is the weight of water which must be supplied per pound of dry air. This equals

$$S.H._2 - S.H._1 = 0.01591 - 0.00808 = 0.00783 \text{ lb water per lb dry air}$$

In the equation of steady flow,

$$h_{m_1} = 0.24(60) + 0.00808[1061.0 + 0.445(60)] = 23.19 \text{ Btu per lb dry air}$$

$$(S.H._2 - S.H._1)h_f = 0.00783(100 - 32) = 0.53 \text{ Btu per lb dry air}$$

and  $h_{m_2} = 0.24(90) + 0.01591[1061.0 + 0.445(90)] = 39.12 \text{ Btu per lb dry air}$

Then the heat added is

$$Q = h_{m_2} - h_{m_1} - (S H_2 - S H_1) h_f \\ = 39.12 - (23.19 + 0.53) = 15.40 \text{ Btu per lb dry air}$$

The weight of dry air present is

$$W_a = \frac{PV}{RT} = \frac{144(14.0 - 0.1794)2000}{53.3(60 + 460)} = 143.6 \text{ lb}$$

and the total heat added equals

$$143.6(15.40) = 2,211 \text{ Btu}$$

Ans

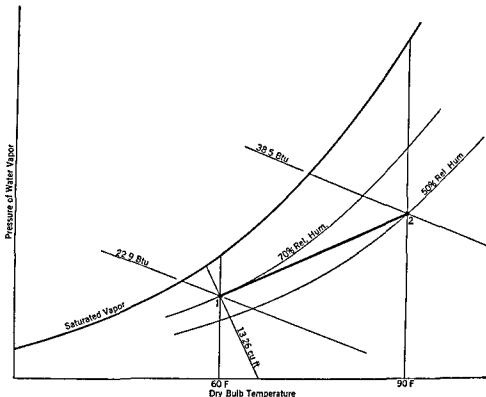


Fig 15-15 Solution of Example 15-11

*Example 15-11* —Solve the preceding example by means of the psychrometric chart in Fig 15-7, using the difference in enthalpies of the saturated air

*Solution* —From the psychrometric chart the enthalpy of saturated air at the original conditions is 22.9 Btu per pound of dry air (see Fig 15-15). At the final conditions the enthalpy, from Fig 15-7, is 38.5 Btu per pound of dry air. By subtracting these two enthalpies, the heat added is found to be

$$38.5 - 22.9 = 15.6 \text{ Btu per lb dry air}$$

This agrees well with the calculated value of 15.40 Btu per pound of dry air, as the error introduced by the pressure being other than atmospheric tends to compensate for the fact that the enthalpy of the entering water is neglected. The specific volume

of the dry air from the chart in Fig 15-7 is 13.26 cu ft. The total weight, using this figure, is

$$\frac{2000}{13.26} = 151 \text{ lb}$$

This differs appreciably from the calculated value of 143.6 lb, and the difference shows the effect of neglecting the influence of pressure on specific volume.

★15-7. **Dehumidifier Systems.**—As was stated in Art. 15-5, when air is dehumidified by cooling, saturated air at low temperature results. Normally, air is desired at a temperature much higher than that at which it leaves the dehumidifying chamber. Conceivably the hot air that is to be dehumidified might be put through a heat exchanger before it entered the dehumidifier and used to heat the cool air leaving the dehumidifier. In this way, the dehumidified air could be heated to the desired tempera-

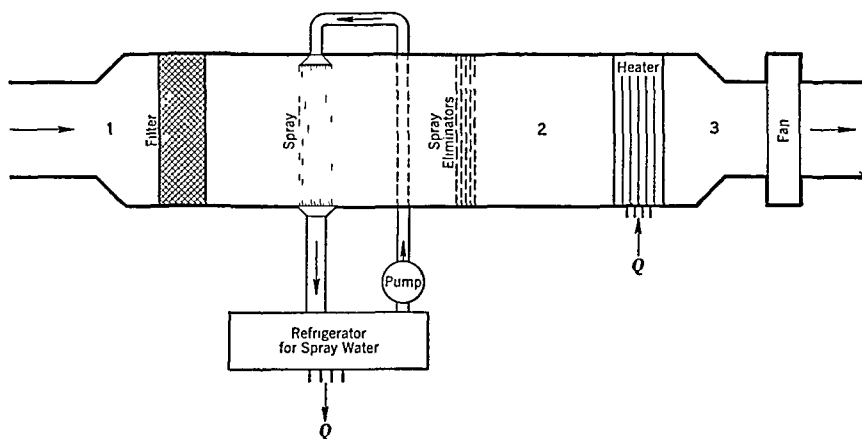


Fig. 15-16 Dehumidifier Unit

ture and the cooling load for the dehumidifier could be greatly reduced. However, heat transfer from gas to gas through a wall is greatly impeded by the gas films that cover the surfaces of the wall. Furthermore, in this heat exchanger the average temperature difference is small. The combined effect of large resistance to heat flow and small temperature difference necessitates so large a heat exchanger as to make its use uneconomical. Common practice is to use a heater, either steam or gas-fired, after the dehumidifying chamber. A line diagram of a typical dehumidifier set-up is shown in Fig. 15-16. The whole unit, including the heater, will be spoken of as a dehumidifier unit.

Up to this point it has been assumed that the air is saturated after dehumidification. This condition is approached if there are many rows of cooling coils or an extensive spray is used. However, when there are a few

★15-8 **Humidifying Systems**—At low temperatures, the saturation pressure of water vapor is low. Hence, the maximum specific humidity of low-temperature air is low. When low-temperature air is heated and supplied to a building without addition of moisture, air with very low relative humidity results. In cold weather, outside air must be heated before sufficient vapor may be introduced into it, if the relative humidity of the air as it is supplied to a building is to be reasonable. Accurate control of the final humidity may be obtained by controlling the amount of heat added to the air before water is introduced into it. If sufficient water is sprayed into the air, it will become saturated. Fig 15-18 shows a sketch of a heating-humidifying system.

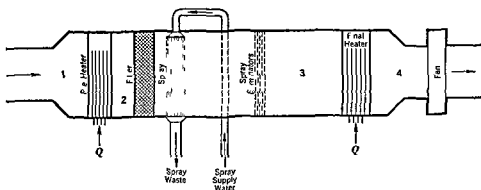


Fig 15-18 Heating-Humidifying System

★*Example 15-13*—Air is to be supplied at 120 F and 20% RH, atmospheric pressure being standard. Outside air is at 0 F and 60% RH. Calculate the heat added in each heater per pound of dry air.

*Solution*—The vapor pressure of the outside air is

$$P_{v_1} = 0.6(0.0185) = 0.0111 \text{ psia}$$

The specific humidity at this point is

$$\text{S.H.}_1 = 0.622 \times \frac{0.0111}{14.7 - 0.0111} = 0.00047 \text{ lb per lb dry air}$$

The vapor pressure leaving the final heater is

$$P_{v_4} = 0.2(1.6924) = 0.3385 \text{ psia}$$

The specific humidity at this point is

$$\text{S.H.}_4 = 0.622 \times \frac{0.3385}{14.7 - 0.3385} = 0.01466 \text{ lb per lb dry air}$$

As the pressure does not change in the heaters, the vapor pressure of the air entering the final heater is  $P_{v_4} = 0.3385 \text{ psia}$ . For saturation at this point the temperature is 68 F.

The enthalpy of the air at preheater entrance is

$$\begin{aligned} h_{m_1} &= 0.24(0) + 0.00047[1061.0 + 0.445(0)] \\ &= 0.50 \text{ Btu per lb dry air} \end{aligned}$$

The enthalpy of the air entering the final heater is

$$\begin{aligned} h_{m_3} &= 0.24(68) + 0.01465[1061.0 + 0.445(68)] \\ &= 32.31 \text{ Btu per lb dry air} \end{aligned}$$

The enthalpy of the air leaving the final heater is

$$\begin{aligned} h_{m_4} &= 0.24(120) + 0.01466[1061.0 + 0.445(120)] \\ &= 45.14 \text{ Btu per lb dry air} \end{aligned}$$

If the enthalpy of the water supplied for humidification is neglected, the heat added in the preheater is

$$Q = h_{m_3} - h_{m_1} = 32.31 - 0.50 = 31.81 \text{ Btu per lb dry air} \quad \text{Ans.}$$

The heat added in the final heater is

$$Q = h_{m_4} - h_{m_3} = 45.14 - 32.31 = 12.83 \text{ Btu per lb dry air} \quad \text{Ans.}$$

The total amount of heat added per pound of dry air is high. To minimize this, recirculation is used in the same manner as in the case of dehumidification.

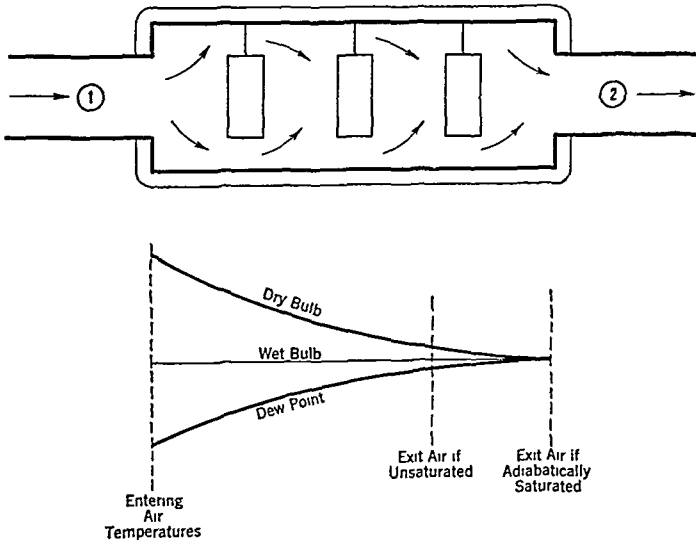


Fig. 15-19. Adiabatic Drying System

**15-9. Drying.**—There are many industrial cases where the product becomes wetted by a liquid in the course of its manufacture and must be finally dried. Drying is accomplished by blowing hot air of low relative humidity over the product. In such a case the process is nearly an

adiabatic one. The fundamental set up is shown in Fig 15-19. The fundamental problem is independent of the nature of the liquid involved. For that matter, the problem is essentially the same if a gas other than air is the agent which picks up the vapor.

*Example 15-14* — Air enters a drying chamber at 120 F dry bulb and 82 F wet bulb. If the air is saturated adiabatically, determine the weight of water vapor picked up per pound of dry air. Assume standard atmospheric pressure.

*First Solution* — By equation 15-5, the vapor pressure at entrance is

$$\begin{aligned} P_{v_1} &= P_s - \frac{(P_s - P_s')(t - t')}{2800 - 1.3t} \\ &= 0.5410 - \frac{(14.7 - 0.5410)(120 - 82)}{2800 - 1.3(82)} \\ &= 0.3413 \text{ psia} \end{aligned}$$

At this point the specific humidity is

$$S H_1 = 0.622 \times \frac{0.3413}{14.7 - 0.3413} = 0.01478 \text{ lb per lb dry air}$$

As was pointed out in Art 15-2 when air is saturated with water vapor adiabatically its temperature is practically equal to its original wet-bulb temperature. (Note: This assumption may not be justified when dealing with liquids other than water and with gases other than air.) Then the vapor pressure leaving the chamber is  $P_{v_2} = 0.5410$  psia. The specific humidity at exit is

$$S H_2 = 0.622 \times \frac{0.5410}{14.7 - 0.5410} = 0.02376 \text{ lb per lb dry air}$$

The weight of water vaporized is the difference between the two specific humidities, or

$$0.02376 - 0.01478 = 0.00898 \text{ lb per lb dry air} \quad \text{Ans}$$

*Alternate Solution* — The energy equation of adiabatic saturation may be used in the solution of the problem. When equilibrium is reached the vaporizing water will be at the wet-bulb temperature. Although its energy is its intrinsic energy, this is practically equal to its enthalpy. The water vaporized is the difference in the specific humidities of the air entering and leaving the chamber. The energy equation is

$$h_{m_1} + (S H_2 - S H_1)h_f = h_{m_2}$$

$$\text{or} \quad 0.24(120) + 0.01478[1061.0 + 0.445(120)] + (S H_2 - 0.01478)(82 - 32) = 0.24(82) + S H_2[1061.0 + 0.445(82)]$$

$$\text{from which} \quad S H_2 = 0.02372 \text{ lb per lb dry air}$$

Then the water vaporized equals

$$S H_2 - S H_1 = 0.02372 - 0.01478 = 0.00894 \text{ lb per lb dry air} \quad \text{Ans}$$

Under most conditions, the air is not saturated when leaving the drying chamber, as it is difficult to bring every particle of air in contact with water. If the temperature of the air at exit is known, the water vaporized per pound of dry air may be found by setting up the equation of steady flow

## CHAPTER 16

### GAS CYCLES

**16-1. Introduction.**—In spite of the fact that its efficiency is of the highest order, the Carnot gas cycle has not been used because the ratio of its mean effective pressure to the maximum pressure is so low that in many cases the engine cannot overcome its own friction. Many other cycles have been proposed to be used in place of the Carnot Cycle. Some of these have certain outstanding merits and are in use today. Others have had little to recommend them and have been discarded. Still others, while excellent thermodynamically, had to be discarded because of the difficulties encountered in designing an actual engine to utilize the advantages of the given cycle.

Factors to be considered in weighing the merits of a cycle are:

- (1) Thermal efficiency
- (2) Ratio of mean effective pressure to maximum pressure
- (3) The maximum temperature and pressure and the length of time they are sustained
- (4) The ease of designing the actual engine to closely approach the theoretical cycle when running at reasonable speeds

The cycle thermal efficiency depends on the temperatures of the hot and cold bodies and on the nature of the processes composing the cycle. Any cycle, receiving heat from the source and rejecting heat to the cold body in a reversible manner, and having the rest of its cycle composed of reversible processes, will have the highest possible thermal efficiency for the given conditions. This thermal efficiency equals that of the Carnot Cycle for the same conditions. (See Chapter 5.)

In the case of either an internal-combustion engine or a gas turbine, heat is not added to the working substance through walls from a source. An equivalent temperature rise is produced by the burning of a fuel which releases its chemical energy. Although "heat" is not added in these cases, it is necessary to maintain conditions similar to those in which heat is actually added, if high thermal efficiency is to be attained. For instance, in order to attain high thermal efficiency in a cycle in which heat is added, the temperature must be high at the start of addition of heat. Likewise in an internal-combustion engine the temperature must be high at the start of combustion, if a high thermal efficiency is to be realized. Although a high temperature at the start of combustion may be obtained by heat exchange, the high temperature is generally obtained by high compression.



One of the early cycles, the Lenoir, had no compression, hence, the thermal efficiency of the Lenoir engine was low. The Lenoir engine was abandoned when engines with compression were introduced.

Early types of "heat" engines were hot-air engines. These engines were designed to approach either the Stirling Cycle or the Ericsson Cycle. Because these engines are not used today, details of these cycles will be omitted. However, they are noteworthy because their theoretical thermal efficiencies were the same as that of the Carnot Cycle for the same temperature limits. Instead of relying on an isentropic compression to attain high temperatures, the Stirling-Cycle engine had its working substance heated in a reversible constant-volume process by heat which had been stored in another part of the cycle. In the Ericsson Cycle the heating was done in a constant-pressure manner. In spite of their high theoretical thermal efficiencies, the hot air engines were not very successful, although they were made and used for many years. The outstanding reason for this poor performance was the necessity of transferring heat from the hot products of combustion through walls to the working substance, air. The great resistance to heat flow offered by the gas films caused three bad effects:

- (1) The temperature drop to cause heat flow was large, resulting in a low over-all temperature range of the working substance and, hence, low thermal efficiency.
- (2) The temperature of those parts of the walls exposed to the flames was high, causing deterioration of those parts.
- (3) In order to transfer appreciable quantities of heat, the engine speed had to be kept low. This meant that the horsepower was low. As horsepower is the product of speed and mep for a given displacement, the relatively feeble power achieved with a huge displacement gave a small ratio of power to engine weight.

Common gas cycles encountered today are the Otto, the Diesel, the Dual or Limited Pressure, and the Brayton or Joule.

**16-2 The Air Standard Method of Cycle Analysis**—Cycles, whether they are gas cycles or vapor cycles, may be regarded as being composed of a series of theoretical processes, such as the constant-pressure, the constant-volume, the constant-temperature, and the isentropic. A theoretical engine operates on the theoretical cycle. Except for the assumption that the working substance is a perfect gas with a given specific heat, nothing is implied in the description of the cycle as to the nature of the working substance, *i.e.*, whether it is a single gas or a gaseous mixture. Neither is the manner of increasing the temperature of the working substance specified, *i.e.*, whether it is to receive heat through the cylinder head (as was

done in the Carnot Cycle) or is to have combustion taking place in the working substance itself.

The value of the analysis of a theoretical cycle is two-fold. First, it shows the inherent characteristics of the cycle and points out the possible advantages and also disadvantages of an actual engine approximating this cycle. Secondly, the analysis should indicate the limits of performance, which the actual engine may approach but may never exceed.

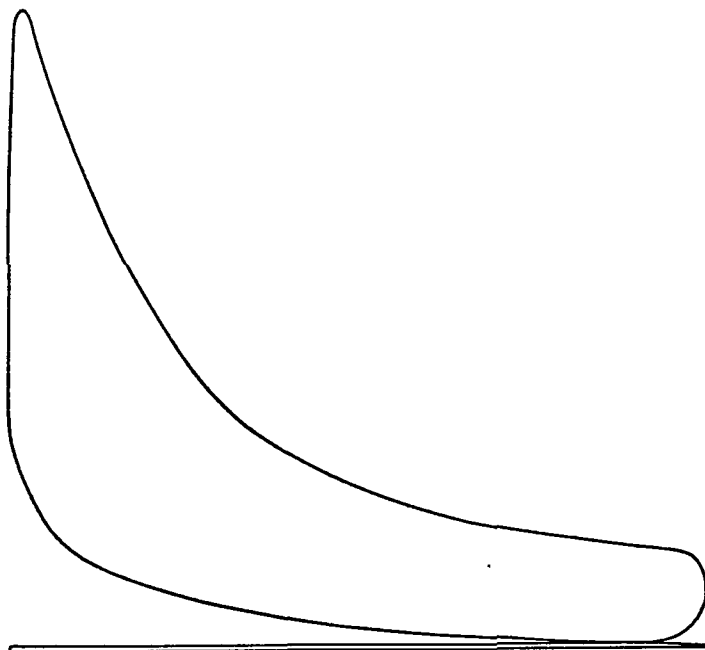


Fig. 16-1. Indicator Card for Slow-Speed Gas or Gasoline Engine

The actual engine, regardless of how nearly perfect it is, must use an actual working substance. Such a working substance may consist of air, fuel, and clearance gas in one part of the cycle and of products of combustion in another part. Since the working substance undergoes a wide range of temperature, its properties differ greatly from those of a perfect gas. The treatment of the actual working substance is a complicated one and will be taken up in Chapter 17 together with the limiting performance of a perfect engine.

One may make many assumptions regarding the performance of the working substance in a simple cycle analysis and still retain the ability to discover the inherent strength and weaknesses of a given cycle. The

simplest of such analyses is the "air standard". The following assumptions are made in the air standard method of analysis

- (1) The working substance is air and air alone
- (2) Air behaves as a perfect gas, having a constant specific heat—as do all perfect gases
- (3) Heat is added through the cylinder head in the manner desired *i.e.*, at constant pressure, at constant volume, etc
- (4) The air remains in the cylinder throughout the cycle at a fixed weight
- (5) All adiabatics within the cylinder are reversible ones

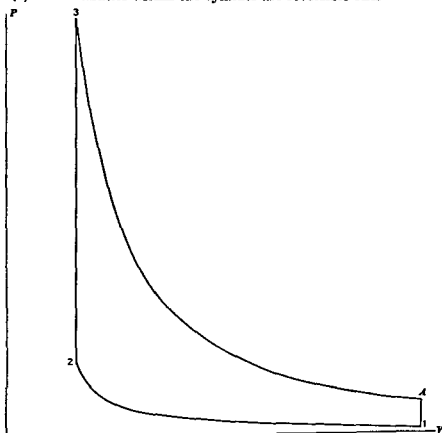


Fig 16-2 P V Diagram for Air Standard Otto Cycle

An analysis of these assumptions shows that conditions assumed for the air standard differ so much from those encountered in the actual engine that the air standard is of only relative value in predicting the limits of performance. The air standard analysis of cycles does show the *inherent characteristics* of the cycles.

**16-3. The Air-Standard Otto Cycle.**—An engine using either gasoline for a fuel or a gaseous fuel draws into the cylinder the air-fuel mixture, compresses that mixture, burns it, expands it, and then pushes out the products of combustion. In the theoretical case, there is no pressure drop through the valves, and hence no net work is done during the suction and exhaust strokes. In the air standard there can be no suction or exhaust

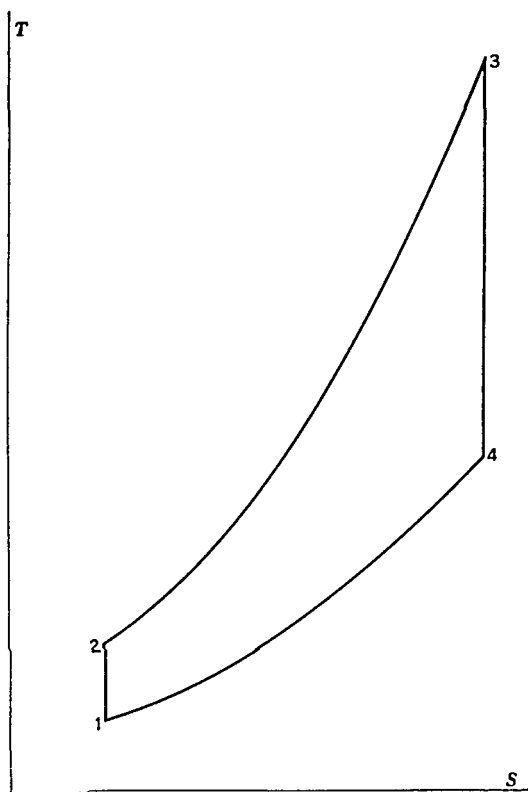


Fig. 16-3. T-S Diagram for Air-Standard Otto Cycle

strokes, as the weight must remain constant. In Fig. 16-1 is shown an indicator card for a slow-speed gas or gasoline engine. Fig. 16-2 shows the theoretical air standard cycle for this type of engine. In the theoretical case, compression is isentropic. The earlier engines were of slow speed; there was very little piston motion during the combustion period. In this case a constant-volume addition of heat in the air standard is an equivalent process approximating the combustion process in the actual engine. Although heat is lost during the expansion stroke, the isentropic expansion

is the limiting process. In a slow speed engine, the exhaust valve is opened just before the end of the expansion stroke and the pressure drops to about that of the atmosphere with very little piston motion. A constant-volume removal of heat in the air standard approximates this process. This combination of processes is known as the Otto Cycle.

Fig. 16-3 shows the air-standard Otto Cycle on the  $T$ - $S$  plane, with the points numbered to correspond to those on the  $P$ - $V$  plane in Fig. 16-2. As indicated, process 1-2 is an isentropic compression, process 2-3 is addition of heat at constant volume, process 3-4 is an isentropic expansion, process 4-1 is rejection of heat at constant volume.

The thermal efficiency of any heat engine is that portion of the heat supplied which is turned into work. In a cycle—such as the Otto—in which work is done in only two processes, the evaluation of the work done is simple. In other cycles work may be done in three or four processes. In general, it becomes desirable to evaluate the cycle work by subtracting the heat rejected from the heat supplied. Thus, the thermal efficiency of the air-standard Otto Cycle becomes

$$\text{Thermal Efficiency} = \frac{\frac{W_k}{J}}{Q_s} = \frac{Q_s - Q_E}{Q_s}$$

But the heat supplied is furnished between points 2 and 3 and equals  $W_{c_v}(T_3 - T_2)$ , and the heat rejected is given up between points 4 and 1 and equals  $W_{c_v}(T_4 - T_1)$ . Thus,

$$\begin{aligned} \text{Thermal Efficiency} &= \frac{W_{c_v}(T_3 - T_2) - W_{c_v}(T_4 - T_1)}{W_{c_v}(T_3 - T_2)} \\ &= \frac{(T_3 - T_2) - (T_4 - T_1)}{T_3 - T_2} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \end{aligned}$$

As processes 1-2 and 3-4 are isentropic ones, the following relations exist

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1}$$

Since  $V_1 = V_4$  and  $V_2 = V_3$ ,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Starting with the quantity  $\frac{T_4 - T_1}{T_3 - T_2}$  and factoring out  $T_1$  in the numerator and  $T_2$  in the denominator, we get

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1 \left( \frac{T_4}{T_1} - \frac{T_1}{T_1} \right)}{T_2 \left( \frac{T_3}{T_2} - \frac{T_2}{T_2} \right)} = \frac{T_1 \left( \frac{T_4}{T_1} - 1 \right)}{T_2 \left( \frac{T_3}{T_2} - 1 \right)}$$

Since  $\frac{T_4}{T_1} = \frac{T_3}{T_2}$ ,

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2}$$

Substituting this value of  $\frac{T_4 - T_1}{T_3 - T_2}$  back into the expression for the thermal efficiency of the air-standard Otto Cycle, we get

$$\text{Thermal Efficiency} = 1 - \frac{T_1}{T_2} = 1 - \left( \frac{1}{\frac{T_2}{T_1}} \right)$$

But  $\frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^{k-1}$ , where  $\frac{V_1}{V_2}$  is the compression ratio  $r$ . Then

$$\text{Thermal Efficiency} = 1 - \frac{1}{r^{k-1}} \quad (16-1)$$

Equation 16-1 shows that the thermal efficiency of the air-standard Otto Cycle is independent of the amount of heat added as well as the pressure and temperature at the start of compression, but *depends solely on the compression ratio*. Although the pressure and temperature at the start of compression have no bearing on the thermal efficiency, they materially affect the mean effective pressure, as this depends on air density.

Equation 16-1 shows that, as the compression ratio is increased, the thermal efficiency of the air-standard Otto Cycle increases rapidly at first and then more slowly, as indicated in Fig. 16-4.

**Example 16-1.**—At the start of compression in an air-standard Otto Cycle, the pressure is 14.0 psia and the temperature is 140 F. The compression ratio is 6 to 1. The heat added per pound of air is 1300 Btu.\* Compute the temperatures, pressures, and specific volumes at all parts of the cycle, the thermal efficiency, and the mean effective pressure.

**Solution.**—The cycle is represented in Fig. 16-2 and Fig. 16-3. The specific volume at the start of compression is

$$V_1 = \frac{WRT}{P} = \frac{1(53.3)(140 + 460)}{144(14.0)} = 15.86 \text{ cu ft per lb}$$

\*Although the energy released per pound of working substance in an actual engine varies greatly, this value of 1300 Btu per lb is a reasonable value. For a chemical energy of 19,300 Btu per lb of fuel, this figure is equivalent to 14.85 lb of working substance per lb of fuel.

At the end of compression,

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^k = 14.0(6)^{1.4} = 172.0 \text{ psia}$$

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = (140 + 460)(6)^{0.4} = 1229 \text{ R}$$

$$V_2 = \frac{V_1}{r} = \frac{15.86}{6} = 2.64 \text{ cu ft per lb}$$

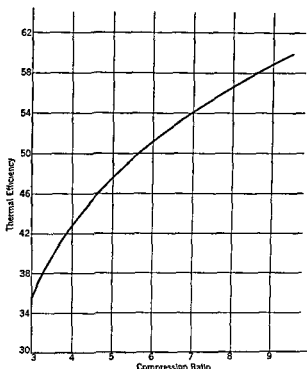


Fig 16-4. Relation of Thermal Efficiency to Compression Ratio for Air-Standard Otto Cycle

The temperature rise between points 2 and 3 is fixed by the amount of heat added. Thus,

$${}_2Q_3 = Wc_v(T_3 - T_2)$$

or

$$T_3 - T_2 = \frac{{}_2Q_3}{Wc_v} = \frac{1300}{0.1715} = 7580$$

Then,

$$T_3 = T_2 + (T_3 - T_2) = 1229 + 7580 = 8809 \text{ R}$$

$$P_3 = P_2 \frac{T_3}{T_2} = 172.0 \times \frac{8809}{1229} = 1233 \text{ psia}$$

$$V_3 = V_2 = 2.64 \text{ cu ft per lb}$$

$$T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{k-1}} = \frac{8809}{6^{0.4}} = 4301 \text{ R}$$

$$P_4 = \frac{P_3}{\left(\frac{V_4}{V_3}\right)^k} = \frac{1234}{6^{1.4}} = 100.4 \text{ psia}$$

$$V_4 = V_1 = 15.86 \text{ cu ft per lb}$$

The heat rejected is

$$W_c(T_4 - T_1) = 1(0.1715)(4301 - 600) = 634.8 \text{ Btu per lb}$$

The net cycle work is

$$Q_s - Q_R = 1300 - 634.8 = 665.2 \text{ Btu per lb or } 517,500 \text{ ft-lb per lb}$$

The thermal efficiency of the cycle is

$$\text{Thermal Efficiency} = \frac{\frac{W_k}{J}}{Q_s} = \frac{665.2}{1300} = 0.5117 \text{ or } 51.17\%$$

By equation 16-1,

$$\text{Thermal Efficiency} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{6^{0.4}} = 0.5117 \text{ or } 51.17\%$$

The mean effective pressure, in lb per sq ft, is numerically equal to the net cycle work in ft-lb per cu ft of piston displacement (see Chapter 8). Then the mep, in psi, is:

$$\text{mep} = \frac{\text{Net Work}}{\text{Piston Disp. (144)}} = \frac{517,500}{(15.86 - 2.64)144} = 272 \text{ psi}$$

An analysis of the results of the foregoing example shows that the thermal efficiency of the air-standard Otto Cycle is high and that the mean effective pressure is very high compared with that of the Carnot Cycle (see Chapter 5). Both the maximum pressure and the maximum temperature are very high, but an inspection of the  $T$ - $S$  and  $P$ - $V$  diagrams shows that these maximum values are instantaneous ones; both the temperatures and pressures drop off quite rapidly from the maximum values.

As was discussed in Art. 16-2, the results obtained by the air-standard analysis far exceed those of even a perfect engine. The fact that the thermal efficiency of the actual engine with a compression ratio of 6 to 1 generally is less than half that indicated by the foregoing calculations does not mean that there is great room for improvement in the actual engine. The temperatures and pressure in the air standard are practically double those of the actual engine. The major reason for these discrepancies is that the actual engine uses an actual working substance the behavior of which differs greatly from that of a perfect gas.

**16-4. The Diesel Cycle.**—Rudolf Diesel, realizing that high efficiencies may be obtained by adding heat at a high constant temperature, tried to design an engine in which combustion was so regulated that it took place at substantially constant temperature. As should have been expected,



the resulting engine was a feeble one, having a low mean effective pressure. To remedy this difficulty, Diesel substituted a constant-pressure process for the constant-temperature one. In order that the fuel may burn at substantially constant pressure, it is necessary to introduce the fuel as it burns and to regulate the rate at which it is introduced. In order that the fuel\* may burn almost as soon as it comes into the cylinder, it is essential that the cylinder air be at a temperature much higher than the self-ignition temperature of the fuel. This condition necessitates a much higher compression ratio than for a gasoline engine.

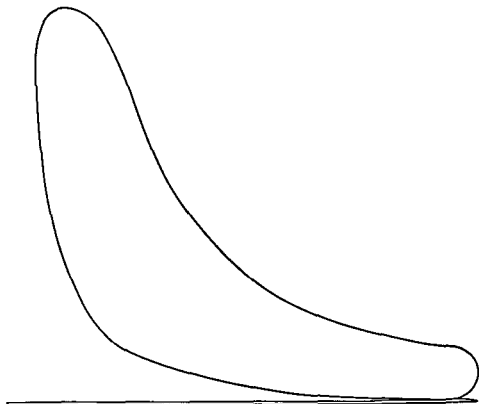


Fig 16-5 Indicator Card for Slow Speed Diesel Engine

Fig 16-5 shows an indicator card for an early diesel engine. Fig 16-6 and Fig 16-7 show the air standard Diesel Cycle on the  $P$ - $V$  and  $T$ - $S$  planes with points numbered similarly. Process 1-2 is an isentropic compression, process 2-3 is addition of heat at constant pressure, process

\* Diesel originally contemplated using powdered coal, of which Germany had large supplies in order to free Germany of the dependence on petroleum fuel which she lacked. However, the many difficulties he encountered in trying to use coal led him to shift to oil.

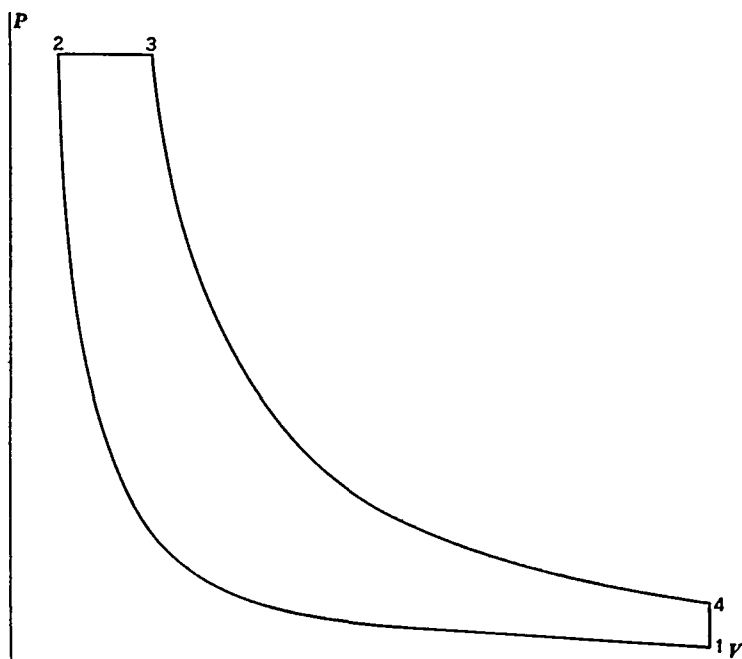


Fig. 16-6. P-V Diagram for Air Standard Diesel Cycle

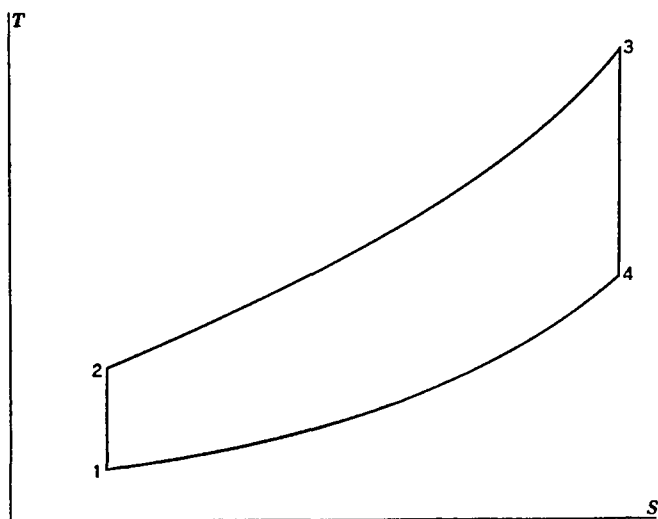


Fig. 16-7. T-S Diagram for Air Standard Diesel Cycle

3-4 is an isentropic expansion, process 4-1 is rejection of heat at constant volume

The thermal efficiency of the air-standard Diesel Cycle is

$$\begin{aligned}\text{Thermal Efficiency} &= \frac{\frac{Wk}{J}}{Q_s} = \frac{Q_s - Q_R}{Q_s} \\ &= \frac{Wc_p(T_3 - T_2) - Wc_v(T_4 - T_1)}{Wc_p(T_3 - T_2)} \\ &= 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}\end{aligned}$$

or

$$\text{Thermal Efficiency} = 1 - \frac{1}{k} \left( \frac{T_4 - T_1}{T_3 - T_2} \right) \quad (16-2)$$

Equation 16-2 cannot be simplified, as was done for the air-standard Otto Cycle, because there is no equality in any property at the various points in the Diesel Cycle like that which exists between the volumes in the Otto Cycle. It is possible to put the expression for thermal efficiency of the air-standard Diesel Cycle in other forms, particularly one involving the constant-pressure expansion ratio ( $r_e$ , the ratio of the volume at the end to the volume at the start of the constant-pressure process) or in terms of the isentropic expansion ratio (the ratio of the volume at the end to the volume at the start of isentropic expansion). In general, however, such expressions are not easier to use than equation 16-2.

*Example 16-2* —At the start of compression in an air standard Diesel Cycle the pressure is 14.2 psia and the temperature is 100 F. If the heat added per pound of air is 800 Btu\* and the compression ratio is 15 to 1, compute the cycle efficiency and the mep.

*Solution* —The cycle is represented in Figs. 16-6 and 16-7. The temperature at the end of compression is

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = (100 + 460)(15)^{0.4} = 1654 \text{ R}$$

The heat added in process 2-3 is

$${}_2Q_3 = Wc_p(T_3 - T_2)$$

from which

$$T_3 - T_2 = \frac{{}_2Q_3}{Wc_p} = \frac{800}{1(0.24)} = 3333$$

and

$$T_3 = T_2 + (T_3 - T_2) = 1654 + 3333 = 4987 \text{ R}$$

The temperature at point 4 depends on  $T_3$  and the isentropic expansion ratio  $\frac{V_4}{V_3}$ .

\* In a diesel engine much excess air must be used to insure clean combustion. This means that the energy released per pound of working substance is much lower than in the gasoline type engine.

Although calculations may be made for these volumes, it is easier to express each of them in terms of  $V_2$ . Since  $\frac{V_1}{V_2} = 15$  and  $V_4 = V_1$

$$V_4 = 15 V_2$$

Also,  $\frac{V_3}{V_2} = \frac{T_3}{T_2}$  or

$$V_3 = \frac{T_3}{T_2} V_2 = \frac{4987}{1654} V_2 = 3.015 V_2$$

Then

$$\frac{V_4}{V_3} = \frac{15 V_2}{3.015 V_2} = 4.975$$

and

$$T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{k-1}} = \frac{4987}{4.975^{0.4}} = 2625 \text{ R}$$

The heat rejected is

$$Q_R = W C_v (T_4 - T_1) = 1(0.1715)(2625 - 560) = 354 \text{ Btu per lb}$$

The net cycle work is

$$Q_S - Q_R = 800 - 354 = 446 \text{ Btu per lb or } 347,000 \text{ ft-lb per lb}$$

The cycle thermal efficiency is

$$\frac{\frac{W_k}{J}}{\frac{Q_S}{J}} = \frac{446}{800} = 55.75\% \quad \text{Ans.}$$

This efficiency may also be found by using equation 16-2. If the temperatures just calculated are substituted in equation 16-2, the thermal efficiency is found to be 55.75%.

The volume at the start of compression is

$$V_1 = \frac{WRT_1}{P_1} = \frac{1(53.3)(100 + 460)}{144(14.2)} = 14.60 \text{ cu ft per lb}$$

The piston displacement in an engine with a compression ratio of 15 to 1 is  $\frac{14}{15}$  times the total cylinder volume. Then the piston displacement equals  $\frac{14}{15}(14.60) = 13.63$  cu ft per lb, and the mean effective pressure is

$$\text{mep} = \frac{347,000}{13.63(144)} = 177 \text{ psi} \quad \text{Ans.}$$

If the amount of heat added per pound of air in the preceding example were reduced, it would be found that the thermal efficiency would be higher than 55.75%. The reason for this is that, with less heat added, isentropic expansion can take place for a larger portion of the stroke. With the larger isentropic expansion after the end of the addition of heat, the temperature has a chance to drop to a lower value before rejection of heat starts. This fact—which is applicable to all “heat” engine cycles—means that, to obtain the necessary temperature range for high thermal efficiency, both the isentropic compression ratio and the isentropic expansion ratio must be high.

**16-5 The Dual or Limited Pressure Cycle**—In the modern engine, burning an oil which is injected into the cylinder, the major portion of the combustion comes closer to a constant-volume process than it does a constant-pressure one, although at heavy loads—particularly at low speeds—much of the combustion approximates a constant-pressure process. This means that combustion in the actual engine takes place in a smaller percentage of the stroke than is indicated by the Diesel Cycle, hence, there is more chance for expansion in the actual engine. If all other losses could be eliminated, the thermal efficiency of the actual engine would exceed that of the theoretical constant-pressure cycle.

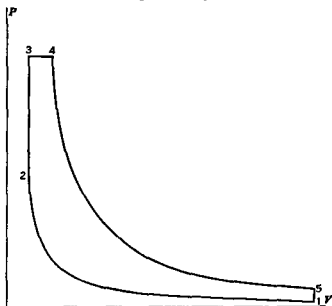


Fig 16-8 P-V Diagram for Air Standard Dual Cycle

Although the action during combustion in the actual engine does not resemble that of the Diesel Cycle, the term "diesel engine" persists for an engine having fuel injection and compression ignition. Some attempts have been made to call such engines "compression ignition" engines. Even though this term properly describes such engines, this text will follow common practice and speak of diesel engines.

The so-called dual or limited pressure cycle has been proposed as a cycle that represents the action in the actual diesel engine more closely than does the Diesel Cycle. Figs 16-8 and 16-9 show the air standard dual cycle. Process 1-2 is an isentropic compression, process 2-3 is addition of heat at constant volume, process 3-4 is addition of heat at constant pressure, process 4-5 is an isentropic expansion, process 5-1 is rejection of heat at constant volume.

It is possible to derive an expression for the thermal efficiency of the air-standard dual cycle in terms of the five temperatures involved and the isentropic exponent  $k$ . Little is gained by doing this, as the thermal efficiency may be found quite readily by finding the heat supplied and the heat rejected per cycle.

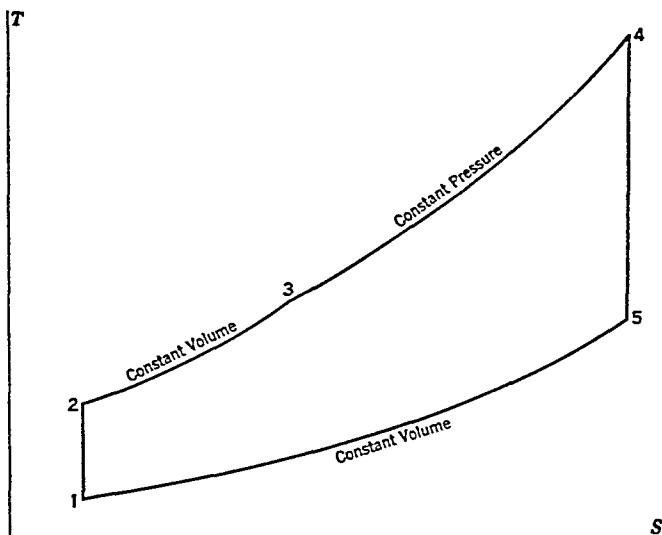


Fig. 16-9. T-S Diagram for Air Standard Dual Cycle

*Example 16-3.*—Assume conditions like those for the air-standard Diesel Cycle in Example 16-2, but use the Dual Cycle, adding 400 Btu per pound of air at constant volume and 400 Btu per pound of air at constant pressure.

*Solution.*—The conditions are represented in Figs. 16-8 and 16-9. The temperature  $T_2$  at the end of compression is the same as that in the air-standard Diesel Cycle in Example 16-2, or 1654 R. Since  ${}_2Q_3 = Wc_v(T_3 - T_2)$ ,

$$T_3 - T_2 = \frac{{}_2Q_3}{Wc_v} = \frac{400}{1(0.1715)} = 2332$$

and 
$$T_3 = T_2 + (T_3 - T_2) = 1654 + 2332 = 3986 \text{ R}$$

Likewise, since  ${}_3Q_4 = Wc_p(T_4 - T_3)$ ,

$$T_4 - T_3 = \frac{{}_3Q_4}{Wc_p} = \frac{400}{1(0.24)} = 1667$$

and 
$$T_4 = T_3 + (T_4 - T_3) = 3986 + 1667 = 5653 \text{ R}$$

Also, 
$$V_5 = V_1 = 15 V_2 = 15 V_3$$

and 
$$\frac{V_4}{V_3} = \frac{T_4}{T_3}$$

from which 
$$V_4 = V_3 \frac{T_4}{T_3} = V_3 \frac{5653}{3986} = 1.418 V_3$$

Then,

$$\frac{V_4}{V_1} = \frac{15 V_2}{1.418 V_1} = 10.58$$

Since  $\frac{T_4}{T_1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$

$$T_4 = \frac{5653}{(10.58)^{0.4}} = 2200 \text{ R}$$

The heat rejected is

$$Q_R = W c_v (T_4 - T_1) = 1(0.1715)(2200 - 560) = 281.3 \text{ Btu per lb}$$

The net cycle work is

$$Q_S - Q_R = 800 - 281.3 = 518.7 \text{ Btu per lb or } 402,400 \text{ ft-lb per lb}$$

The thermal efficiency equals

$$\frac{\text{Wk}}{Q_S} = \frac{518.7}{800} = 0.6488 \text{ or } 64.88\% \quad \text{Ans}$$

Finally

$$\text{mep} = \frac{402,400}{13.63(144)} = 205.0 \text{ psi} \quad \text{Ans}$$

These calculations show that both the thermal efficiency and the mean effective pressure are higher for the Dual Cycle than for a Diesel Cycle having the same compression ratio. One factor that should not be overlooked, however, is that both the maximum pressure and the maximum temperature are higher in the Dual Cycle and that this higher pressure is maintained for an appreciable length of time. The maximum temperatures have been calculated in Examples 16-2 and 16-3. The maximum pressures will be calculated in the following example.

**Example 16-4**—Calculate the maximum pressures for the air standard Diesel and Dual Cycles in Examples 16-2 and 16-3.

**Solution**—The pressure at the end of compression in either cycle is

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 14.2(15)^{1.4} = 629.2 \text{ psia}$$

This pressure is the maximum pressure encountered in the Diesel Cycle.

The maximum pressure in the Dual Cycle is

$$P_3 = P_2 \frac{T_3}{T_2} = 629.2 \times \frac{3986}{1654} = 1520 \text{ psia}$$

This maximum pressure, while much higher than that in the Diesel Cycle, is low in comparison with the maximum pressure of an air-standard Otto Cycle having the same compression ratio.

The limitation on the use of the Dual Cycle as a standard for the actual diesel engine lies in the fact that the per cent of the additional heat which is added at constant volume must be continuously adjusted to approximate the variations in the actual engine. To follow the actual high-speed engine, particularly at light loads, the per cent of the additional heat which is added at constant pressure becomes insignificant and the cycle becomes a constant-volume one for all practical purposes.

In retrospect, it may be said that combustion takes place in an actual diesel engine in a better manner than is indicated by the Diesel Cycle; it may be in a better manner than indicated by the Dual Cycle if in the Dual Cycle too much heat is added at constant pressure. The real limit for indicating the performance of the actual diesel engine is the Otto Cycle. In fact, the combustion process in a high-speed diesel engine may come closer to an Otto Cycle than does that in a high-speed gasoline engine.

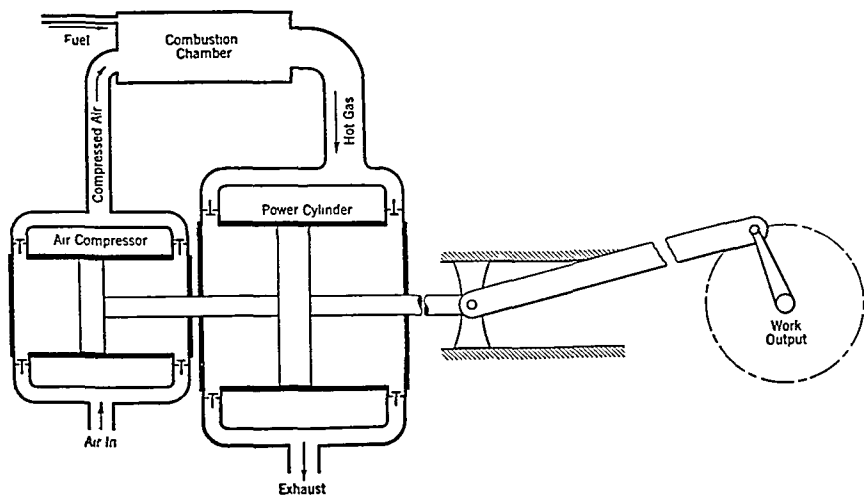


Fig. 16-10. Equipment for Brayton Cycle

**16-6. The Brayton or Joule Cycle.**—The cycle conceived by Brayton and by Joule independently was not carried out in a single cylinder but needed several pieces of equipment for its operation. Although this cycle was proposed for a hot-air engine, it had to overcome the same difficulties which were encountered in the use of other cycles for hot-air engines. The Brayton Cycle proved somewhat more successful when fuel was burned in air that became the working substance. In Fig. 16-10 is shown a line diagram of the equipment required in this case.

Air was drawn into an air compressor and delivered by it at moderate pressure to a combustion chamber. Fuel was injected into the combustion chamber, where it burned at substantially constant pressure. The hot products of combustion entered the power cylinder at only slightly lower pressure than that of the air leaving the compressor; their temperature was considerably higher and hence their volume was large. The work done in the power cylinder was more than was necessary to run the compressor, and the excess energy was delivered to the shaft as useful work.



Fig 16-11 shows theoretical  $P$ - $V$  diagrams for the compressor and the power cylinders, wherein pressure drops outside the cylinders are neglected. When the  $P$ - $V$  diagram for the compressor is superimposed on that of the power cylinder, as in Fig 16-12, the net area represents the work delivered by the unit. In Fig 16-13 is shown a  $P$ - $V$  diagram of an air-standard Brayton Cycle which is equivalent to the net diagram in Fig 16-12. The  $T$ - $S$  diagram is shown in Fig 16-14.

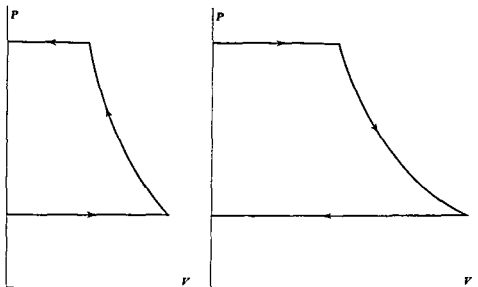


Fig 16-11 Theoretical  $P$ - $V$  Diagrams for Compressor and Power Cylinders

Although the enclosed areas in Figs 16-12 and 16-13 represent net cycle work, the similarity ends here. Fig 16-12 represents the difference between two areas and is not a complete thermodynamic cycle in itself. In fact, the cycle of operations in Fig 16-10 is an open cycle. Air is drawn in from the atmosphere, and products of exhaust are pushed into the atmosphere. This is also true for internal combustion engines. On the other hand, the *air standard cycle* in Figs 16-13 and 16-14 is a closed cycle. Isentropic compression takes place from 1 to 2, process 2-3 is heating at constant pressure, process 3-4 is isentropic expansion, process 4-1 is cooling at constant pressure.

The thermal efficiency of the air-standard Brayton Cycle is

$$\begin{aligned} \text{Thermal Efficiency} &= \frac{\frac{W_k}{J}}{Q_s} = \frac{Q_s - Q_R}{Q_s} = \frac{W_{c_p}(T_3 - T_2) - W_{c_p}(T_4 - T_1)}{W_{c_p}(T_3 - T_2)} \\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2} \end{aligned}$$

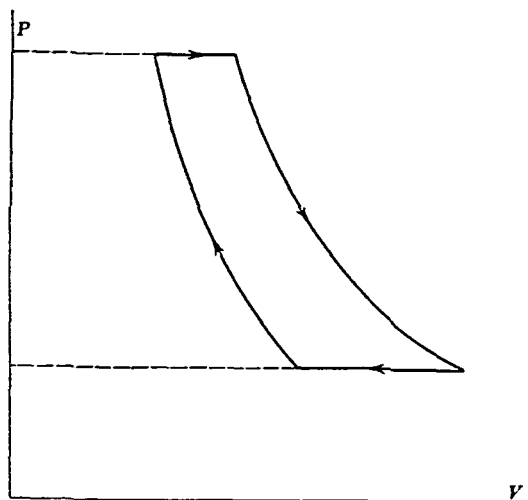


Fig. 16-12. Combined P-V Diagrams for Compressor and Power Cylinders

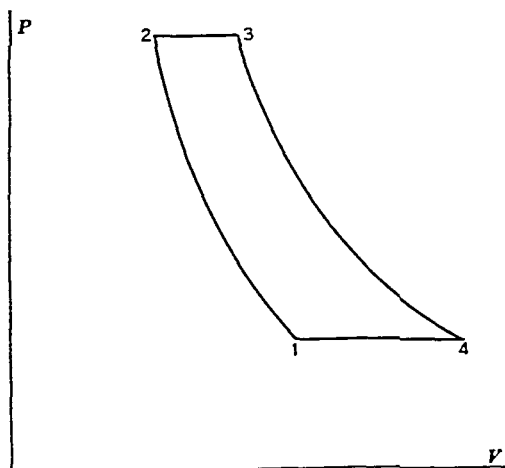


Fig. 16-13. P-V Diagram for Air-Standard Brayton Cycle

As processes 1-2 and 3-4 are isentropics,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}}$$

Since  $P_2 = P_3$  and  $P_1 = P_4$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{and} \quad \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2}$$

The intermediate steps are given in Art 16-3

$$\text{Then} \quad \text{Thermal Efficiency} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{k-1}} \quad (16-3)$$

where  $r$  is the isentropic compression ratio

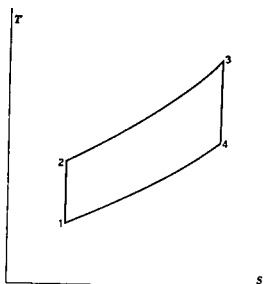


Fig 16-14 T S Diagram for Air Standard Brayton Cycle

It is to be noted by comparing equations 16-1 and 16-3 that the air standard thermal efficiency is the same for the Otto Cycle as for the Brayton Cycle if the *same isentropic compression ratio* is used. This fact can likewise be extended to the Carnot Cycle *within the same limits of isentropic compression ratio*. In fact, these three cycles are symmetrical cycles which may have a common thermal efficiency expression based upon the isentropic compression ratio. This expression does not fit the Diesel Cycle and the Dual Cycle because they are not symmetrical cycles.

*Example 16-5.*—At the start of compression in an air-standard Brayton Cycle, the pressure is 14.0 psia and the temperature is 140 F. The isentropic compression ratio is 6 to 1. The heat added per pound of air is 1,300 Btu.\* Determine the temperatures throughout the cycle, the thermal efficiency, and the mep. Note that these conditions are the same as those for the Otto Cycle in Example 16-1.

*Solution.*—The cycle is represented in Figs. 16-13 and 16-14. The temperature  $T_2$  at the end of isentropic compression is the same as it was for the Otto Cycle, namely, 1229 R. Since  ${}_2Q_3 = W_{Cp}(T_3 - T_2)$ ,

$$T_3 - T_2 = \frac{{}_2Q_3}{W_{Cp}} = \frac{1300}{1(0.24)} = 5417$$

and

$$T_3 = T_2 + (T_3 - T_2) = 1229 + 5417 = 6646 \text{ R}$$

But,

$$T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{k-1}} \text{ and } \frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{\frac{1}{k}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{k}} = \frac{V_1}{V_2} = 6$$

Therefore,

$$T_4 = \frac{6646}{(6)^{0.4}} = 3245 \text{ R}$$

The heat rejected is

$$Q_R = W_{Cp}(T_4 - T_1) = 1(0.24)(3245 - 600) = 634.8 \text{ Btu per lb}$$

The net cycle work is

$$Q_S - Q_R = 1300 - 634.8 = 665.2 \text{ Btu per lb or 517,500 ft-lb per lb}$$

The thermal efficiency is

$$\frac{W_k}{J} = \frac{665.2}{1300} = 0.5117 \text{ or } 51.17\%$$

This compares with the value of 51.17% obtained when using equation 16-3.

The cylinder volume is

$$V_4 = V_1 \frac{T_4}{T_1} = 15.86 \left( \frac{3245}{600} \right) = 85.76 \text{ cu ft}$$

( $V_1$  is taken from Example 16-1. From the same source,  $V_2 = 2.64 \text{ cu ft.}$ )

Finally,

$$\text{mep} = \frac{517,500}{(85.76 - 2.64)144} = 43.2 \text{ psi}$$

The mean effective pressure of the Brayton Cycle, although much larger than that of the Carnot Cycle, is very low in comparison with that obtained in the Otto, Diesel, and Dual Cycles. When reciprocating machinery is used, this low mean effective pressure is a very serious problem. This fact, plus the requirement of two pieces of machinery in the Brayton Cycle rather than one, prevented extensive use of the Brayton Cycle.

Rotating machines, such as gas turbines, can handle very large volumes per unit of time; hence, for them low mean effective pressures are not a

\* As the temperatures in the stationary parts of the system remain constant, the maximum temperatures must be kept down. This is done by using much excess air. This high value of heat added per pound is used here to allow for comparison with the air-standard Otto Cycle.

serious handicap. In very recent years, great improvements in the elements of the gas turbine have made them a competitor in the power generation field. The elementary gas turbine follows the Brayton Cycle quite closely. For a detailed discussion of the gas turbine, see Chapter 18.

The reversed Brayton Cycle is the basis of the air refrigeration system. See Chapter 12.



## CHAPTER 17

### INTERNAL-COMBUSTION ENGINES

17-1. **Introduction.**—In a steam power plant, the products of combustion are not used in the power cycle; heat must be transferred from the products of combustion through walls to the working substance. Steam power plants, while having many advantages, have some serious disadvantages which arise mainly from the fact that they are of the external-combustion type. To obtain high thermal efficiency in steam power plants, there must be many pieces of equipment such as boilers, furnaces, turbines, condensers, fans, pumps, economizers, air preheaters, and feed-water heaters. Such plants must be bulky and also expensive to operate in the low power field (below about 10,000 hp).

Another drawback in the steam power plant is that there must be a large temperature difference between the products of combustion and the working substance to cause heat to flow into the working substance at the desired rate. The degradation in energy by this heat transfer limits the thermal efficiency (see Chapter 5).

Today, the working fluid of the external-combustion power plant is a vapor. The major portion of the heat supplied is added at substantially constant temperature, as the fluid changes state. This temperature is the boiling temperature of the liquid for the allowable vapor pressure; and, for a given fluid, it can be increased only by increasing the allowable vapor pressure. It is desirable to have a fluid which not only boils at high temperatures for relatively low pressures but which condenses at approximately condensing-water temperatures without exceeding commercially obtainable vacua. No known fluid is satisfactory as far as the desired pressures are concerned. Thus, the temperature range, and hence the efficiency, of the external-combustion engine power plant is limited. To overcome this limitation, attempts have been made to use a binary vapor cycle in which the efficiency is increased by the use of two fluids (see Chapter 7). However, in spite of improved efficiency, this type of plant is used only to a limited extent, for reasons given in Chapter 7.

The internal-combustion engine, on the other hand, is compact, as combustion takes place within the working cylinder. Heat is transferred through walls only to keep internal surfaces cool. If this heat-transfer rate is sufficiently high, the maximum temperature reached in internal-combustion engines is limited only by temperatures possible of attainment by combustion. Thus, the temperature range, and hence the efficiency, of the internal-combustion engine is inherently high.

One disadvantage of the internal combustion engine is that it requires a higher grade fuel than does the combustion equipment of an external-combustion power plant. This is so because the combustion in an internal-combustion engine is intermittent and takes place in a very short period of time in an extremely limited space. In addition, a poor fuel burned in an internal-combustion engine gives rise to problems of detonation and also ash removal. In the furnace of a steam power plant, on the other hand, the combustion takes place continuously in a large combustion space with such devices as ignition arches to help burn a low grade fuel.

Another disadvantage of any reciprocating engine is that the weight of the reciprocating parts increases with the cylinder size faster than does the horsepower output. Rotative speeds must therefore be reduced to hold the inertia forces to reasonable values. With decreased rotative speeds there is a decreased power output. Thus, internal-combustion engines with high power output per cylinder are bulky and heavy. For internal combustion engines having very large cylinders, the weight may exceed that of a steam power plant, including all of its auxiliaries, having the same output.

*Except for the smaller sizes,* the initial cost of an internal combustion engine power plant is generally greater than that of a steam power plant. For this reason, the internal-combustion engine is not used as a stationary power plant, except where favorable location may mean that fuel oil of a good grade may be obtained at prices comparable with those of coal.

The outstanding field of service for the internal-combustion engine is transportation. For automotive, truck, bus, and airplane services, its light weight,\* low cost, small bulk, and high efficiency have made it predominant. In the last few years, internal-combustion engines have been used to an increasing extent for marine service (below about a 20 000-hp power plant). There has been a pronounced shift to the use of internal-combustion engines in the railroad field more recently.

Internal-combustion engines are classified as four-stroke-cycle (or four-cycle) and two stroke-cycle (or two-cycle) engines. During the suction stroke of the four cycle spark ignition† engine, the piston *a* moves down in the cylinder *b*, as shown in Fig. 17-1(*a*). The inlet valve *c* is open, and the exhaust valve *d* is closed. The suction created by the downward movement of the piston draws a mixture of air and fuel into the cylinder. Shortly after the piston reaches the bottom of its stroke, or bottom dead center (BDC), the inlet valve closes. As the piston moves

\* In the transportation field cylinder sizes are kept small to minimize the weight. If high power outputs are required many cylinders must be used.

† The term spark ignition as here used will refer to that type of engine using a gaseous or a light liquid fuel. It does not include oil engines having spark ignition.

upward, as shown in Fig. 17-1(b), it compresses the charge. Somewhat before the piston reaches top dead center (TDC), the spark occurs, initiating combustion. The combustion then spreads throughout the

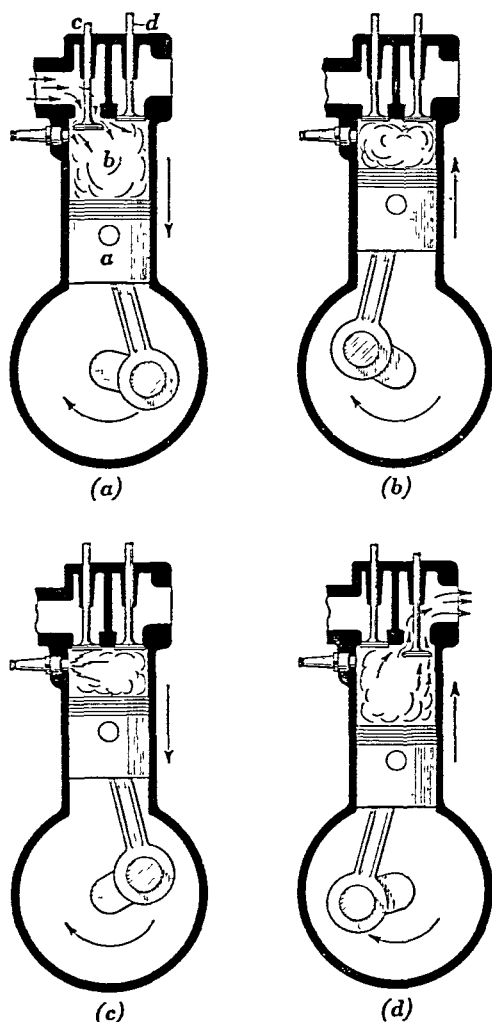


Fig. 17-1. Suction, Compression, Expansion and Exhaust Operations in Four-Stroke-Cycle Engine

combustion chamber, building up the temperature, and hence the pressure, within the cylinder. Because of this increase in pressure, much work is done on the piston as it moves downward on the expansion stroke, as indicated in Fig. 17-1(c). Just before the end of the expansion stroke,



the exhaust valve opens and some of the products of combustion rush out. As the piston moves upward on the exhaust stroke, as indicated in Fig 17-1(d), it pushes out the major portion of the remaining cylinder contents. Near top dead center, the exhaust valve closes and the intake valve opens. The cycle is then repeated.

In order to retain high thermal efficiency in the two cycle engine, the compression and expansion strokes must be retained. If the intake and exhaust strokes are to be eliminated, ways must be found to get the charge into the cylinder and to get the exhaust gases out without depending on the piston. These results are attained by supplying the charge to the cylinder under pressure. The charge may be put under pressure by a

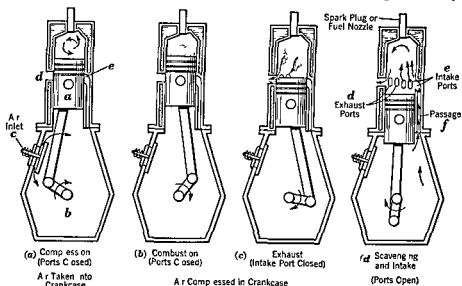


Fig 17-2 Operation of Two Stroke Cycle Internal Combustion Engine

separate compressor or by crank-case compression. Fig 17-2 shows a two cycle gasoline engine with crank-case compression. In engines having crank-case compression, it is common practice to eliminate the valves and to use ports which are covered and uncovered by the piston. In Fig 17-2(a), the piston 'a' is moving upward, compressing the charge above it. At the same time this upward movement of the piston creates a suction in the crank case 'b', drawing the charge into it through valve 'c' (valve 'c' is pressure actuated, opening when the pressure in the crank case is less than atmospheric and closing when it exceeds this value). In Fig 17-2(b), the piston is starting its expansion stroke, the spark having initiated combustion previously. As the piston moves downward, it compresses the charge in the crank case to a moderate value, say 5 psi gage. In Fig 17-2(c), the left hand side of the piston has uncovered the exhaust

port *d*, and the exhaust gases start to rush out. With a slight additional piston movement, the intake port *e* is cracked, and the charge starts to enter the space above the piston from the crank case through the passage *f*. Fig. 17-2(*d*) shows the position of the piston with the crank just after bottom dead center, both ports being open. A crown is often put on the top of the piston to deflect the incoming charge upward into the cylinder. As the charge is under pressure, it tends to sweep the exhaust gases out ahead of it (*i.e.*, to scavenge the cylinder). As the piston moves upward, it closes first the intake port and then the exhaust port. Compression then starts and the cycle is repeated.

The fundamental principle of operation of the four-stroke diesel\* engine is the same as that of the four-stroke spark-ignition engine. However, during the intake stroke (see Fig. 17-1) air alone is drawn into the cylinder. This air is compressed to a much higher pressure and temperature during the compression stroke than is the case in the spark-ignition engine. Just before the end of the compression stroke, fuel under high pressure is injected into the cylinder. As the temperature of the air exceeds the self-ignition temperature of the fuel, combustion starts. There is no need for a spark plug.

The difference between the spark-ignition engine and the diesel engine of the two-cycle types is the same as the difference in the four-cycle types. As no unburned fuel is present in the cylinder during the exhaust period of a two-cycle diesel engine, none can be lost to the exhaust. In the two-cycle spark-ignition engine the fuel is mixed with the air as it enters the cylinder, and there may be a serious loss of fuel to the exhaust. For this reason, except in small sizes, two-cycle spark-ignition engines are seldom used. However, work is being done on injection of gasoline into engines of the two-cycle type after exhaust closure. It is too early to arrive at a conclusion regarding the success of this design.

Internal-combustion engines may be classified on the basis of the types of fuel they use. Thus, there are gasoline engines, oil engines, kerosene engines, and gas engines. Attempts have been made to burn other substances, such as powdered coal, but such engines are not used commercially.

In the smaller sizes, internal-combustion engines are generally vertical, single-acting engines. Some automotive, rail-car, and marine engines have two banks of cylinders, being called V-type engines from the arrangement of the two banks. Aircraft engines may have their cylinders arranged in-line, in-line inverted, radial (in one or more banks), or opposed (quite often horizontal). See the Appendix.

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\* Although the term "compression ignition" is truly descriptive of an engine depending on compression for its ignition, this text will follow common practice and call such engines "diesel" engines.

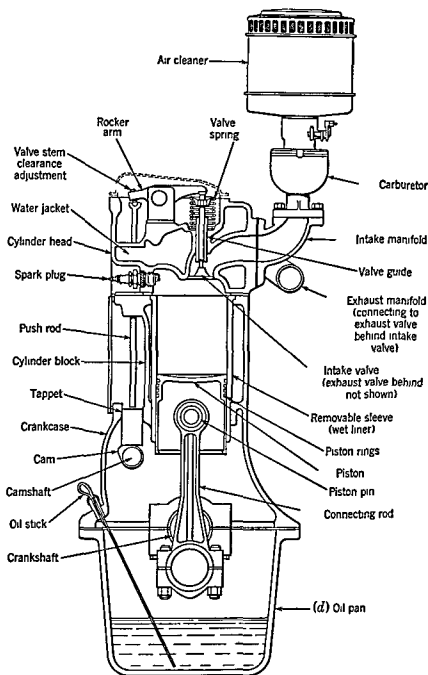


Fig 17-3 Schematic Diagram of Four-Stroke Cycle Gasoline Engine

In the larger sizes, internal-combustion engines may be either vertical or horizontal, the larger of the horizontal ones quite often being double-acting. Some large vertical engines are also double-acting. A special type of engine is the opposed-piston engine; here the cylinder is vertical and contains a piston in either end.

Fig. 17-3 is a schematic diagram of a vertical four-cycle gasoline engine, with the principal parts labeled.

★17-2. Combustion in the Spark-Ignition Engine.—In the spark-ignition engine, there is reason to believe that the fuel either is vaporized before the start of the compression process or becomes vaporized during the compression. The spark initiates a flame which spreads throughout the air-vapor mixture until the walls of the cylinder are reached, as indicated in Fig. 17-4. This is a time-taking process, the length of time

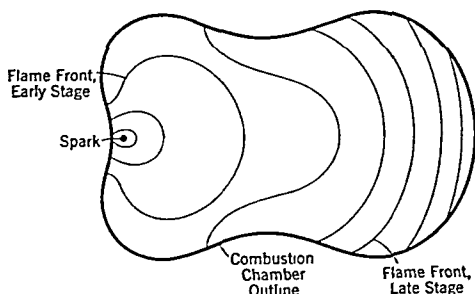


Fig. 17-4. Spread of Flame Front, Starting from Spark, in Non-Detonating Cylinder

depending on many factors. Flame propagation in a spark-ignition engine has been studied by various investigators and by various means. Spread of the flame is accelerated by any internal turbulence of the charge. In fact, were it not for the increased flame velocity at high rotative speeds due to this turbulence, modern high-speed spark-ignition engines could not have been developed. However, as the flame velocity does not increase so fast as does the engine speed, it is necessary to advance the spark as the speed is increased, to permit the major portion of the charge to burn near top dead center and thus produce high thermal efficiency.

A second important factor affecting flame velocity is the amount of dilution of the charge by clearance gas. The clearance gas, being inert, slows down the flame. Any item which will increase the ratio of clearance gas to fresh mixture will reduce the flame velocity and will therefore necessitate an earlier spark. Thus, at any given speed, a reduction in load necessitates a spark advance, for the following reason. It is essential in

a spark-ignition type of engine to keep the air-fuel ratio within rather narrow limits, to insure flame propagation. When the load is reduced, the fuel input is reduced. To keep the desired air-fuel ratio within the desired range for flame propagation, the air flow must also be reduced.\* This reduction in the amount of charge means that the amount of clearance gases is relatively large and that flame propagation will be slower.

As shown in Chapter 16, an increase in compression ratio results in an increase in thermal efficiency, as well as power output. Early in the development of the spark ignition type of engine, it was found that an increase in compression ratio resulted in *detonation* or *knock*. When detonation becomes severe, both the thermal efficiency and power output fall off. Continued operation of a detonating engine results in a tendency toward sticking rings and burning of exposed parts such as valves and pistons.

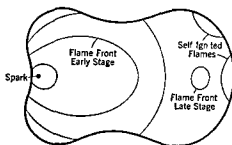


Fig 17-5 Self Ignition Occurring in Advance of Flame Front in Detonating Cylinder

The mechanism of combustion in an internal combustion engine is complex, the mechanism of detonations is more so. It appears that there is more than one type of detonation. However, it is fairly well agreed that when detonation occurs pressure waves travel through the cylinder with very high velocity and produce audible metallic sounds as they strike the piston and cylinder walls. The most common type of detonation occurs when part of the charge in the cylinder self-ignites before the flame front can reach it. See Fig 17-5. To minimize this type of detonation it is desirable to try to have the flame front move through the charge and burn it progressively before it can self-ignite. Thus, in order to increase the compression ratio and obtain the higher thermal efficiencies and power outputs, it has been necessary to develop fuels that resist the tendency to self-ignite, *i e*, fuels which have higher self ignition temperatures.

\* The reduction in air fuel flow is accomplished by means of throttling. Like any other throttling this is a loss of available energy and causes a rapid reduction in the thermal efficiency of a spark ignition engine with load at any given engine speed.

It is relatively easy to determine the self-ignition temperature of a fuel at atmospheric pressure. However, there is not good correlation of the self-ignition temperature of a fuel at atmospheric pressure with its tendency to self-ignite in an engine cylinder. Hence, the only satisfactory way to determine the tendency of a fuel to self-ignite in a cylinder is to test it in an engine cylinder. To obtain comparable results, it has become standard practice to use a single-cylinder engine run under specified conditions. This engine is known as a C.F.R. (Cooperative Fuels Research) engine. Since several problems are introduced if an attempt is made to measure the absolute amount of detonation produced when a part of a given fuel charge self-ignites, this is not done in the United States. The standard procedure is to compare the self-ignition characteristics of the given fuel with those of standard reference fuels. The two reference fuels selected were normal heptane, which detonates readily in a spark-ignition engine, and iso-octane, which greatly resists detonation.

The tendency of a fuel to resist self-ignition, and hence detonation, is expressed by means of its *octane number*. The octane number of a fuel is that percentage by volume of iso-octane in a mixture of iso-octane and normal heptane which knocks with the same intensity as does the given fuel under the same conditions. In recent years, fuels have been developed for high-compression engines, particularly of the aircraft type, which have better anti-detonating properties than those of pure iso-octane. To measure the detonating properties of these gasolines, known quantities of tetraethyl lead are added to pure iso-octane.\*

In addition to the anti-knock properties of a gasoline, a very important property is its volatility (*i.e.*, its ease of vaporization). The gasoline must *not be so volatile that it will vaporize sufficiently in the fuel system between the fuel tank and the carburetor jet to interfere with the flow of the liquid fuel*. On the other hand, the gasoline should have sufficient volatility to form in the cylinder the inflammable mixture needed for flame propagation. This inflammable mixture should be formed under all conditions of operation. Because no gasoline has the correct volatility under all conditions, it is sometimes necessary (as when starting a cold engine) for the carburetor to supply more gasoline than is needed for combustion in order that the partial vaporization of the larger amount of gasoline will produce the air-vapor mixture required for flame propagation.

The discussion in this article has been centered on gasoline. However, other fuels should be considered for spark-ignition engines. Kerosene has been used to some extent for spark-ignition engines, but its volatility is poor and so are its anti-knock properties. Benzene, which is a coal-tar

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\* In practice, the cost of normal heptane and iso-octane has led to the use of carefully calibrated secondary reference fuels to determine the octane number of a fuel.

distillate, has been used for blending with gasoline, as it is quite volatile and has excellent anti knock properties. Various alcohols have been tried in internal-combustion engines. Although they are excellent anti-knock fuels, their heating values are roughly two-thirds of that of gasoline and they are much more expensive than gasoline under present conditions.

★17-3. **Combustion in the Diesel Engine**—As there are no spark plugs in a diesel engine, the fuel must self-ignite after it is heated by the highly compressed, high temperature air. To minimize the amount of fuel burning at once and causing knocking, the fuel cannot be put into the diesel engine before the compression stroke begins but must be put in just before the end of this stroke. In the ideal case, the fuel should burn as it

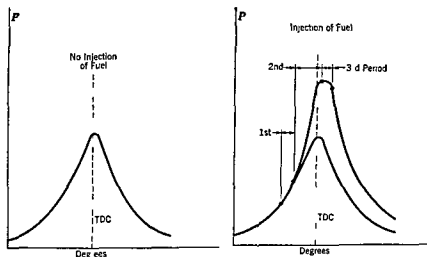


Fig 17-6 Pressure-Crank Angle Diagrams of Diesel Engine, Illustrating Three Stages of Combustion

comes into the cylinder. In this way, the rate of combustion could be controlled by controlling the rate of injection.\* In the actual case, however, the fuel is not ready for combustion as it comes into the cylinder. It is cold and in the liquid state. To transfer heat to the oil with sufficient rapidity to bring it to ignition conditions within the time period available, there must be much surface for heat transfer. To provide the required surface, the fuel must be highly atomized. In high-speed engines, the mean diameter of the oil droplets is approximately half that of a human hair.

The entire combustion process in a diesel engine has been divided into three stages or parts by Ricardo. See Fig 17-6. The first period is the

\* It should be kept in mind that theoretically the maximum efficiency is obtained in both the spark ignition and diesel engines when combustion takes place entirely at top dead center. In both types of engines however, this must be avoided because of detonation.

ignition lag or delay period. This is the period between the start of injection and the start of combustion. During this period, the fuel is being heated and vaporized. After it reaches its self-ignition temperature, there is an additional delay, as a fuel must be held at its self-ignition temperature for a short period of time before combustion starts. The second period is the period of rapid combustion. Once combustion starts, the temperatures in the cylinder are greatly increased. Heat is transferred at a rapid rate not only to the oil in the cylinder but also to that coming into the cylinder during the second period. Therefore, the fuel in the cylinder at the start of combustion burns rapidly, and the fuel coming into the cylinder after combustion has started burns shortly after it comes into the cylinder. The combined result is that there is a rapid rate of pressure rise during this period. After the major portion of the fuel present in the cylinder at the start of combustion has burned, the rate of combustion will depend on the rate of injection. The burning of the additional fuel is the third period of combustion. As the piston is generally moving down from top dead center during this period, there is little or no pressure rise during this period; in fact, the pressure may drop. In case the load is light, the injection of fuel may stop\* before the end of the period of rapid combustion. In this case the third period will be very short, and may be non-existent, as only that fuel, if any, which is still unburned at the end of the second period can be burned during the third period.

In case the rate of combustion is too high during the second period of combustion, the engine will knock. The primary factor affecting the rate of combustion during this period is the amount of fuel present at the start of the period. For a given rate of injection, the amount of fuel present at the start of the second period depends on the length of the ignition lag period. In general, then, any factor which reduces the length of the ignition lag period will reduce the rate of pressure rise during the second period and hence will reduce the tendency to knock.

It should be noted that flame propagation is not essential to the operation of a diesel engine; rather, the diesel engine depends on self-ignition of the fuel. Because the diesel engine does not depend on flame propagation, it is not necessary to reduce the air flow to a diesel engine as the fuel flow is reduced at part loads. Hence, the throttling loss at part loads is avoided in a diesel engine and the air-fuel ratio is high; thus, the efficiency of a diesel engine does not decrease so rapidly with a decrease in load as does the efficiency of a spark-ignition engine.

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\* The decrease in the amount of oil injected into the cylinder required by a decrease in load is obtained by decreasing the length of the injection period in most diesel engines. This decrease is normally obtained by changing the time at which injection of the fuel is terminated.



Whereas self-ignition of a fuel is to be avoided in a spark ignition engine, it is essential for a diesel engine. Thus, the more suitable a fuel is to resist detonation in a spark ignition engine, the more unsuitable is the fuel for a diesel engine because of its tendency to knock in the diesel engine. This self ignition property of the fuel under engine conditions is one of the most important properties of a diesel fuel. As in the case of a fuel for spark ignition engines, the ability to resist knock depends on its self ignition property under engine conditions. This property is spoken of as the ignitability of the oil.

As in the case of a gasoline engine, the only satisfactory\* way of predicting the ignitability of the oil is to test it in an engine. The engine used is a specially designed, specially operated single-cylinder engine. The ignitability of the oil is compared with that of various mixtures of cetane and alpha methyl naphthalene by measuring either the lowest compression ratio at which the fuel will self ignite or the compression ratio necessary for a fixed ignition lag. The result reported is the cetane number of the fuel. This is defined as that percentage by volume of cetane in a mixture of cetane and alpha methyl naphthalene which has the same ignitability as does the given fuel under the same conditions. It must be emphasized that the volatility of the fuel is not an indication of its ignitability. Gasoline will partly vaporize at very low temperatures and can be readily ignited by a spark, but its self ignition temperature is much higher than that of kerosene or a diesel fuel.

Another important property of a diesel fuel is its viscosity. A viscous fuel resists atomization and the resulting droplets have a tendency to be so large that they will not vaporize in the time available. Although it is true that many types of fuel including dusts and powdered coal may be burned after a fashion the fuel for a high-speed diesel engine must have high ignitability and must be capable of being broken down into very small particles so that it may be brought to its self ignition temperature in a very short period of time. At the present time, only a high grade of fuel oil satisfies the requirements.

**17-4 Theoretical Air-Fuel Method of Cycle Analysis**—The air standard method of cycle analysis is very valuable in that the inherent characteristics of various cycles may be determined with relatively few simple calculations. Thus, it may be shown by use of the air standard analysis that, for a given maximum pressure the Carnot Cycle has an inherently low mean effective pressure as compared with that of the corresponding Otto Cycle. The air standard analysis of the Lenoir

\* There are some means of correlating fairly easily measured properties of a fuel oil with its tendency to resist knock but the correlation is not perfect.

Cycle shows that this cycle has a very low thermal efficiency because of the lack of compression of the charge before combustion. Internal-combustion engines the cycles of operations of which were patterned after the Lenoir Cycle were discarded with the development of the Otto-Cycle engine, which includes compression of the charge and hence has high efficiency. Another value of the air standard analysis is that it may be used to predict the relative effects of changes made in actual engines. For instance, a change in the compression ratio of a spark-ignition engine produces, roughly, the same percentage change in efficiency and maximum pressure as that shown by the air standard calculations.

Because the air standard analysis assumes an ideal working substance (air, which is treated as a perfect gas with constant specific heat), the efficiency calculated by the air standard analysis greatly exceeds that which may be attained by a perfect engine.\* Likewise, the temperatures and pressures calculated by the air standard analysis are very much higher than those which would be reached in a perfect engine.† Thus, it may be seen that the air standard analysis is not satisfactory in determining the degree of perfection of an actual engine.

Many attempts have been made to modify the air standard method to obtain results closer to those which could be attained in a perfect engine. No satisfactory solution of the problem appeared until the work of Goodenough and Baker‡ was published. In their method, the efficiencies, temperatures, and pressures were determined as nearly as possible for a perfect engine using an actual working substance. In this perfect engine, charge is drawn into the cylinder and mixed with the hot clearance gases left in the clearance space, the mixture coming to a common temperature. The variable specific heats of the clearance gases, air, and fuel vapor are taken into account. At the end of compression, the charge is burned at constant volume (Otto Cycle), the temperature and pressure greatly increasing; but the temperature is limited to a certain maximum value by dissociation of the products of combustion. The products of combustion expand isentropically, and during this process the dissociated products recombine. The variable specific heats of the individual substances are taken into account, as before. The exhaust starts at the end of the expansion stroke and continues for the entire exhaust stroke.

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\* A perfect engine is considered to be one in which all mechanical imperfections have been eliminated.

† In comparing the calculated quantities of the air standard analysis with similar quantities in the actual engine, it is assumed that the engine is an internal-combustion engine. For the gas turbine, with its low temperature range, deviations are not very great. This is also the case for a lightly loaded diesel engine.

‡ Goodenough and Baker, "A Thermodynamic Analysis of Internal Combustion Engine Cycles," Bulletin 160, Engineering Experiment Station, University of Illinois.

As dissociation of the products limits the maximum temperature, and hence the efficiency, a brief discussion of it is in order. At high temperatures there is a tendency for the products of combustion to dissociate or break down, the carbon dioxide dissociating into carbon monoxide and oxygen, and the water vapor into hydrogen and oxygen. This tendency increases with an increase in temperature, dissociation being insignificant at temperatures around 2000 F but becoming quite rapid at temperatures around 4000 F to 5000 F. In the formation of the products by combustion, energy is liberated. When the products dissociate, energy must be supplied. In a combustion process, the air-fuel mixture and the products—dissociated or otherwise—come to a state of dynamic equilibrium at a temperature known as the chemical equilibrium temperature. At this temperature the tendency of the fuel, air, and dissociated products to combine and liberate energy is exactly balanced by the tendency of the products thus formed to dissociate and absorb energy. No higher temperature than this can be reached. Unless energy is removed by heat transfer or in the form of work, the temperature remains that of the chemical equilibrium.

Calculations of equilibrium temperatures are quite involved. Fortunately, charts have been prepared by Hershey, Eberhardt, and Hottel,\* which take into account not only the effect of dissociation but also the specific heats of the products of combustion of  $C_8H_{18}$  (or other hydrocarbons having the same hydrogen-carbon ratio) and the variable specific heats of the air-fuel mixture. By use of these charts, the determination of thermal efficiencies, pressures, temperatures, and mean effective pressures for the theoretical case are relatively simple.

The chart of efficiency prepared by Goodenough and Baker was based on specific heat and dissociation data that are now recognized as inaccurate. In addition, their efficiencies are based on the lower heating value of the fuel, whereas it is often the practice in the United States to base efficiency on the higher heating value. For these reasons the authors have prepared the chart in Fig. 17-7, showing the thermal efficiency for the theoretical air-fuel Otto Cycle, based on the higher heating value of the fuel. Use was made of the charts of Hershey, Eberhardt, and Hottel in these calculations.

From the discussion in Art. 17-3, it may be seen that the combustion in an actual diesel engine differs greatly from the process resulting from the constant pressure addition of heat as assumed for the theoretical Diesel Cycle. The period of rapid combustion tends to make the thermal efficiency of the actual cycle higher than that of the constant-pressure Diesel Cycle. In fact, as the third period of combustion becomes mini-

\* Hershey, Eberhardt, and Hottel "Thermodynamic Properties of the Working Fluid in Internal Combustion Engines." S.A.E. Journal 39 409 (1936)

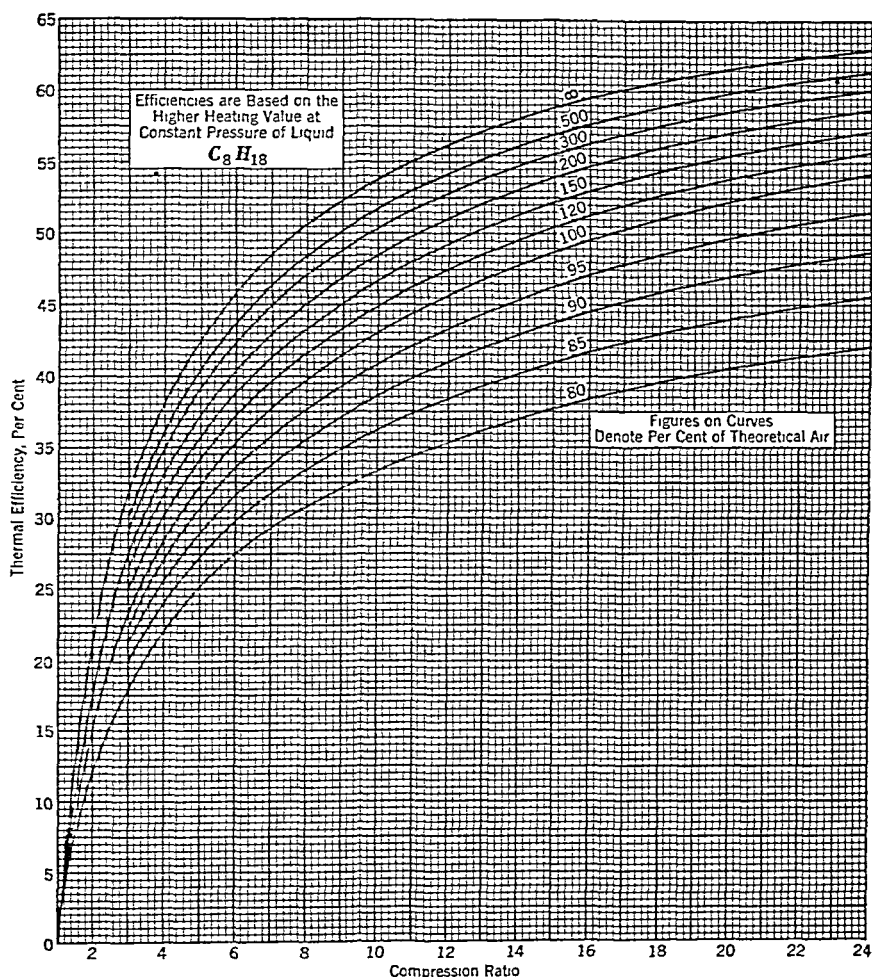


Fig. 17-7. Theoretical Thermal Efficiencies vs. Compression Ratio for Various Air-Fuel Mixtures (Otto Cycle)

mized, the efficiency of the actual engine should approach that of the theoretical air-fuel cycle, other losses being neglected. Thus, the theoretical air-fuel Otto Cycle should be the limit of performance of an actual diesel engine, as well as the spark-ignition engine.\* To cover a wide range,

\* It is realized that diesel fuels differ materially from  $C_8H_{18}$ . Furthermore, in the actual diesel engine the fuel vapor is not compressed along with the air, as assumed in the calculation of values used in Fig. 17-7. Since these deviations are small and have some tendency to compensate for one another, Fig. 17-7 may be used for diesel engines as well as spark-ignition engines for ordinary engineering work.

the efficiencies in Fig 17-7 are carried to compression ratios of 24 to 1, although compression ratios in actual diesel engines seldom exceed 17 to 1

**17-5 Deviations of the Actual Engine From the Theoretical Air-Fuel Cycle**—As nearly as possible, the theoretical air-fuel method of cycle analysis deals with a perfect engine using an actual working substance. The thermal efficiency of the actual engine falls short of that of the theoretical cycle because of imperfections in the actual engine, such as the following

- (1) There are friction losses
- (2) There are heat losses
- (3) Distribution of fuel and air to the various cylinders is not perfect
- (4) Combustion takes time
- (5) Combustion may not take place at the right time
- (6) Throttling action and turbulence occur through valves and manifolds
- (7) Valves do not open and close at dead center
- (8) There is incomplete combustion (due to improper mixing)

All these losses, except possibly friction, are thermodynamic losses. Hence, they produce an indicated thermal efficiency less than the theoretical value. Friction losses cause the brake thermal efficiency to be lower than the indicated value.

Approximately one-fourth to one third of the heating value of the fuel is removed as heat by the jacket water of an internal combustion engine. Offhand it may appear that the thermal efficiency would be greatly improved if it were possible to minimize the amount of heat transferred to cooling water. Actually, however, this would not be true. The available portion of the heat passing into the cooling water cannot exceed the thermal efficiency of the theoretical air-fuel cycle. The *available* portion of heat transferred is actually only a small portion of the total heat transferred, as only a small portion of the heat is transferred at the time of maximum availability (*i.e.*, at top dead center). Therefore, complete suppression of heat transfer to the cooling water of an engine having a thermal efficiency of 25% may increase the thermal efficiency to only about 29%.

As a result of the various losses, the indicated thermal efficiency usually is between about 70 and 90 per cent of that of the theoretical cycle. Thus it may be said that the indicated *relative efficiency*\* of an engine may

\* The term relative efficiency means the ratio of the actual thermal efficiency to the thermal efficiency of the theoretical air-fuel cycle. As the thermal efficiency may be based on either indicated horsepower or brake horsepower, there may be indicated and brake relative efficiencies. Care should be taken to specify which relative efficiency is being used. It may be recalled that the engine efficiency is likewise on an indicated or brake basis (see Chapter 7).

range from 70 to 90 per cent. Although it shows the over-all engine performance, brake relative efficiency may allow certain excellent engine features to mask poor ones. Thus, a swirl chamber diesel engine may have a poor mechanical efficiency, but because of an excellent thermodynamic performance may have a reasonable brake relative efficiency. By a study of the indicated relative efficiency and the mechanical efficiency, rather than the brake relative efficiency, it will be apparent that because of the low mechanical efficiency the over-all engine performance is by no means so satisfactory as the brake relative efficiency indicates.

**17-6. Factors Affecting Mean Effective Pressure and Power Output.** The energy delivered to a piston of an internal-combustion engine depends on the following items:

- (1) Heating value of the charge supplied to the engine.
- (2) Amount of charge drawn into the engine (volumetric efficiency).
- (3) Efficiency of transformation of the heating value of the fuel into mechanical energy (indicated thermal efficiency).

Air and fuel are supplied separately to the engine. Thus, in dealing with the heating value per cubic foot of charge supplied to the engine, it becomes necessary to speak of the heating value of an equivalent charge, *i.e.*, a charge formed by assuming the fuel and air to be mixed at supply conditions. The heating value of this equivalent charge, expressed in Btu per cu ft, is equal to the heating value of the fuel as it approaches the engine divided by the total volume of air and fuel approaching the engine. This is true, regardless of whether the engine has a carburetor or liquid fuel is injected into either the manifold or engine cylinder. In case the fuel is supplied in the liquid state, its volume is extremely small and may be neglected.

The heating values of theoretical mixtures of air and liquid fuel used in spark-ignition engines do not differ greatly, being about 100 Btu per cu ft of charge at standard conditions. A change in either the temperature or pressure at which the charge is supplied to the engine, or a change in the per cent of theoretical air used, may greatly change this heating value. As a gaseous fuel occupies appreciable volume, the heating value of a charge having a gaseous fuel is lower than one with a liquid fuel, varying from about 95 Btu per cu ft for a fuel with little inert matter to below 60 Btu per cu ft for a gaseous fuel having much inert matter (such as blast-furnace gas). These figures refer to the theoretical air-fuel mixtures at standard conditions of 14.7 psia and 60 F.

The piston of an engine sweeps out a given space during the suction stroke. Theoretically this space should be filled up with charge under

The rest of the volume, or  $6.52 - 0.42 = 6.10$  units, is occupied by the charge. If it is assumed that the entire charge would be in the gaseous state under outside conditions, its volume would equal

$$6.10 \left( \frac{14.2}{14.7} \right) \left( \frac{520}{710} \right) = 4.31 \text{ units}$$

The portion of this volume of 4.31 units which is air volume may be found by writing the combustion equation for the fuel (assumed to be  $C_8H_{18}$ ). Thus



This shows that the air volume is  $\frac{12.5 + 47.0}{12.5 + 47.0 + 1} \times 100 = 98.4$  per cent of the charge volume, if the charge is all gas. The air volume is

$$4.31(0.984) = 4.24 \text{ units}$$

In reality the fuel approaching the engine is liquid, and hence its volume may be neglected. Then the volume approaching the engine may be taken as the air volume, or 4.24 units. The volumetric efficiency of the engine is

$$\frac{\text{Volume Approaching Engine}}{\text{Piston Displacement}} \times 100 = \frac{4.24}{6} \times 100 = 70.7\% \quad \text{Ans}$$

The efficiency of transformation of the heating value of the fuel into mechanical energy (indicated thermal efficiency) is equal to the product of the theoretical efficiency and the relative efficiency. The factors affecting these quantities were discussed in Arts 17-4 and 17-5, to which the student should refer. The method of using the various factors just discussed to compute the power output of an engine is shown in the following example.

**Example 17-2**—Compute the power output and the indicated mean effective pressure of a 6-cylinder, 4-cycle, single-acting engine with a compression ratio of 7 to 1 and a speed of 3600 rpm. The indicated relative efficiency is 80%, and the volumetric efficiency is 69.3%. The engine is 4 in  $\times$  5 in. The theoretical air fuel ratio is used, the heating value of the fuel is 20,200 Btu per lb, and standard atmospheric conditions are assumed.

**Solution**—The volume of air used under atmospheric conditions is

$$\frac{(4\pi)^4 (5)}{1728} \left( \frac{6}{2} \right) (3600)(0.693) = 272 \text{ cu ft per min}$$

The weight of air used is

$$\frac{(14.4)(14.7)(272)}{(53.3)(520)} = 20.77 \text{ lb per min}$$

If the molecular weight of air is taken as 28.97 and the fuel is assumed to be  $C_8H_{18}$ , the air fuel ratio is, by weight,

$$\frac{(12\frac{1}{2} + 47)28.97}{(8)(12) + (1)(18)} = 15.12 \text{ lb air per lb fuel}$$

Thus, the weight of fuel is

$$\frac{20.77}{15.12} = 1.373 \text{ lb per min}$$

The heating value supplied to the engine is

$$1.373(20,200) = 27,730 \text{ Btu per min}$$

From Fig 17-7, the theoretical efficiency is 37.7%.

The indicated thermal efficiency is 37.7 (0.80) = 30.2%.

The indicated horsepower becomes

$$\text{ihp} = \frac{27,730(0.302)}{42.4} = 197.5 \quad \text{Ans}$$

The indicated mean effective pressure of the engine may be calculated from the ihp, or it may be calculated directly as follows:

The volume of air per lb of fuel is

$$V = \frac{WRT}{P} = \frac{(15.12)(53.3)(520)}{(144)(14.7)} = 198 \text{ cu ft}$$

The heating value per cu ft of charge (outside conditions) is

$$\frac{20,200}{198} = 102 \text{ Btu}$$

The heating value per cu ft of piston displacement is

$$102(0.693) = 70.7 \text{ Btu}$$

The work per cu ft of piston displacement is

$$70.7(0.302)(778) = 16,610 \text{ ft-lb}$$

The indicated mep is

$$\frac{16,610}{144} = 115 \text{ psi} \quad \text{Ans}$$

*Example 17-3.*—A diesel engine has a brake mep of 95 psi when using 0.42 lb of fuel per bhp-hr. The heating value of the fuel is 19,200 Btu per lb. The engine is supplied with 270 cu ft of air per lb of fuel. Calculate the volumetric efficiency of the engine.

*Solution.*—The product of the heating value of the charge supplied to the engine, in Btu per cu ft, the volumetric efficiency, and the brake thermal efficiency equals the brake work in Btu per cubic foot of piston displacement. The brake work in foot-pounds per cubic foot of piston displacement is numerically equal to the brake mep in pounds per square foot.

The heating value of the charge supplied to the engine equals

$$\frac{19,200}{270} = 71.1 \text{ Btu per cu ft}$$

$$\text{Brake Thermal Efficiency} = \frac{2544}{0.42(19,200)} \times 100 = 31.6\%$$

$$\text{Then,} \quad (71.1)(\text{Vol. Eff.})(0.316)(778) = 95(144)$$

$$\text{from which} \quad \text{Vol. Eff.} = 0.783 \text{ or } 78.3\% \quad \text{Ans}$$

A knowledge of the heating value  $Q_{HP}$  of the fuel is not necessary for the solution of this problem, as may be seen by combining all steps. Thus,

$$\left(\frac{Q_{HP}}{270}\right)(\text{Vol. Eff.})\left(\frac{2544}{0.42(Q_{HP})}\right)(778) = 95(144)$$

$$\text{and} \quad \text{Vol. Eff.} = 0.783 \text{ or } 78.3\% \quad \text{Ans.}$$

**17-7. Engine Performance.**—It is customary to show the performance of a variable-speed engine by plotting its characteristics against engine rpm. The two chief characteristics are brake horsepower and brake specific fuel rate (lb fuel per bhp-hr). It is, however, desirable to study those factors that influence these two main characteristics; and auxiliary characteristics, such as volumetric efficiency, imep, bmep, torque, ihp, fhp,



and mechanical efficiency, are often plotted. The primary plot is made for the engine with wide-open throttle, although part-throttle results are often shown. The characteristics discussed here are for wide-open throttle. Typical performance curves are shown in Fig 17-9.

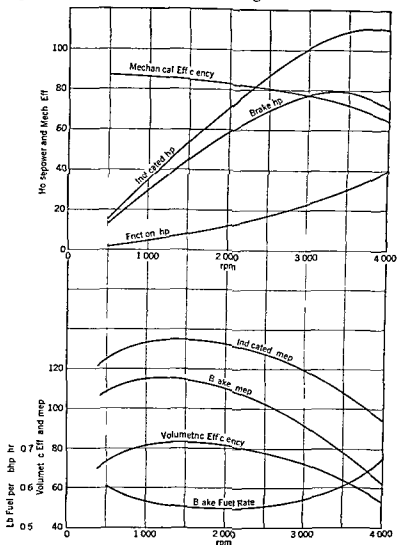


Fig 17-9 Typical Performance Curves for Automotive Engine at Full Throttle

**Volumetric Efficiency** The first characteristic that should be studied is volumetric efficiency, as the power output depends directly on the amount of charge drawn into the cylinder. The shape of the volumetric-efficiency curve depends on the timing of the intake valve. In an automotive engine, the time of intake-valve closure is a compromise. At high speeds

maximum use is not made of the inertia of the incoming charge; whereas, at low speeds some of the charge may be pushed back into the intake manifold. For this reason, the volumetric efficiency of an automotive engine generally peaks at a medium speed, say 1200 to 2000 rpm. The volumetric efficiency falls off gradually as the rpm is reduced below this value. As speeds are increased above the medium speed, the volumetric efficiency falls off gradually at first; and then at high speeds, as the pressure across the valves becomes excessive, there is a rapid decrease in volumetric

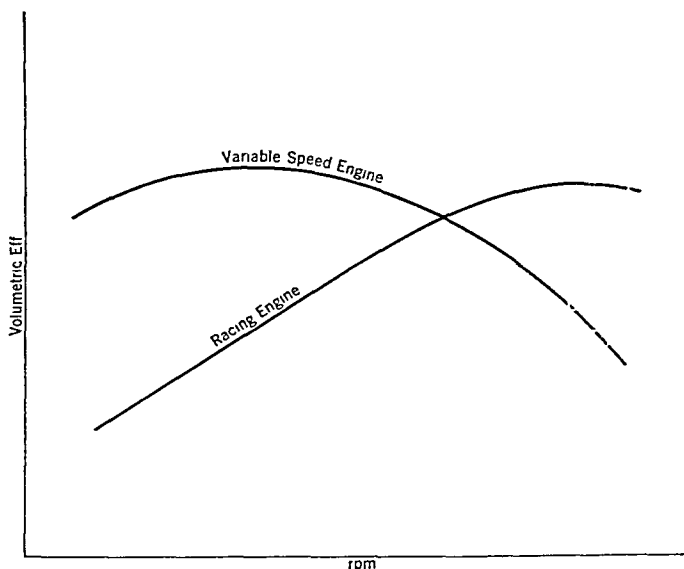


Fig. 17-10. Contrast in Volumetric Efficiency vs. rpm for Variable-Speed and Racing Engines

efficiency with rpm. See Fig. 17-9. If the engine is to be designed primarily for operation at high speed, say for a racing car, the volumetric efficiency should be sacrificed at low speeds and the inlet valve should not be closed until the cylinder pressure is as high as possible at high speeds. In this case, the maximum volumetric efficiency occurs at high speed. See Fig. 17-10. The volumetric efficiency of an average slow-speed gasoline engine may reach a value as high as 90%, but in a high-speed engine that is properly timed to take advantage of inertia effects, the volumetric efficiency may approach 100%.

*Indicated mep:* Although volumetric efficiency is an outstanding factor in determining the indicated mep produced by an internal-combustion engine, the heating value of the charge and the indicated thermal efficiency are equally important. With the same fuel and with constant supply conditions, the heating value of the charge will vary only if the air-fuel

ratio is varied. With full throttle, there is little need to vary the air fuel ratio as the speed is varied. However, if the carburetor cannot supply a constant air-fuel ratio, the heating value of the charge will change with a change in speed. As the engine is not designed for full load operation at very low speed, a poor thermal efficiency under these conditions does not constitute a design problem. Several minor factors contribute to a somewhat lower thermal efficiency under these conditions. In general, the indicated mep of an internal combustion engine at full throttle follows the volumetric-efficiency curve rather closely, but at low speed it falls off a little more than does the volumetric efficiency.

**Indicated Horsepower** For a given engine the indicated horsepower is directly proportional to the product of the indicated mep and the rpm. At low and moderate speeds, there is not a pronounced change in indicated mep. Hence, over this range, the ihp curve is almost a straight line, the ihp being practically proportional to the rpm. At higher speeds the decrease in indicated mep causes the ihp curve to fall away from a straight line, and at very high speeds the indicated mep falls off faster than the rpm increases, and the ihp decreases.

**Friction Horsepower** The friction horsepower of a given engine is a function of the product of the frictional resistance and the rpm. The major portion of engine friction in an internal combustion engine is the friction between the rings and cylinder walls, and between the pistons and cylinder walls. Here, because of the acceleration and deceleration of the piston, the thickness of the oil film is continually changing. Although at times of reversal of motion, the oil film may be extremely thin for the major portion of the time the oil film is thick enough to be treated as a thick oil film. With a thick oil film the frictional resistance is directly proportional to the speed. Hence, the frictional horsepower should increase faster than the rpm. Test results show that this is true.

**Brake Horsepower** Brake horsepower is the difference between the indicated and friction horsepowers. As the friction horsepower increases faster than the rpm, the brake horsepower reaches a maximum at a speed somewhat lower than that of maximum ihp. The speed for maximum bhp is known as the peak speed of the engine. This is the speed at which automotive engines usually are rated.

**Mechanical Efficiency** Mechanical efficiency is the ratio of the brake and indicated horsepowers or it is the ratio of the brake horsepower to the sum of the brake and friction horsepowers. Since the friction horsepower increases faster than the rpm and since the brake horsepower fails to increase as fast as the rpm, the mechanical efficiency must decrease as the speed increases. The decrease is gradual at low speeds, but becomes very rapid at high speeds.

**Brake mep:** The brake mep is the mep obtained by setting the bhp equal to  $\frac{PLAN}{33,000}$  and solving for  $P$ . The indicated mep is an indication of the amount of work per unit of piston displacement per cycle. As it is difficult to determine accurately either the indicated mep or the ihp for a high-speed internal-combustion engine, the brake mep is calculated and used instead. Brake mep is also equal to the product of the indicated mep and the mechanical efficiency. Hence, the curve for brake mep is quite similar in shape to that for the indicated mep, but it falls off faster at high speeds.

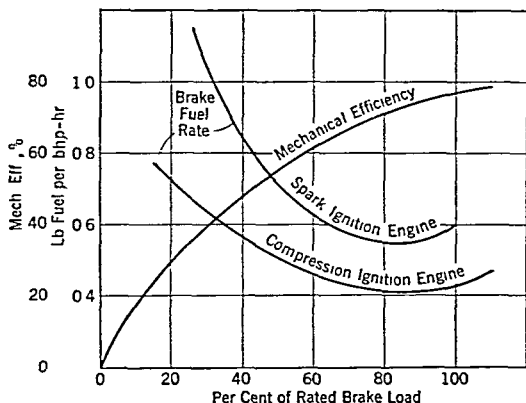


Fig. 17-11. Typical Brake Fuel Rates and Mechanical Efficiency for Constant-Speed Engines

**Torque:** Torque is the turning effort produced by an engine. It may be thought of as the force, acting at a radius of 1 ft from the center of the shaft, which will produce the same work that is produced by the engine. For a given engine, torque is a direct function of brake mep and, as such, the torque curve must have the same shape as the mep curve.

**Brake Fuel Rate:** The brake fuel rate (sometimes called brake specific fuel consumption, bsfc) is the weight of fuel required per bhp-hr. Brake fuel rate is inversely proportional to brake thermal efficiency. Since the indicated thermal efficiency falls off at low speed, the brake fuel rate becomes relatively high. At high speed, although the indicated thermal efficiency remains high, the excessive frictional losses cause a decrease in the brake thermal efficiency and an increase in the brake fuel rate. Although the curves that have been discussed are those of a variable-speed spark-ignition engine, the curves for a diesel engine are similar.

For a constant-speed engine, the curve most commonly plotted is the

brake fuel rate vs load, although curves of mechanical and thermal efficiencies may also be plotted. See Fig 17-11. For both the diesel and the spark-ignition types of engines, the brake fuel rate increases at heavy loads primarily because of the large amount of incomplete combustion that accompanies the low air fuel ratios used to obtain the heavy loads. At light loads, the brake fuel rates for both types of engines become rather large, primarily because the friction hp, being substantially constant at a

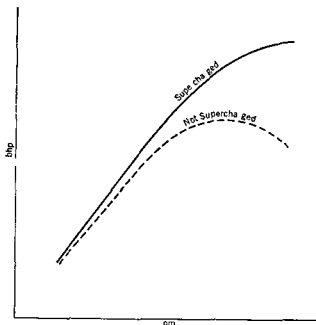


Fig 17-12 Effect of Supercharging on bhp

given rpm, is such a large portion of the indicated power output at light loads, hence, much more fuel must be used per brake hp at these light loads. In addition, as the spark-ignition type of engine is governed by throttling at light loads, the efficiency is poor because of the large throttling losses.

**Supercharging** The limiting factor on power output of a non-supercharged engine is the amount of charge or air it can draw in. Because of the great decrease in atmospheric density with an increase in altitude, it is customary to supercharge aircraft engines to enable them to deliver sea-level power at high altitudes without increasing the weight of the engines. Diesel engines for both marine and railroad purposes are supercharged to a large extent, as supercharging materially reduces both the size and weight of the engine. In the case of slow speed diesel engines of the stationary type, there has been a tendency to resort to supercharging to decrease the bulk of the engine.

A supercharger is a compressor (generally of the rotative type to minimize size and weight) which draws into itself more air or charge than can the non-supercharged engine to which it is to be attached. Superchargers may be of either the displacement type or the centrifugal type. See Chapter 11 for descriptions of the various types of rotating compressors.

The power output of an engine equipped with a supercharger depends on the amount of charge delivered by the supercharger, and this amount in turn depends on the speed and volumetric efficiency of the supercharger. A supercharger of the displacement type will deliver more charge at all speeds than would the normally aspirated engine, but the supercharged engine is particularly effective at high speeds as its volumetric efficiency falls off less rapidly at high speeds than does that of the non-supercharged engine. See Fig. 17-12.

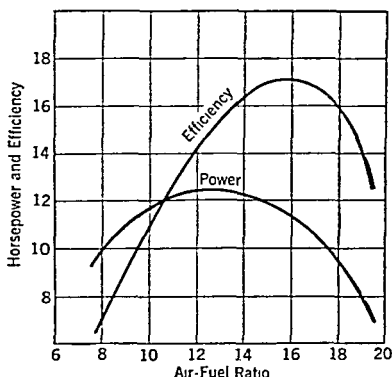


Fig. 17-13. Effect of Air-Fuel Ratio on Power Output and Efficiency

★17-8. Carburetion.—The purpose of the carburetor of an internal-combustion engine is to supply the engine with the desired air-fuel ratio with the fuel well atomized. Normally the time available is not sufficient to provide for much vaporization of the fuel in the carburetor, even though the fuel may be very volatile. Hence, the vaporization must be done in the intake manifold and engine cylinder.

The air-vapor ratio must be kept within a rather narrow range, in order that the flame will spread throughout the combustion chamber. The carburetor should supply an air-fuel ratio such that at all times, regardless of conditions of operation, the air-vapor ratio produced will be within this narrow range. A good carburetor will, however, do more, because a relatively small change in air-fuel ratio may change both the thermal efficiency and the power output, as shown in Fig. 17-13. The curves in Fig. 17-13 are drawn for a particular engine with given speed and given inlet conditions. A change in any of these items will change these curves and may change the air-fuel ratio at which these curves peak. It should be noted that there is little variation in the power output from its maximum value as the air-fuel ratio is varied from 10 to 14.5; but over the same range the efficiency varies materially, reaching its maximum with a lean mixture. As the air-fuel ratio approaches either the lean limit or the rich limit, the thermal efficiency and the power output fall off quite rapidly.

At full-throttle conditions, an automotive carburetor should supply that air fuel ratio which produces maximum power. At any other throttle opening with steady operating conditions, the carburetor should supply that air fuel ratio which gives the highest thermal efficiency. To maintain the desired air-fuel ratio as the air flow through the carburetor increases, certain "leaning" devices must be incorporated into the carburetor, otherwise, the air fuel ratio would decrease. In addition, the

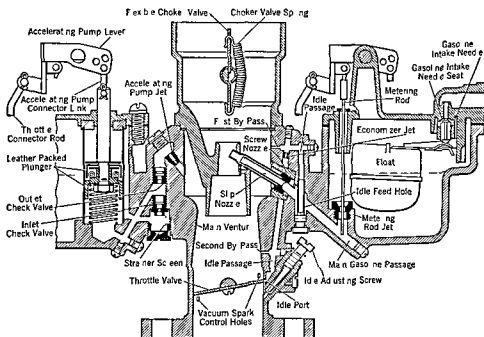


Fig 17-14 Typical Automobile Carburetor

carburetor must have an idling device, a device to provide additional fuel during acceleration, and a choke to provide a rich mixture for starting. Thus, to perform its functions satisfactorily, an automotive carburetor must contain many parts. Fig 17-14 shows the cross section of a typical automotive carburetor.

The rapid decrease in air density with altitude will cause the air fuel ratio of an airplane carburetor to decrease rapidly unless steps are taken to correct this. Simple airplane carburetors have manual means of decreasing the fuel flow at high altitude to maintain desired air-fuel ratios, whereas carburetors for the larger airplane engines are designed to do this automatically.

Because of several problems peculiar to airplane carburetors, it has been found desirable in airplane work to use an injection type of car-

buretor in which the fuel is injected under pressure at the supercharger entrance or into the intake manifold. In some cases, the carburetor has been eliminated entirely and gasoline has been injected directly into the engine cylinder during the suction stroke.

★17-9. **Diesel Fuel Injection Systems.**—There are two general types of fuel injection systems for the diesel engine, namely, the air injection and the mechanical (formerly known as the solid injection) system.

In the air injection system, a three-stage or four-stage air compressor delivers air to the injection valve, where the air entrains oil and carries it into the cylinder. The air injection system gives good atomization; but

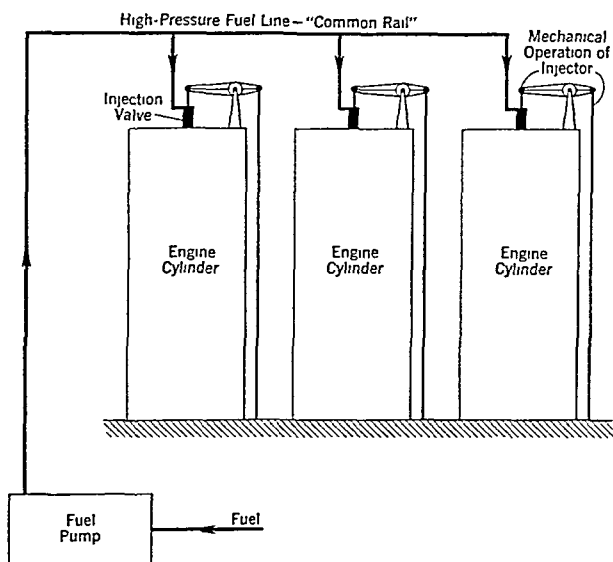


Fig. 17-15. Common-Rail Type of Diesel Fuel Injection System

the air compressor required is a drawback, as it occupies space, increases original costs, consumes power, and adds to the maintenance problems. In addition, the injection nozzle is complicated and expensive. The earlier diesel engines were air injection engines; but, as the mechanical injection system was improved, it gradually displaced the air injection system until today the use of the air injection system is confined almost entirely to the older engines.

The mechanical injection system consists of a fuel pump and an injection nozzle. Normally the fuel pump is located near the engine shaft and hence must be connected to the injection nozzle by means of a pipe line. In some cases the injection nozzle and the pump are built together



as one unit, known as the unit injector. There are two general types of mechanical injection systems, the common-rail system and the jerk pump system. In the common rail system, which is represented in Fig 17-15, a multi cylinder pump delivers oil to a common header or rail at pressures ranging from about 2000 to 5000 psi. Fuel lines lead from the header to the injection nozzles of the various cylinders. The fuel from the injection nozzle enters the engine cylinder through one or more small openings (orifices). In the line between the header and the injection nozzle is located a control valve, which may be operated mechanically or magnetically. This control valve controls the start and the length of the injection period.

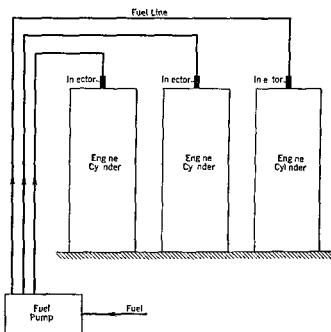


Fig 17-16 Jerk-Pump Type of Diesel Fuel Injection System

The jerk-pump system, represented in Fig 17-16, is used much more extensively than is the common-rail system. In the jerk pump system, there is an individual pump for each cylinder, although it is common practice to group the various pumps of a multi cylinder engine in a single casting, the pump plungers being operated by cams mounted on a common camshaft. The pump plunger is long in comparison with its diameter. The plunger being without rings, clearances between the plunger and its barrel must be carefully controlled to minimize leakages. This requirement calls for excellent manufacturing equipment and workmanship.

A system which cannot be classified completely as either a jerk-pump system or a common-rail system is the Cummings system. In this system, oil is delivered to a distributing unit which meters the fuel. From the distributor, the oil is delivered in turn to the various cylinders before the time for injection. At the time injection is desired, a plunger in the injection valve is mechanically pushed down on the oil beneath it, building up the high oil pressure required for satisfactory injection.

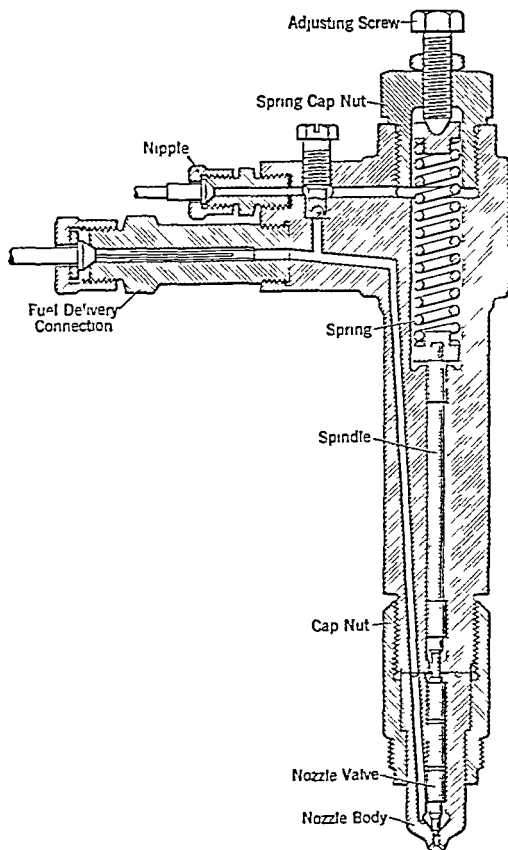


Fig 17-17. Section of Bosch Nozzle Holder  
With Open Pintle Nozzle

Most of the injection nozzles used for the jerk-pump system are closed injection nozzles. Such a nozzle is shown in Fig. 17-17. The closed injection nozzle has a valve held on its seat by a sufficiently heavy spring to prevent its opening until the oil pressure reaches the desired amount. This desired oil pressure working on part of the bottom of the valve is

sufficient to move the valve against the spring pressure, causing it to open and allow flow

To be satisfactory, a diesel fuel injection system must meet the following requirements at all times (1) It must deliver the correct amount of oil, (2) it must inject the oil at the right part of the cycle, (3) it must provide proper atomization, (4) it must provide distribution of the fuel in the cylinder. In the common-rail system, the metering of the fuel and the timing are controlled at the control valve. In the jerk-pump system, these two operations are controlled in the pump. The oil pressure and the design and location of the injection nozzle control the atomization and distribution of the fuel in the cylinder.

The most common way of controlling the amount of oil delivered by the jerk pump to the line, and hence to the injection nozzle, is by varying the effective length of the pump stroke. The Bosch pump, shown in Fig 17-18, is typical of this method. In the Bosch pump, the pump plunger is rotated to control the effective length of the stroke. Delivery starts when the top *A* of the pump plunger closes the suction and by-pass ports. Delivery continues until the edge of the helical cut *B* in the plunger starts to uncover the by-pass port (which leads to the suction line). A study of Fig 17-19 will show the method of rotating the pump plunger to change the time at which the helix starts to uncover the by-pass port.

Fuel is atomized in the cylinder as the high-velocity oil stream shoots through the dense cylinder air, the air peeling off the outside layer of the fuel stream. Surface tension causes these shorn-off particles of oil to assume droplet form. If the flow through the orifices is turbulent, the individual particles have a cross-stream velocity as well as a forward velocity. In this case, as the stream loses its restraining walls upon passage into the cylinder, the cross stream velocity carries the individual particles out of the stream, thus producing additional atomization.

Distribution of the fuel in the cylinder is accomplished by penetration (shooting straight ahead through the dense air) and by dispersion (spreading out of the fuel jet as it enters the cylinder). The relative amounts of dispersion and penetration desired depend on the shape of the combustion chamber, as well as on the location of the injection nozzle. It should be emphasized that the injection nozzle and its location must be chosen to fit the combustion chamber.

Most large engines are direct injection engines, i.e., fuel is injected directly into the main combustion chamber. Many small engines however, use the precombustion arrangement shown in Fig 17-20 to reduce the delay period, to limit the pressure in the main part of the cylinder, and to make the engine less sensitive to nozzle performance. Proper atomization and distribution of the fuel may be obtained with a lower injection

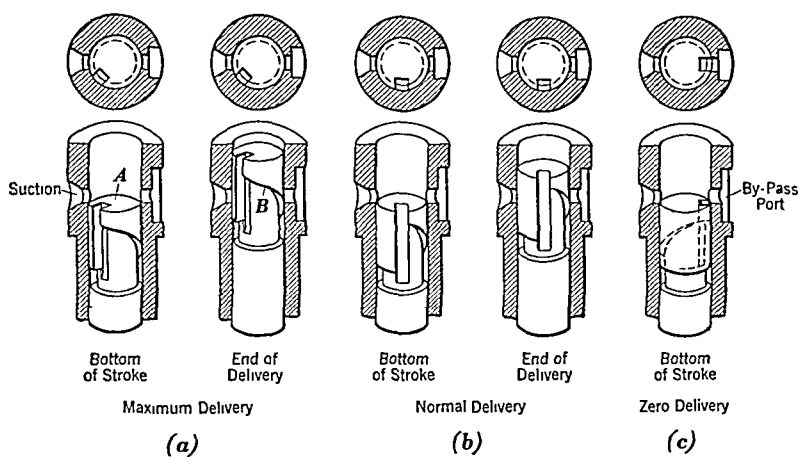


Fig. 17-18. Pump Barrel With Various Plunger Positions

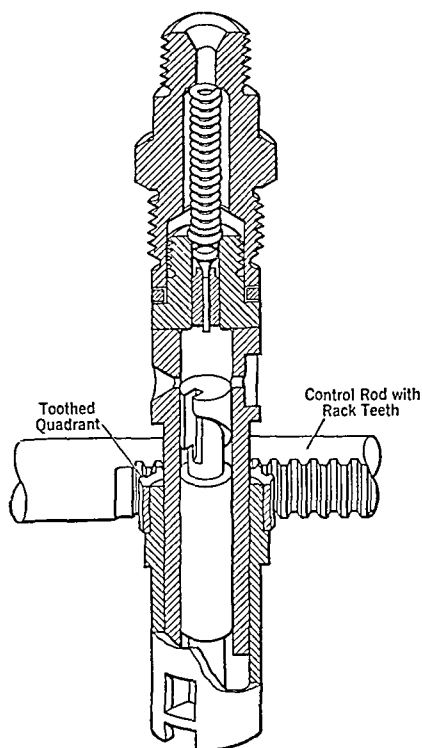


Fig. 17-19. Pump Element in Section

pressure if a precombustion chamber is used. This is an auxiliary chamber connected to the main chamber by a small opening. Oil is injected into the precombustion chamber at pressures ranging from 900 to 1500 psi or more. As the walls of the precombustion chamber are not well cooled, vaporization and partial combustion of the fuel take place, accompanied by a rapid pressure rise. This high pressure causes a rapid flow of the mixture of hot gases and fuel through the small opening into the main combustion chamber, thus producing the desired atomization and distribution of the fuel.

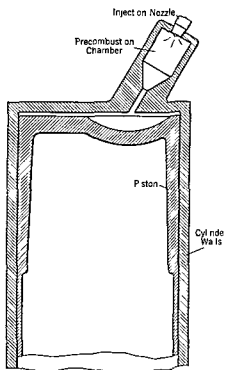


Fig 17-20 Precombustion Chamber of Diesel Cylinder

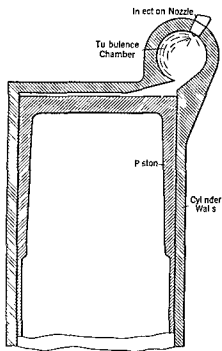


Fig 17-21 Turbulence Chamber of Diesel Cylinder

Another type of combustion chamber that is used to a large extent is the turbulence chamber, shown in Fig 17-21. This is also a combustion chamber attached to the engine cylinder, but it is larger than the precombustion chamber. In contrast to the precombustion chamber, which is designed to have only sufficient combustion to provide good combustion in the main combustion chamber, the turbulence chamber is designed to burn the major portion of the fuel within itself. To minimize the amount of excess air required and, at the same time, to insure good combustion, the turbulence within the combustion chamber must be high.

## CHAPTER 18

### GAS TURBINES

**18-1. Application of the Brayton Cycle to Gas Turbines.**—As stated in Chapter 16, the Brayton Cycle is poorly suited to piston engines for several reasons, chief of which is its low mep. Because rotating machinery (especially of high rpm) can handle large volumes of gases without excessive weight and space requirements (as stated in Chapter 11), the low mep of the Brayton Cycle is not a determining factor in the gas-turbine unit.

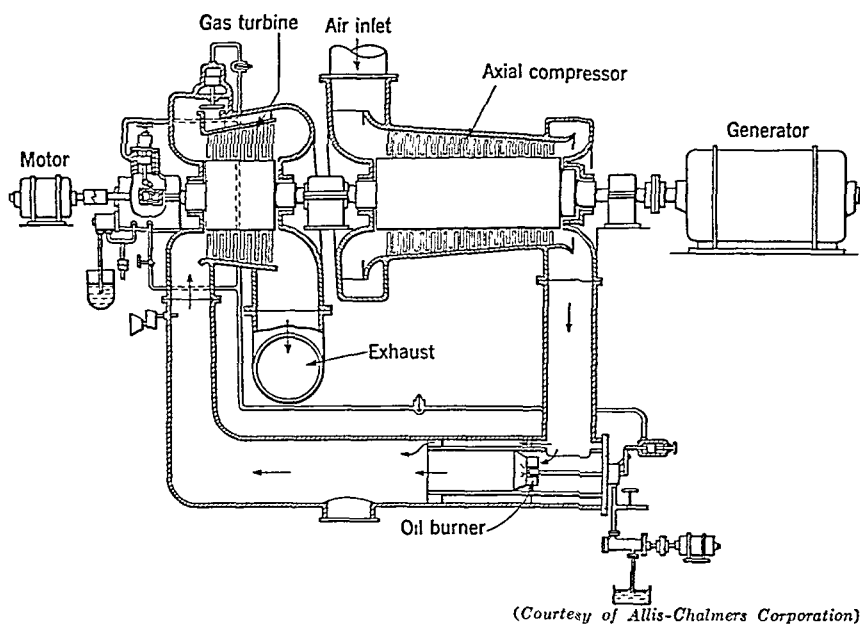


Fig. 18-1. Diagram of Simplest Form of Combustion-Turbine Plant

The Brayton Cycle, modified, is adaptable to gas turbines for the following reasons: (1) A gas turbine uses a separate compressor and combustion chamber; (2) combustion occurs at nearly constant pressure; (3) expansion can be carried to nearly exhaust pressure (whether in the turbine itself or in a jet exhaust); (4) generally low over-all pressures are used, so that it is possible to use a high-speed centrifugal or turbo-compressor having a relatively low weight and a high mechanical efficiency. It may be asked why the gas turbine was not in the limelight long ago. One answer lies in the compressor. As will be seen later in this chapter, a large part of the

gross work produced by the turbine is consumed by the compressor, and, unless the compressor is very efficient, the net work output of the unit will approach the vanishing point. Another feature is that the over-all thermal efficiency is greatly affected by the maximum temperature reached, until comparatively recently, the maximum temperature that could be safely used on turbine blades because of metallurgical limitations was too low. The inherent advantages of an efficient, light weight, compact power unit such as the modern gas-turbine plant are further intensified when one considers the question of fuel. By not requiring a highly refined fuel—in fact, by not necessarily requiring a liquid fuel but by being capable of operating on pulverized coal—the gas turbine possesses many inherent advantages.

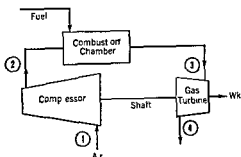


Fig 18-2 Diagrammatic Layout of Gas Turbine Plant

Fig 18-1 shows a section sketch of a gas turbine and compressor unit having a 5-stage turbine rotor on the same shaft as the 15-stage compressor which it drives. Fig 18-2 shows a diagrammatic sketch of the compressor combustion chamber, and turbine. Air enters at point 1 and is compressed to point 2, at which point it enters the combustion chamber. Here fuel is added and burned, the products of combustion leaving at point 3 and entering the turbine, expanding to exhaust at point 4. Compare Fig 18-2 with Fig 16-10. Figs 18-3 and 18-4 show corresponding points 1, 2, 3 and 4 on diagrams constructed by using  $P$ - $V$  and  $T$ - $S$  coordinates, respectively.

It will be noted that there is lightly sketched through points 1, 2', 3', and 4' a Brayton Cycle operating between the same *extremes of temperatures*. (Familiarity with the air-standard Brayton Cycle of Chapter 16 is here assumed, otherwise, that chapter should be reviewed.) The departure of the actual cycle from the air-standard Brayton Cycle is caused by the following conditions:

(a) Compression, even though it may be adiabatic is not *reversibly* adiabatic and hence is not isentropic.

(b) Flow through the combustion chamber is accompanied by a *pressure drop*. (It should be recalled that irreversible processes do not mean that the area on a  $T$ - $S$  plane denotes  $Q$ , even though lines are solid.)

(c) Even though the operation of the turbine is adiabatic, it is not *reversibly* adiabatic, and hence is not isentropic. (Recall to mind steam turbines considered in Chapter 10.)

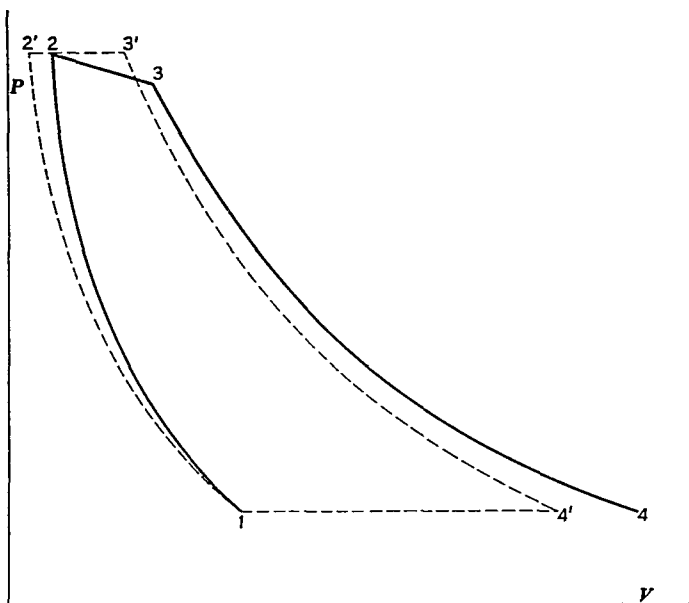


Fig. 18-3. P-V Diagram for Gas-Turbine Plant

(d) The cycle is usually *open*, taking in air, adding fuel, and expelling exhaust gases at a pressure sometimes equal to, but often above, that at which air entered.

(e) The working substance is not a perfect gas.

**18-2. Energy Analysis.**—Figs. 18-3 and 18-4 are sketched so that both the actual cycle and the Brayton Cycle here depicted operate with the same compressor *pressure ratio*. In Chapter 16, the air-standard thermal efficiency of the Brayton Cycle (based on the nomenclature of Figs. 18-3 and 18-4) was shown to be as follows:

$$\text{Brayton Thermal Eff} = 1 - \frac{T_1}{T_2'} \quad (18-1)$$

$$\text{or} \quad \text{Brayton Thermal Eff} = 1 - \frac{1}{\left(\frac{P_2'}{P_1}\right)^{\frac{k-1}{k}}} \quad (18-1a)$$



In the latter form, the air-standard thermal efficiency shows that the greater the pressure ratio, the greater the efficiency. This is not so in the actual cycle, as will presently be demonstrated.

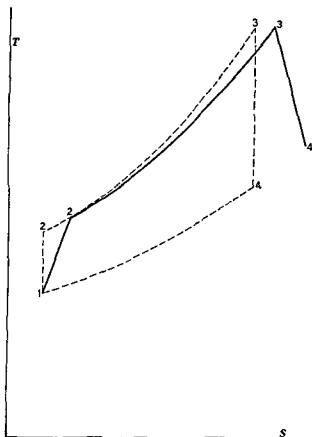


Fig 18-4 T S Diagram for Gas Turbine Plant

The thermal efficiency of the actual cycle is not so simply determined as that of the air-standard cycle, the five deviations listed in Art 18-1 enter into the picture, particularly the factors causing entropy increases in the compressor and turbine. Briefly stated, the actual thermal efficiency based on the net shaft work output is

$$\begin{aligned} \text{Actual Thermal Eff} &= \frac{\text{Net Work Output}}{\text{Heat Supplied}} \\ &= \frac{\text{Turbine Shaft Work} - \text{Compressor Shaft Work}}{\text{Heat Supplied in Fuel}} \end{aligned}$$

The turbine shaft work may be found by multiplying the turbine internal work by the turbine mechanical efficiency, the compressor shaft work, by dividing the compressor internal work by the compressor mechanical

efficiency: and the heat supplied in the fuel, by multiplying the weight of fuel used by its heating value.

While there are several methods whereby explicit figures may be used in determining the actual thermal efficiency, a simple method is to base the energy quantities on 1 pound of air *entering the compressor*. If  $W_f$  represents the fuel-air ratio by weight, or the number of pounds of fuel per pound of air, the weight of the products of combustion will be  $(W_f+1)$  lb per pound of air. Let  $\text{Eff}_{\tau T}$  represent the mechanical efficiency of the

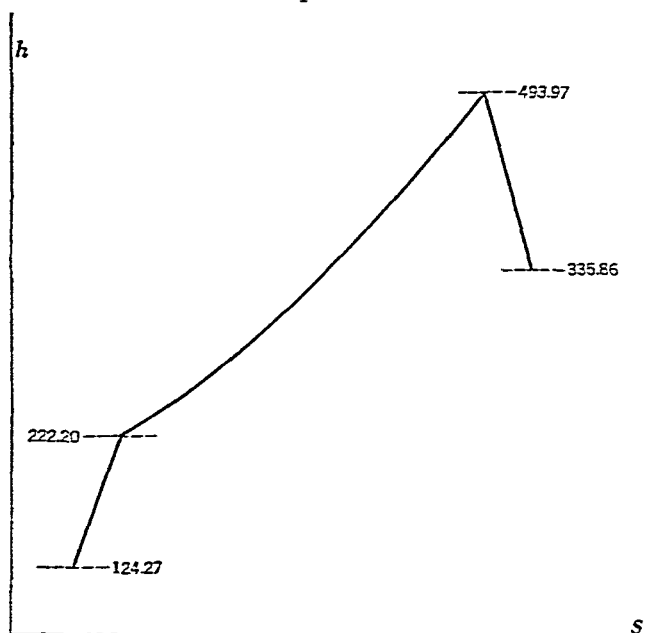


Fig. 18-5. Diagram for Example 18-1

turbine and  $\text{Eff}_{\tau c}$  that of the compressor. If both the turbine and the compressor are considered as steady-flow adiabatic machines, the internal work of each will be equivalent to the change in enthalpy between entrance and exit. Hence,

$$\text{Act. Thermal Eff} = \frac{(W_f+1)(h_3-h_4)(\text{Eff}_{\tau T}) - (h_2-h_1) \div (\text{Eff}_{\tau c})}{W_f Q_P} \quad (18-2)$$

where  $Q_P$  is the fuel's heating value at constant pressure, and the subscripts of the enthalpy terms are in accordance with Figs. 18-3 and 18-4. Either  $Q_{HP}$  or  $Q_{LP}$  may be used. Although, as stated in Chapter 13, most engineers in general work in the United States use the higher heating value, gas-turbine engineers have shown a preference for the lower value.

**Example 18-1**—Calculate the actual thermal efficiency of a gas-turbine unit having a compressor pressure ratio of 6 if the specific enthalpies at points 1, 2, 3, and 4 in the cycle are respectively 124.27, 222.20, 493.97, and 335.86 Btu per lb. The fuel-air ratio by weight is 0.0159 and the fuel's heating value is 19,800 Btu per lb. Each machine has a mechanical efficiency of 98%.

**Solution**—See Fig. 18-5 for a sketch of the cycle constructed by using  $h$ - $S$  coordinates as will appear later. These coordinates have distinct advantages. Substituting in equation 18-2 we obtain

$$\text{Act. Thermal Eff} = \frac{(1.0159)(493.97 - 335.86)(0.98) - (222.20 - 124.27) - (0.98)}{(0.0159)(19,800)} \quad \text{Ans.}$$

0.182 or 18.2%

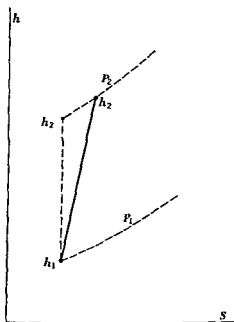


Fig. 18-6. Diagram for Compressor

It is of interest to note that the corresponding air standard Brayton Cycle with a pressure ratio of 6 would have an efficiency of 40%. It is also of interest to note the small fuel-air ratio, which is a characteristic of gas turbines in order to keep down the temperature of the gas entering the turbine. It should be recalled that a large amount of excess air will accomplish this. Metallurgical considerations limit present gas turbine temperatures to approximately 1800 F, but higher temperatures should be expected in the near future, especially with cooled blades.

**18-3 Internal Efficiencies**—As shown in Chapters 10 and 11 the internal efficiencies of turbines and compressors are based on the ratio of the internal work actually performed to that which would be performed by

an ideal machine operating *between the same pressure limits*. For gas turbines and compressors (either radial or axial flow), the ideal is taken as the isentropic work between the pressures under consideration, the position of the isentropic being located at conditions entering the machine. Fig.

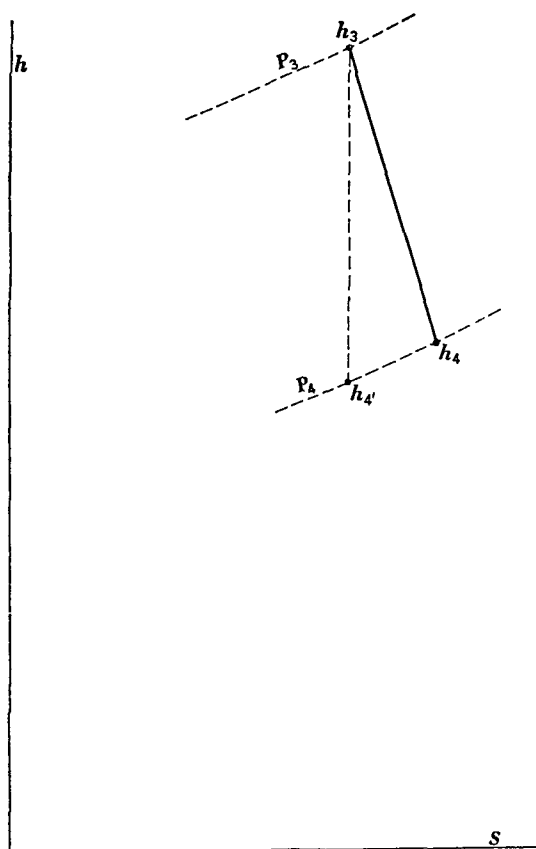


Fig. 18-7. Diagram for Gas Turbine

18-6\* shows the conditions for a compressor and Fig. 18-7 is for a turbine, the nomenclature used on these two diagrams corresponding to that on Figs. 18-3, 18-4, and 18-5.

For the compressor, the internal efficiency (sometimes called "adiabatic efficiency") is

$$\text{Compressor Internal Eff} = \frac{h_2' - h_1}{h_2 - h_1} \quad (18-3)$$

\* Compare Fig. 18-6 with Fig. 11-10.

For the turbine, the internal efficiency is

$$\text{Turbine Internal Eff} = \frac{h_3 - h_4}{h_3 - h_4} \quad (18-4)$$

Typical values for compressors are from 78 to 85%, and those for gas turbines are from 80 to 90%. In the evaluation of enthalpies, the properties of air are customarily used, since the products of combustion contain a preponderance of excess air. A simple analysis of the gas-turbine cycle can be approximated by using perfect-gas laws ( $c_p = 0.24$ ), but, since the influence of temperature on specific heat is important, it is preferable to evaluate the expression  $dh = c_p dT$  by the methods of Chapter 14 for variable specific heat. Fortunately, while gas turbines involve high temperatures and high velocities, the pressures are low, hence, the effect of pressure on specific heat can be neglected, compensation being solely for temperature.

★18-4. Gas Tables —With the idea of providing a convenient set of properties of air based on the available data on specific heats as a function

TABLE 18-1  
THERMODYNAMIC PROPERTIES OF AIR\*

T (abs)	h	$p_r$	$c_p$	$c_v$	k
300	71.61	0.17795	2395	1709	1.401
350	83.57	0.3048	2395	1710	1.401
400	93.53	0.4858	2396	1711	1.401
450	107.50	0.7329	2397	1712	1.400
500	119.48	1.0590	2399	1714	1.400
550	131.46	1.4779	2402	1717	1.399
600	143.47	2.005	2406	1721	1.398
650	155.50	2.655	2412	1726	1.397
700	167.56	3.446	2418	1733	1.396
750	179.66	4.396	2426	1741	1.394
800	191.81	5.526	2436	1751	1.392
850	204.01	6.856	2447	1762	1.389
900	216.26	8.411	2460	1775	1.386
1000	240.98	12.298	2488	1803	1.380
1100	265.99	17.413	2518	1833	1.374
1200	291.30	24.01	2550	1864	1.368
1300	316.94	32.39	2582	1896	1.361
1400	342.90	42.88	2614	1928	1.355
1500	369.17	55.86	2644	1959	1.350
1600	395.74	71.73	2673	1988	1.345
1700	422.59	90.95	2701	2016	1.340
1800	449.71	114.03	2728	2042	1.336
1900	477.09	141.51	2753	2067	1.332
2000	504.71	174.00	2776	2090	1.328

\* Reprinted by permission from Gas Tables by Keenan and Key published by John Wiley & Sons Inc

*Solution* — As indicated by the actual path in Fig. 18-8 there is an increase in entropy from the initial temperature to the final temperature because of irreversibility. From Table 18-1 the value of  $h$  at 850 R is 204.01. Substitution of proper values in equation 18-3 then gives

$$\text{Compressor Internal Eff} = \frac{191.81 - 119.48}{204.01 - 119.48} = 0.856 \text{ or } 85.6\% \quad \text{Ans.}$$

The remaining columns in Table 18-1 are self-explanatory and are included to show the variation in these items from perfect-gas values. Since tables for low-pressure air are based on the assumption that the gas constant  $R$  is truly invariable, accurate values of specific volume, intrinsic energy, and entropy changes can be computed with the given tabular values.

**★18-5 Combustion Chamber Energy Balance** — As the compressed air leaves the compressor, it enters the combustion chamber, where a continuous firing of fuel takes place. As this operation is merely another application of the general energy equation for steady flow, one need only balance the energies flowing in against those flowing out. By applying the equation for a steady-flow calorimeter for determining the heating value at constant pressure, and recalling that here there is a large per cent of excess air with a small heat loss and there is a possible kinetic energy of consequence of air and (or) gas, the burner energy equation *per pound of air entering* is as follows:

$$W_f(\text{Chem En.} + h) + h_2 + \frac{\overline{Vel}^2}{2gJ} = (W_f + 1) \left( h_3 + \frac{\overline{Vel}^2}{2gJ} \right) + Q_{loss} \quad (18-5)$$

where the quantity  $(\text{Chem En.} + h)$  is for the fuel and  $Q_{loss}$  represents the (usually small) heat transfer through the combustion-chamber walls per pound of air.

Frequently, combustion is incomplete, in spite of the huge amount of excess air, when this is so, the condition is customarily corrected for by multiplying the chemical energy by the so-called "efficiency of combustion." This efficiency ranges from 85 to 100%.

It will also be noted that the introduction of kinetic-energy terms in equation 18-5 causes the air and the products of combustion to have a total energy greater than that due to enthalpy, this total energy has given rise to a new set of properties, designated by some engineers "stagnation" properties, the most common of which is "stagnation enthalpy." Briefly stated, this enthalpy is the sum of the conventional  $h$  and the kinetic energy, and is designated by the subscript zero. Thus,

$$h_0 = h + \frac{\overline{Vel}^2}{2gJ} \quad (18-6)$$

The stagnation temperature is that temperature which corresponds to  $h_0$ ; and the stagnation pressure is that which would be reached by *isentropically* raising the temperature from the true temperature  $T$  to  $T_0$ . Fig. 18-9 shows the relationship between the true properties and the stagnation properties, designated by the subscript 0.

While the stagnation properties are *fictitious*, their convenience to the designer must not be overlooked, especially where high-speed rams and jets are involved. The details of the ram, for instance, cause complications where ram efficiency is involved; but these complications are easily simplified by the use of stagnation properties.

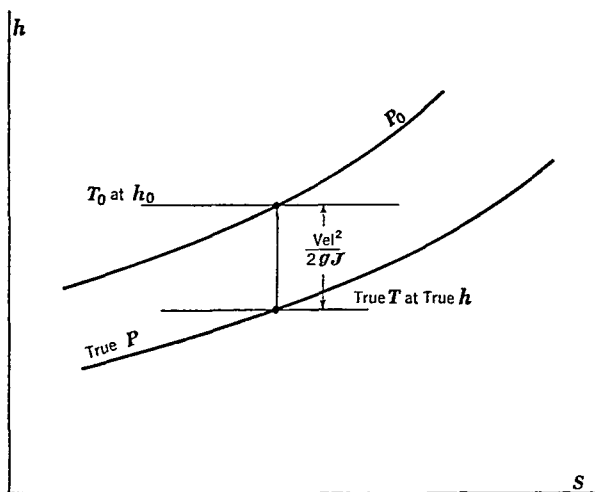


Fig. 18-9. Relation Between True Properties and Stagnation Properties of Gas

★18-6. **Regeneration as an Aid to Efficiency.**—So far, all discussion of the subject of the gas-turbine cycle has been based on the fact that the air went from the compressor directly to the burner. Fig. 18-10 shows diagrammatically the use of a heat exchanger between the compressor and the burner to heat the air by extracting energy from the products of combustion leaving the turbine. The fact that the air will thus reach the burner at a greatly elevated temperature tends to decrease  $W_f$ ; reference to equation 18-5 will show that, since  $h_3$  is limited by the allowable maximum temperature in the turbine, any increase in  $h_2$  will decrease  $W_f$ . When this fact is properly applied, it can have an enormous influence on the thermal efficiency. The difficulties of application are chiefly concerned with the amount of heat-transfer surface and with the pressure drop caused by the fluid flow over so much surface. The large surface is

occasioned by the fact that the heat transfer is through a gas film on each side of the metal wall through which the heat flows, and gas films naturally cause poor heat transfer. A given gas turbine unit without a regenerator

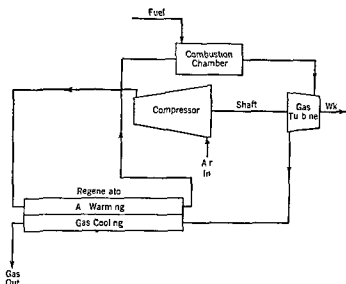


Fig 18-10 Use of Heat Exchanger Between Compressor and Burner

can be made to have a greater thermal efficiency by the addition of the regenerator, but the unit with the regenerator will have slightly less power owing to the regenerator pressure drop (the maximum temperature remaining the same)

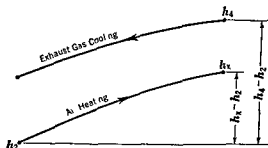


Fig 18-11 Effectiveness of Regenerator

The extent to which the air is heated to approach the exhaust-gas temperature is called regenerator efficiency or, better, regenerator effectiveness. This relation may be stated more explicitly as follows. The effectiveness of a heat transfer apparatus, such as a regenerator, is the ratio of the heat transferred to the amount that would be transferred if,



in this case, the cooler fluid reached the maximum temperature of the hotter fluid.

In Fig. 18-11 is shown a counterflow temperature distribution between two fluids; they flow in opposite directions, and the "warm" fluid rejects heat to the "cool" fluid. Obviously, if the exit temperature of either fluid were to equal the entering temperature of the other fluid, the maximum heat transfer would take place. Then,

$$\text{Regenerator Effectiveness} = \frac{\text{Heat Actually Transferred}}{\text{Maximum Possible Heat Transfer}}$$

When effectiveness is defined in terms of enthalpies, it may be expressed as follows: The enthalpy of the air leaving the compressor is  $h_2$ . If that air reached the temperature of the turbine exhaust, its enthalpy would be  $h_4$ . By introducing the symbol  $h_x$  for the enthalpy of the air actually emerging from the regenerator, we obtain:

$$\text{Regenerator Eff} = \frac{h_x - h_2}{h_4 - h_2} \quad (18-7)$$

An alternate definition of effectiveness is based on temperatures. This definition is

$$\text{Regenerator Eff} = \frac{T_x - T_2}{T_4 - T_2} \quad (18-7a)$$

*Example 18-4.*—Consider that the gas-turbine unit in Example 18-1 is equipped with a generator having an effectiveness of 60%. If other conditions remain unchanged, what is the thermal efficiency?

*Solution.*—Substituting in equation 18-7 to determine  $h_x$ , we obtain:

$$0.60 = \frac{h_x - 222.20}{335.86 - 222.20}$$

from which

$$h_x = 290.41$$

It is now necessary to determine the new fuel-air ratio,  $W_f$ . This is found by equation 18-5, for which additional data are needed. The sum of the fuel's chemical energy and its enthalpy is 18,385 Btu per lb of fuel; the burner heat loss and combustion inefficiency amount to 784 Btu per lb of fuel. (With no regeneration, substitution of these values in equation 18-5 will give  $W_f$  equal to 0.0159, as previously used.) However, using  $h_x$  in place of  $h_2$  for air entering the burner, and considering any velocity effects on both sides of the equation to be contained in the enthalpies, we get:

$$W_f(18,385) + 290.4 = (W_f + 1)(493.97) + (784)(W_f)$$

from which

$$W_f = 0.0119 \text{ lb fuel per lb air}$$

By equation 18-2,

$$\begin{aligned} \text{Act. Thermal Eff} &= \frac{1.0119(493.97 - 335.86)(0.98) - (222.20 - 124.27) \div (0.98)}{(0.0119)(19,800)} \\ &= 0.242 \text{ or } 24.2\% \end{aligned}$$

Ans.

This is a considerable improvement over the previous value of 18.2%.

The subject of regeneration is so important that, at low pressure ratios, an actual gas-turbine unit with a regenerator having an effectiveness of

90% can have a thermal efficiency greater than that of an air-standard Brayton Cycle of the same pressure ratio, as shown in Fig 18-12. In this figure, the efficiency of the air-standard Brayton Cycle increases with the pressure ratio, as expected, each curve for the actual gas turbine unit

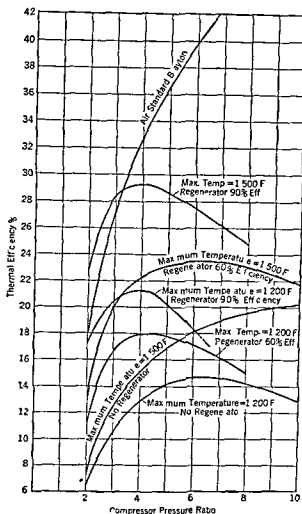


Fig 18-12 Effect of Regeneration on Thermal Efficiency

has a peak, indicating one pressure ratio for which the efficiency is a maximum. The data common to these curves are as follows: compressor internal efficiency, 0.85, turbine internal efficiency, 0.87, regenerator pressure loss on air side, 1.5% for a regenerator effectiveness of 60% and 5% for an effectiveness of 90%. The various pressure losses naturally cause slight changes in the turbine exhaust conditions, so that the net

work output per pound of air entering the compressor is slightly reduced as regenerator pressure losses increase.

Another consideration—besides the influence of regeneration—that should be significant in indicating a great future for the gas turbine is the effect of metallurgy on the maximum temperature which can be allowed. In Fig. 18-13 are shown the materials of construction and the operating temperatures for a gas turbine used for ship propulsion.

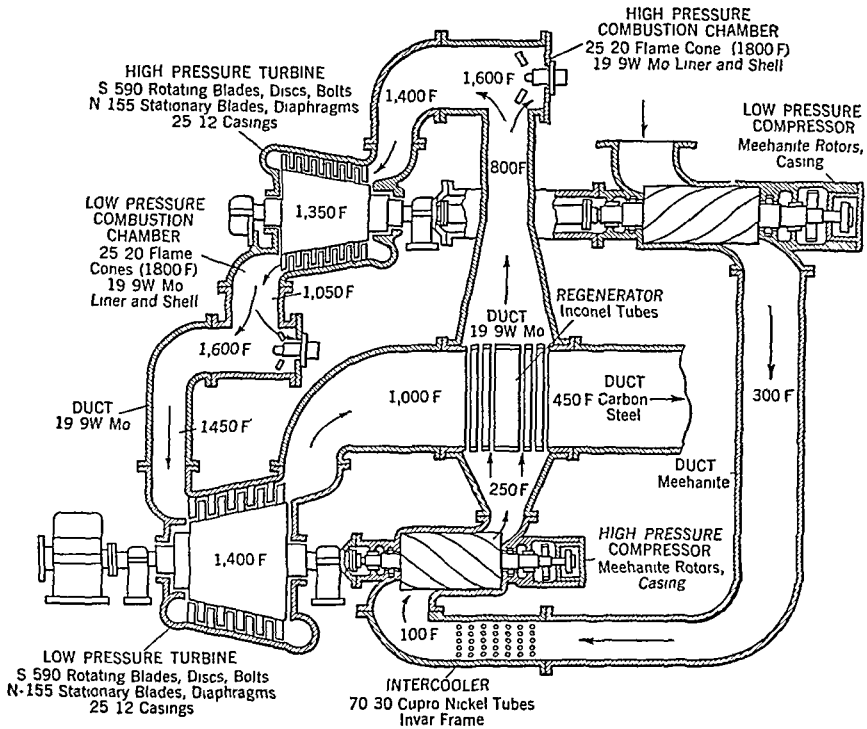


Fig. 18-13. Diagram of 2500-Hp Ship-Propulsion Gas-Turbine Cycle, Showing Materials of Construction and Operating Temperatures

In summation, let it be reiterated that the fundamental thermodynamic fact regarding the gas turbine and its compressor is that of the influence of temperature on work done between two pressures. The designer's goal is to obtain a high temperature of gas entering the turbine for maximum output and a low temperature of air entering the compressor for minimum input. The fundamental thermodynamic validity of this goal is contained in the expression

$$\left(\frac{dh}{ds}\right)_P = T$$

This expression, first noted in a footnote of Art 6-10 has had numerous applications throughout this text. Stated in words, it can be said that the "flare" of the constant pressure lines on the  $H$ - $S$  plane is due to the fact that they become steeper as the temperature rises. Note the many instances of the use of this *fundamental* fact in dealing with thermodynamic problems.

## CHAPTER 19

### STEADY STATE HEAT TRANSFER

**19-1. Fundamental Concepts.**—As previously stated, heat is defined as energy transferred from one substance to another substance because of a temperature difference existing between the two substances. The time rate of this energy transfer, usually expressed in Btu per hour, has its analogies in both the field of hydraulics and the field of electricity. In heat transfer, a flow of Btu per hour against a thermal resistance is due to a temperature difference. In hydraulics, a flow of pounds of fluid per hour against a resistance is due to a pressure difference. In electricity, a flow of amperes against a resistance is due to a voltage difference. Thus, the fundamental concept of heat transfer may be based upon that of flow of Btu through a conductor which offers a resistance.

One often hears of the three methods of heat transfer—conduction, convection, and radiation. Conduction within a substance is described as the transfer of energy by molecular contact. Convection is described as the transfer of energy by mass movements of fluids. Radiation is a transfer of energy by electromagnetic waves. Convection is not actually a separate method, as conduction to or from the fluid is really what constitutes the heat transfer, and the movement of the fluid carries the energy transferred to another location. A household hot-water heating system is a good example of this type of transfer.

**19-2. Ohm's Law Analogy.**—To return to the electrical analogy, consider Ohm's Law, which may be expressed in the form

$$\text{Current, in amperes} = \frac{\text{Potential Change, in volts}}{\text{Resistance, in ohms}} \quad (19-1)$$

Let this law be applied to a d-c circuit, noting carefully the values in Fig. 19-1.

The current in a *series circuit* is the same throughout; and the voltage drop across *parallel* resistance is the same. Now substitute a thermal circuit for an electric circuit; and consider the 100 Btu per hr flow from 1100 F to 710 F through a circuit to which Fig. 19-2 applies. This circuit is similar to that in Fig. 19-1, but temperatures are substituted for voltages.

The part of the circuit in Fig. 19-2 at the left might represent the temperature distribution when a source of heat energy at 1100 F transfers

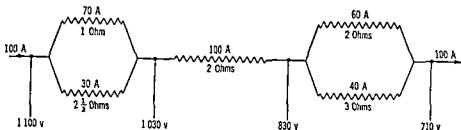


Fig 19-1 Application of Ohm's Law to Series and Parallel Circuits

Btu through a fluid film by conduction and radiation to a wall of a given material with its nearer surface at 1030 F. The resistance of the wall causes a drop of 200 deg before the other surface is reached. Then conduction and radiation through another fluid film cause the Btu to reach the ultimate receiver of heat at 710 F. Compare temperatures and voltages in the two diagrams.

**19-3. Heat Conduction**—Now concentrate on the wall itself in Fig. 19-2. In the application of Ohm's Law in Fig 19-1, the units, volts, amperes, and ohms can be directly applied. In a circuit like that in Fig 19-2, when Btu per hour are substituted for amperes, and degrees temperature are substituted for volts, the resistance can be handled as follows

$$\frac{q}{A} = \frac{\Delta T}{\text{Res}} \quad (19-2)$$

where  $q$ =heat transfer, in Btu per hour,

$A$ =wall area normal to path of heat flow, in square feet,

$\Delta T$ =temperature drop across wall, in degrees F or Rankine,

Res=resistance to heat flow, in square foot-hour-degrees per Btu

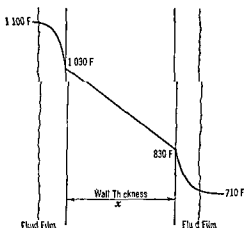


Fig 19-2 Heat Transfer Application of Ohm's Law Analogy

At this point, the units of resistance seem to have little significance. Now consider the following relation between this resistance and the wall thickness  $x$ :

$$\text{Res} = \frac{x}{\lambda} \quad (19-3)$$

where  $x$  = wall thickness, in feet;

$$\lambda = \text{thermal conductivity,}^* \text{ in } \frac{\text{Btu}}{\text{sq ft hr } \frac{\text{deg}}{\text{ft}}}$$

This thermal conductivity  $\lambda$  is a specific item which is a property of the material of which the wall is composed. The units of conductivity may be rearranged as follows:

$$\frac{\text{Btu ft}}{\text{sq ft hr deg}}$$

It is inadvisable to further simplify the units of  $\lambda$ , as an incorrect concept of the nature of this term may result.

Let us return to the quantity  $\frac{q}{A}$ , the units of which are  $\frac{\text{Btu}}{\text{hr sq ft}}$ . This quantity is often called either *heat flux* or *thermal current*.

In the fundamental statement of the units of  $\lambda$ , the denominator contains the fractional term representing "degrees per foot." This can be written  $\frac{dT}{dx}$ , and is known as the *temperature gradient* at any point  $x$ . If the temperature drops at a uniform rate throughout the entire thickness of the wall, as in Fig. 19-2,  $\frac{dT}{dx}$  can be replaced by  $\frac{\Delta T}{x}$ . Under steady flow conditions, this occurs when  $\lambda$  has a constant value, or when an average value of  $\lambda$  is used as a constant value. This latter procedure is quite well justified, as the variation of  $\lambda$  with temperature is nearly linear. In general, when there is only one resistance,

$$\frac{q}{A} = \frac{\Delta T}{\text{Res}} = \frac{\Delta T}{\frac{x}{\lambda}} = \frac{\lambda \Delta T}{x} \quad (19-4)$$

For several resistances in series, as in Fig. 19-3(a) or (b),

$$\frac{q}{A} = \frac{\lambda_1 \Delta T_1}{x_1} = \frac{\lambda_2 \Delta T_2}{x_2} = \frac{\Delta T_{\text{over-all}}}{\Sigma \text{Res}} \quad (19-5)$$

\* The symbol (lambda)  $\lambda$  has been chosen to represent conductivity rather than  $k$ , which appears in much heat transfer literature, to avoid confusion with the adiabatic exponent. Lambda is also used to represent conductivity in Keenan and Kaye's Gas Tables.

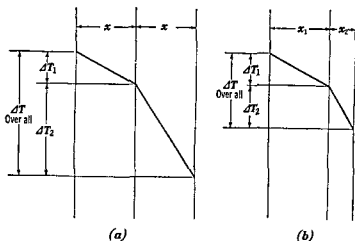


Fig 19-3 Temperature Distribution in Thick and Thin Composite Walls

In either (a) or (b) there are two dissimilar substances. In (a) each substance has the same thickness  $x$ . In (b) the substance with the flatter gradient has a greater thickness than the other substance. Hence, the total change in temperature  $\Delta T$  over-all is greater in (a) than in (b). The effect of insulation thickness on plane walls can be seen here. For instance, let  $\frac{q}{A} = 100 \frac{\text{Btu}}{\text{sq ft hr}}$ ,  $\lambda_1 = 0.4$ , and  $x_1 = 1$  ft. Then, by equation 19-4,  $\Delta T_1 = 250$  deg. If  $\lambda_2 = 0.2$  and  $x_2$  also is 1 ft, as in Fig 19-3 (a), then  $\Delta T_2 = 500$  deg and  $\Delta T_{\text{over-all}} = 750$  deg. If, however,  $\frac{q}{A}$  is unchanged and  $x_2$  is only 0.5 ft, as in (b), then  $\Delta T_2 = 250$  deg and  $\Delta T'_{\text{over-all}} = 500$  deg.

Stated differently, the resistance  $\frac{x_1}{\lambda_1}$  is  $\frac{1}{0.4}$ , or  $2.5 \frac{\text{sq ft hr deg}}{\text{Btu}}$ , for the conditions in Fig 19-3 (a), the resistance  $\frac{x_2}{\lambda_2}$  is  $\frac{1}{0.2}$ , or  $5 \frac{\text{sq ft hr deg}}{\text{Btu}}$ , whereas for Fig 19-3 (b) the resistance  $\frac{x_2}{\lambda_2}$  is  $\frac{0.5}{0.2}$ , or  $2.5 \frac{\text{sq ft hr deg}}{\text{Btu}}$ .

★19-4. Generalized Treatment of Heat Conduction in Solids—The foregoing sections have treated the simplified case of conducted heat transfer. In the first place, the problem was treated as one-dimensional heat flow (i.e., the slab transferring heat was thin in comparison with its length and width and, hence, substantially all the heat passed through the slab and none "leaked" out at the ends). In the second place, steady state heat transfer was also assumed. This means that the solid transferring heat had reached equilibrium conditions. In other words, at no point in



the solid was there any change in temperature with time. This one-dimensional steady state heat flow is encountered in many engineering problems, such as heat flow through building walls, through furnace walls, and through pipes and their insulation.

A differential equation may be written for the very general case of heat transfer through solids. This expression for three-dimensional heat conduction in the unsteady state is:

$$\frac{\partial}{\partial x} \lambda(x, y, z, T) \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \lambda(x, y, z, T) \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} \lambda(x, y, z, T) \frac{\partial T}{\partial z} + q(x, y, z, T, \Theta) = \frac{c}{v}(x, y, z, T) \frac{\partial T}{\partial \Theta} \quad (19-6)$$

where  $x$ ,  $y$ , and  $z$  are linear dimensions,  $\Theta$  (theta) is time,  $c$  is specific heat, and  $v$  is specific volume; the properties  $\lambda$  and  $\frac{c}{v}$  are functions of the coordinates and of the temperature; and  $q$ , a rate of heat supply per unit volume, may also be a function of time. The solution of this expression is quite involved. However, if  $\lambda$ ,  $q$ , and  $\frac{c}{v}$  are substantially constant, the following simplification will be obtained:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{\lambda} = \frac{c}{v\lambda} \frac{\partial T}{\partial \Theta} = \frac{1}{\alpha} \frac{\partial T}{\partial \Theta} \quad (19-7)$$

This introduces  $\alpha$  (alpha), which is called the thermal diffusivity and is  $\frac{v\lambda}{c}$ ; it is expressed in square feet per hour.

If there is no temperature change with respect to time and there is no heat storage, the following further simplification, known as Laplace's equation, results:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (19-8)$$

If boundary conditions can be established, solutions may be obtained. These often involve infinite series. To save time and avoid repetitious calculations, graphs have been constructed for some of the more common engineering applications. These are available in those texts dealing extensively with unsteady state heat flow.

**19-5. Heat Conduction Through Cylinders.**—Up to here, quantitative examples have been confined to steady state heat flow where  $A$  has been constant. The steady state studies will be continued, but with the area variable. One immediate application of this is in the field of thick-walled

cylinders with radial heat flow, as in Fig 19-4 Here the inside and outside diameters are  $D_i$  and  $D_o$ , and the wall thickness is

$$x = \frac{D_o - D_i}{2}$$

Hence the resistance to heat flow is

$$\text{Res} = \frac{x}{\lambda} = \frac{D_o - D_i}{2\lambda}$$

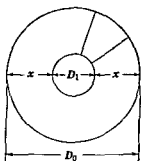


Fig 19-4 Thick Walled Cylinder

The heat flux, or thermal current,  $\frac{q}{A}$ , is based on the mean area  $A_m$  (mean circumference times length of pipe) so that

$$\frac{q}{A_m} = \frac{\Delta T}{\text{Res}} = \frac{\Delta T}{\frac{D_o - D_i}{2\lambda}} = \frac{2\lambda \Delta T}{D_o - D_i} \quad (19-9)$$

The mean circumference, as found by integration, is

$$\pi D_m = \pi \left[ \frac{D_o - D_i}{\ln \left( \frac{D_o}{D_i} \right)} \right] \quad (19-10)$$

Therefore, the heat transfer, in Btu per hour, is

$$q = \frac{2\lambda \Delta T}{D_o - D_i} A_m = \frac{2\lambda \Delta T}{D_o - D_i} L \pi \left[ \frac{D_o - D_i}{\ln \left( \frac{D_o}{D_i} \right)} \right] = \frac{2\pi \lambda L \Delta T}{\ln \left( \frac{D_o}{D_i} \right)} \quad (19-11)$$

where  $L$  is the length of pipe, in feet This equation is also very convenient for considering the heat transfer per foot of length

\* The symbol  $\ln$  is used to designate the natural logarithm or  $\log_e$ .

Since  $\frac{q}{A_m}$  usually is not the ultimate object desired, but  $\frac{q}{A_o}$  or  $\frac{q}{A_i}$  frequently is sought, the following conversions are of interest:

$$\frac{q}{A_o} = \frac{q}{A_m} \times \frac{\pi D_m L}{\pi D_o L} = \frac{q}{A_m} \times \frac{D_m}{D_o} \quad (19-12)$$

and

$$\frac{q}{A_i} = \frac{q}{A_m} \times \frac{D_m}{D_i} \quad (19-12a)$$

A similar relation may also be derived by dividing the heat transfer per foot of length by the appropriate circumference. Thus,

$$\frac{q}{L\pi D_o} = \frac{2\pi\lambda L\Delta T}{L\pi D_o \ln\left(\frac{D_o}{D_i}\right)} = \frac{2\lambda\Delta T}{D_o \ln\left(\frac{D_o}{D_i}\right)} \quad (19-13)$$

At this point it is well to keep in mind the fact that the *Btu per hour* are the same for both outside and inside, and common sense should show that

$$\frac{q}{A_o} < \frac{q}{A_m} < \frac{q}{A_i} \quad (19-12b)$$

Likewise, consider the magnitudes of the figures concerning  $A_m$ . As  $D_i$  approaches  $D_o$  in value, the logarithm of  $\frac{D_o}{D_i}$  approaches 0, and the logarithmic mean approaches the arithmetic mean. In fact, personal errors may make the logarithmic method inaccurate; it is well to check one's figures by keeping the value of the arithmetic mean diameter in mind. When the inner diameter is within 80 per cent of the outer diameter, the arithmetic mean will serve for most purposes.

As an illustration of heat flow through a thick cylinder, let  $\lambda=0.06$ ,  $T_i=400$  F,  $T_o=200$  F,  $D_i=6$  in., and  $D_o=12$  in. Then the wall thickness will be  $x=3$  in., or 0.25 ft. The thermal current based on mean area is

$$\frac{q}{A_m} = \frac{400-200}{\frac{0.25}{0.06}} = 48 \frac{\text{Btu}}{\text{sq ft hr}}$$

The arithmetic mean diameter would be 0.75 ft, and the true mean diameter is

$$D_m = \frac{1-0.5}{\ln\left(\frac{1}{0.5}\right)} = \frac{0.5}{\ln 2} = 0.721 \text{ ft}$$

The heat flow per linear foot of pipe length would be

$$48 \times 0.721\pi = 109 \frac{\text{Btu}}{\text{ft hr}}$$

The thermal current based on outside area would be  $\frac{109}{\pi}$ , or  $34.6 \frac{\text{Btu}}{\text{sq ft hr}}$

Note that this result would have been obtained by correcting  $\frac{q}{A_m}$  by the diametral ratio, according to equation 19-12. Thus,  $48 \times \frac{0.721}{1} = 34.6$ . Likewise,  $\frac{q}{A_1}$  would be  $48 \times \frac{0.721}{0.5}$ , or  $69.2 \frac{\text{Btu}}{\text{sq ft hr}}$

**19-6. Complex Non-Plane Walls**—When complex plane walls were being studied, it was seen that since  $A$  was constant, the thermal current could be evaluated as

$$\frac{q}{A} = \frac{\Delta T_{\text{over-all}}}{\Sigma \text{Res}}$$

When  $A$  is variable, the thermal current may be computed by converting each resistance to the same area basis. The area chosen, for example, may be the outer cylindrical surface  $A_3 = \pi D_3 L$

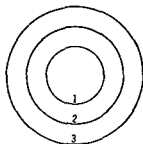


Fig 19-5 Complex Non-Plane Walls

For the conditions in Fig 19-5, let  $D_{1m2}$  be the mean diameter between  $D_1$  and  $D_2$ , and let  $D_{2m3}$  be the mean diameter between  $D_2$  and  $D_3$ . Then,

$$\begin{aligned} \frac{q}{A_3} &= \frac{\Delta T_{\text{over-all}}}{\left(\frac{D_2 - D_1}{2 \lambda_2}\right) \left(\frac{D_3}{D_{1m2}}\right) + \left(\frac{D_3 - D_2}{2 \lambda_3}\right) \left(\frac{D_3}{D_{2m3}}\right)} \\ &= \frac{\Delta T_{\text{over-all}}}{\left(\frac{D_2 - D_1}{2 \lambda_2}\right) \left[\frac{D_3}{\frac{D_2 - D_1}{\ln\left(\frac{D_2}{D_1}\right)}}\right] + \left(\frac{D_3 - D_2}{2 \lambda_3}\right) \left[\frac{D_3}{\frac{D_3 - D_2}{\ln\left(\frac{D_3}{D_2}\right)}}\right]} \end{aligned}$$

TABLE 19-1  
TYPICAL VALUES OF  $\lambda$ , EXPRESSED IN  $\frac{\text{Btu}}{\text{sq ft hr deg F}}$

Substance	Temperature, deg F					
	100	400	700	1000	1500	2500
	Thermal Conductivity, $\lambda$					
Aluminum	132	131	131	.....	....	....
Copper	220	215	210	205	....	....
Mild steel	26.4	25.6	24.0	21.9	....	....
4-6% Cr, $\frac{1}{2}$ % Mo alloy steel	.....	20.6	19.7	19.2	....	....
First quality firebrick	.358	.466	.575	.683	.866	1.23
Insulating firebrick	.....	.129	.146	.163	.191	....
High-temperature block insulation	.057	.059	.061	.062	.064	....
85% magnesia insulation	.034	.043	.052	.....	....	....
Liquid water (saturated)	.36	.38	.23	.....	....	....
Mercury (liquid)	4.85	4.85	4.85	.....	....	....
Steam (low pressure)	.014	.020	.028	.050	....	....
Air	.0158	.0227	.0292	.0362	....	....
Hydrogen	.108	.....	.....	.....	....	....

or

$$\frac{q}{A_3} = \frac{2 \Delta T_{\text{over-all}}}{D_3 \left[ \frac{\ln\left(\frac{D_2}{D_1}\right)}{1\lambda_2} + \frac{\ln\left(\frac{D_3}{D_2}\right)}{2\lambda_3} \right]} \quad (19-14)$$

Again it can be said that if the temperature change across any of the individual resistances is known, the heat flow can be evaluated by considering that resistance alone. Values of  $\lambda$  for various materials are given in Table 19-1.

**19-7. Convection.**—When heat is transferred by convection, currents are set up in a fluid (liquid or gas) so that mass movement results in a continual sweep of the fluid over solid boundaries. The conduction of heat from the molecules of the fluid to those of the solid differs from the previous concept of conduction in that the fluid molecules are in mass motion.

Convection may be divided into two classes: natural convection and forced convection. If a sphere is heated above room temperature and suspended by a wire from the ceiling, the molecules of air surrounding the sphere will undergo an increase in temperature. The heated air together with the cooler room air will set up an upward air flow around the sphere because of a natural density difference. This air flow will

continue as long as the sphere is warmer than the room air, and heat will continually be removed by mass flow of the molecules. This form of convection is known as natural, or free, convection.

If air were blown over the sphere by a fan, forced convection would result. In all cases of forced convection, some external means is employed to set up mass movements of the heating or cooling fluid. The hot gases sweeping over the convection elements of a steam generator are set in motion by stack draft or fans, this is an example of forced convection.

**19-8 Natural Convection**—When heat is removed from, or imparted to, the surface of a body by natural convection currents which it sets up in a surrounding fluid at a different temperature, the rate of transfer depends on its size, shape, and inclination to the vertical, as well as on the nature of the fluid and the temperature difference. Furthermore, the heat transfer may be modified by the presence of other surfaces so disposed as to interfere with convection streams.

It is possible, by means of dimensional analysis, to combine those variables affecting the heat transfer by convection into several dimensionless groups. (A dimensionless group is such a combination of variables that when consistent units are used for them all, the units of the variables cancel out, leaving a quantity without dimensions.) The dimensionless groups may be treated as variables, and the number of variables involved is thus reduced. After the dimensional groups are set up, it becomes necessary to establish the relationship between the groups by experiment. Such dimensional groups may be set up for natural convection heat transfer. However, these expressions are rather involved and there is considerable doubt about the proper relationships between the dimensionless groups under many conditions. For this reason empirical or semi empirical equations are frequently used for natural-convection heat transfer, particularly when the fluid involved is air.

Several investigators have developed expressions for the heat loss from bodies of different shapes surrounded by air. R. H. Heilmann gives the following expression for natural convection loss in still atmospheric air

$$\frac{q}{A} = \frac{C(\Delta T)^{1/4}}{(d)^{0.2}(T_{avg})^{0.181}} \frac{\text{Btu}}{\text{sq ft hr}} \quad (19-15)$$

Where  $T_{avg}$  = average of the absolute surface and ambient air temperatures, in degrees Rankine,

$d$  = diameter or height or width of the surface, in inches

$C$  = shape factor. A change in the dimension  $d$  has little effect when the value exceeds 24 in., i.e., the heat lost by an object of large dimensions is assumed to be the same as that for  $d = 24$  in.

The shape factor  $C$  has the following values:

Horizontal cylinders	1.016
Long vertical cylinders	1.235
Vertical flat plates	1.394
Horizontal plates facing upward	1.79
Horizontal plates facing down	0.89
Spheres	1.82

If we divide the thermal current by  $\Delta T$ , we get a value designated by  $\bar{h}_c$ . Thus,

$$\bar{h}_c = \frac{q}{A\Delta T} = \frac{C(\Delta T)^{0.266}}{(d)^{0.2}(T_{avg})^{0.181}} \frac{\text{Btu}}{\text{sq ft hr deg}} \quad (19-16)$$

This quantity  $\bar{h}_c$  is the convection heat-transfer film coefficient at the surface where convection is taking place. *Note carefully the units in which  $\bar{h}_c$  is expressed* and how they differ from those in which  $\lambda$  is expressed. Some authorities further distinguish between these terms by pointing out that one refers to *conductance* and the other to *conductivity*.

The symbol  $h$  is now so widely used as the term for enthalpy as well as the term for heat-transfer film coefficient that a word of caution in the application of nomenclature is included here. In this text a "bar" is used above the letter  $h$  designating the heat-transfer film coefficient. Although the ultimate aim should be to enable the student to understand the terms without being tied down to the symbols used, it is considered helpful to free the novice from the handicap of nomenclature confusion. Once mastery of the subject has been gained, the "bar  $h$ " may be dropped in favor of the standard  $h$  widely used in current engineering literature.

**19-9. Radiation.**—This third mechanism for heat transfer may be defined as the transfer of energy by means of electromagnetic waves due to temperature excitation. These waves require no medium for their propagation, being as readily transferred across a void as across an air space. They include X-rays and radio waves as well as the infra-red rays commonly assigned to the band titled "heat radiation." Further examination of this band would show that most of the radiant heat transfer occurs at wavelengths that are not visible to the eye. A large number of engineering problems involve the transfer of radiant energy from one surface to another across an intervening space. If a surface is in contact with a gas, energy is also transferred by convection; then both processes are operating in *parallel* and should be separately calculated for the total heat transfer.

Boltzman showed by thermodynamic reasoning that energy is radiated from a body in proportion to the fourth power of its absolute temperature.

Stefan had previously indicated this by use of experimental data. The resulting Stefan-Boltzman equation is

$$\frac{q}{A} = 17.30 \times 10^{-10} T^4 \quad (19-17)$$

where  $\frac{q}{A}$  is the energy radiated, in Btu per sq ft hr, and  $T$  is the temperature of the radiating body, in degrees F abs. The constant  $17.30 \times 10^{-10}$  has been determined experimentally. The expression in this form is for "black body" radiation only.

A "black body" is an ideal emitter and absorber, in that all radiant energy falling upon it is absorbed. Some substances approach the ideal "black body," while others are far from being perfect emitters or absorbers.

Radiant energy falling upon a transparent body may be absorbed, reflected, or transmitted through the body. If  $a$  is the proportion of the radiant energy received which is absorbed,  $r$  is the proportion reflected, and  $t$  is the proportion transmitted, then  $a + r + t = 1$ . A glass plate would be an illustration of this case. Most substances are opaque to radiation, and that portion not absorbed is reflected. In such a case,  $a + r = 1$ .

The ratio of the portion absorbed to the total radiation falling upon a body is known as the absorptivity. The maximum rate of emitting energy is then had with a "black body." When other materials are used, the Stefan-Boltzman expression is modified by an emissivity factor which is less than unity. Then

$$\frac{q}{A} = 17.30 \times 10^{-10} e T^4$$

The factor  $e$  varies with the material, temperature, and character of the emitting surface and must be determined experimentally. Values for various surfaces are given in Table 19-2.

This emissivity factor compares the energy emitted by an actual body to that of a "black body" at the same temperature. (It should be noted that the absorptivity equals the emissivity when the body temperature is the same in each case and for radiation of the same wavelength.)

**19-10 Interchange by Radiation**—A body at any temperature above absolute zero radiates heat to its surroundings, even though they may exist at a higher temperature than the body itself. The surroundings would then radiate to the body at a greater rate, with the resulting transfer of energy to the body from its surroundings. This transfer leads to the following general expression for calculating the net energy interchange by radiation

$$q = 17.30 \times 10^{-10} F_1 F_2 A (T_1^4 - T_2^4) \quad (19-18)$$



TABLE 19-2  
EMISSIVITY RATIOS FOR SURFACES

Surface	Temperature Degrees F	Emissivity Ratio
Aluminum plate, polished	73	0.040
Brass plate, polished	100-600	0.096
Brass plate, dull	120-660	0.22
Brass plate, oxidized	1110	0.59
Cast-iron plate, polished	392	0.21
Cast-iron plate, rusted	67	0.68
Cast-iron plate, rough and oxidized	100-480	0.95
Steel plate, rough and oxidized	100-700	0.94-0.97
Silver, polished	100-700	0.022-0.031
Asbestos paper	100-700	0.93-0.945
Plaster, rough lime	50-190	0.91
Roofing paper	69	0.91

In this equation  $T_1$  is often taken as the higher of the two surface temperatures;  $F_a$  is an arrangement factor, having a value between zero and unity, which must be used to allow for the relationship of the two surfaces with regard to the shape and position of each; and  $F_e$  is evaluated by properly considering the emissivity and arrangement of both surfaces. (It is customary practice for engineering problems to take the individual emissivity values at the higher of the two surface temperatures.) Since the expression now includes an area term  $A$ ,  $q$  is the energy radiated, in Btu per hour.

**19-11. Arrangement Factor.**—When dealing with parallel plane surfaces (theoretically infinite in extent), the radiating area of one is taken equal to the corresponding area of the other. Hence either area may be used for evaluating  $A$ . The total energy lost by one surface is then gained by the other. When dealing with enclosed bodies,  $A$  is the area of the enclosed body and not the area of the surrounding surfaces. In this case  $A$  is usually associated with  $T_1$  (whether the higher or lower of the surface temperatures).

Two common cases are (1) parallel planes and (2) the small body completely surrounded by a large enclosure. For both of these cases  $F_a = 1$ . Then,

$$q = 17.30 \times 10^{-10} A_1 (T_1^4 - T_2^4) \left[ \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2} \left( \frac{1}{e_2} - 1 \right)} \right] \quad (19-18a)$$

where  $A_1$  and  $e_1$  are the area and emissivity of the small body, and  $A_2$  and  $e_2$  refer to the large body.

If  $A_2$  is very much larger than  $A_1$ , the term in the bracket, which represents  $F_e$ , reduces to  $e_1$ . Hence, for the enclosed body case,  $F_e$  is equal to the emissivity factor of the enclosed body

For parallel planes,

$$F_e = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \quad (19-19)$$

where  $e_1$  and  $e_2$  are the surface emissivity values for the two planes. However, if both  $e_1$  and  $e_2$  exceed 0.90, it may be assumed that

$$F_e \approx e_1 e_2 \quad (19-19a)$$

There are many published values of the emissivity of different materials determined at various temperatures. The character of the surface is important. A polished surface may have a low emissivity factor, but if the surface is oxidized, roughed, or in any other way loses its polish the magnitude will undergo a marked increase. Some judgment is required to make the proper selection. For non-metallic materials, if no information is available, a value of 0.9 may be used. Because of surface conditions, no general statement can be applied to metals.

The following two forms of the Stefan Boltzman equation are of convenience mathematically

$$q = 0.173 F_e F_a A \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \quad (19-18b)$$

$$q = 1730 F_e F_a A \left[ \left( \frac{T_1}{1000} \right)^4 - \left( \frac{T_2}{1000} \right)^4 \right] \quad (19-18c)$$

The use of these forms will simplify slide rule calculations. Note the ciphers carefully.

If we divide the energy by  $A \Delta T$ , we obtain a quantity designated as  $\bar{h}_r$ . Thus

$$\bar{h}_r = \frac{q}{A \Delta T} = \frac{0.173 F_e F_a \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]}{T_1 - T_2} \frac{\text{Btu}}{\text{sq ft hr deg}} \quad (19-20)$$

This equation defines the surface-film coefficient  $\bar{h}_r$  due to radiation, just as equation 19-16 defines the coefficient  $\bar{h}_c$  due to convection. If, as often occurs, radiation and convection occur in parallel heat flow, the combined film coefficient  $\bar{h}$  is  $\bar{h}_r + \bar{h}_c$ . It is often valuable to know the approximate range of values of this combined  $\bar{h}$  in making trial and error solutions. The following set of experimental values of combined  $\bar{h}$  from a 3.5 in. O.D. horizontal tube to still air at ordinary room temperature is of interest.

$\Delta T$	100	200	300	400	500	600
$\bar{h}$	2.25	2.73	3.31	4.03	4.85	5.71

The hourly heat loss by both radiation and convection is:

$$q = \bar{h} A \Delta T \quad (19-21)$$

*Example 19-1.*—A 6-in. steel pipe (O.D. = 6.625) is carrying saturated steam at 140 psia. The pipe is covered with an insulation having a thickness of 5 in. The outer surface temperature is 100 F, and the room air temperature is 80 F. The pipe is horizontal and the surface emissivity is 0.92. For each square foot of outer surface of insulation, calculate:

- Hourly heat loss by natural convection.
- Hourly heat loss by radiation.
- Total hourly heat loss.
- Combined air-film coefficient  $\bar{h}$ .
- Mean conductivity of insulation.

*Solution.*—(a) The outside of the steel pipe is the inside diameter of the insulation. Therefore,  $x$  being 5 in., the insulation O.D. is 16.625 in. The convective heat loss from the outer surface, per square foot of outer surface, is

$$\frac{q_c}{A_o} = \bar{h}_c(100 - 80) = \frac{1.016(20)^{0.266}(20)}{(16.625)^{0.2}(550)^{0.181}} = 8.23 \frac{\text{Btu}}{\text{sq ft hr}}$$

- (b) The radiant heat transfer from the same surface is

$$\frac{q_r}{A_o} = 0.173 \times 0.92 \left[ \left( \frac{100 + 460}{100} \right)^4 - \left( \frac{80 + 460}{100} \right)^4 \right] = 21.55 \frac{\text{Btu}}{\text{sq ft hr}}$$

- (c) The total heat loss is

$$\frac{q}{A_o} = 29.78 \frac{\text{Btu}}{\text{sq ft hr}}$$

- (d) The combined air-film coefficient is

$$\bar{h} = \frac{29.78}{20} = 1.489 \frac{\text{Btu}}{\text{sq ft hr deg}}$$

(e) In the absence of data to the contrary, it is practical to assume that the inside temperature of the insulation is the same as that of the steam, or 353 F. (As the steam film and the steel pipe wall have comparatively little resistance to heat transfer, the above assumption is valid.) By equation 19-13,

$$29.78 = \frac{2\lambda(353 - 100)}{\frac{16.625}{12} \ln \left( \frac{16.625}{6.625} \right)}$$

from which

$$\lambda = 0.075 \frac{\text{Btu ft}}{\text{sq ft hr deg}}$$

**19-12. Forced Convection.**—Forced convection takes heat transfer into the field of fluid mechanics, where the flow is divided into that of laminar type and that of turbulent type. Forced convection with laminar flow is the less common type, and usually results in an uneconomical design; hence, this work will be confined to turbulent flow, which has a wide application. By dimensional analysis, certain variables of flow and physical properties have been found to so group themselves that their application to heat-transfer work has become widespread. These groups, when used with consistent units, form dimensionless numbers, as follows:

$$N_{Nu} = \text{Nusselt Number} = \frac{\text{Film Coefficient} \times \text{Linear Dimension}}{\text{Conductivity of Fluid}} = \frac{\bar{h}D}{\lambda}$$

$$N_{Re} = \text{Reynolds Number} = \frac{\text{Linear Dimension} \times \text{Mass Flow Velocity}}{\text{Dynamic Viscosity of Fluid}} = \frac{DG}{\mu}$$

$$N_{Pr} = \text{Prandtl Number} = \frac{\text{Specific Heat of Fluid} \times \text{Dynamic Viscosity}}{\text{Conductivity of Fluid}} = \frac{c_p \mu}{\lambda}$$

In these dimensionless numbers,  $G$  represents the mass flow velocity. It is obtained by dividing linear velocity (feet per hour) by specific volume (cubic feet per pound), giving pounds per hour per square foot, it may also be visualized as dividing the flow in pounds per hour by the cross sectional area of the channel in square feet. Dynamic viscosity  $\mu$  (mu) is called absolute viscosity by some writers, and is easily handled in the units of pounds per hour foot. When using unfamiliar viscosity tables from outside sources, it is advisable to put down all units, if the resulting expression is not dimensionally sound, the use of a proper gravitational constant will usually rectify it. Some sources, for instance, give dynamic viscosity values in pound hours per square foot.

In computing the Reynolds number, some students of fluid mechanics prefer to use linear velocity and kinematic viscosity. As kinematic viscosity is defined as dynamic viscosity divided by density giving square feet per hour, the computation of the dimensionless feature of the Reynolds number is here left as an exercise.

**19-13 Forced Convection with Turbulent Flow of a Fluid Flowing Inside a Tube**—This is one of the most frequently encountered cases and one upon which much experimental work has been performed. Turbulent flow of a fluid which completely fills a section of circular tube occurs whenever the value of the Reynolds number exceeds 3 000.

The most common modification of the Nusselt expression (in terms of Reynolds and Prandtl numbers) for turbulent flow of gases, water, and oils flowing in circular pipes when the Reynolds number exceeds 3 000 is

$$N_{Nu} = C N_{Re}^m N_{Pr}^n \quad (19-22)$$

or

$$\frac{\bar{h}D}{\lambda} = 0.023 \left( \frac{DG}{\mu} \right)^{0.8} \left( \frac{c_p \mu}{\lambda} \right)^n \quad (19-22a)$$

The symbols have the following meaning

$\bar{h}$  = heat-transfer film coefficient, in  $\frac{\text{Btu}}{\text{sq ft hr deg}}$ ,

$D$  = internal diameter, in feet,

$\lambda$  = thermal conductivity, in  $\frac{\text{Btu ft}}{\text{sq ft hr deg}}$ ,

TABLE 19-3  
VALUES FOR AIR

Absolute Temperature Degrees Rankine	Dynamic Viscosity $\mu$ $\frac{\text{lb}}{\text{hr ft}}$	Thermal Conductivity $\lambda$ $\frac{\text{Btu}}{\text{sq ft hr } \frac{\text{deg}}{\text{ft}}}$	Prandtl Number $\frac{c_p \mu}{\lambda}$
400	0.0360	0.0118	0.73
500	0.0425	0.0143	0.71
600	0.0486	0.0168	0.70
700	0.0544	0.0191	0.68
800	0.0598	0.0213	0.68
900	0.0644	0.0237	0.67
1000	0.0691	0.026	0.66
1100	0.0738	0.028	0.66
1200	0.0785	0.030	0.66
1300	0.0828	0.032	0.66
1400	0.0871	0.035	0.65
1500	0.0911	0.037	0.65

$G$  = mass flow velocity, in  $\frac{\text{lb}}{\text{hr sq ft (of cross-section)}}$ ;

$\mu$  = dynamic viscosity, in  $\frac{\text{lb}}{\text{hr ft}}$ ;

$c_p$  = specific heat, in  $\frac{\text{Btu}}{\text{lb deg}}$ ;

$n = 0.4$  for heating and  $0.3$  for cooling.

Values of  $\mu$ ,  $\lambda$ , and the Prandtl number for air and water at various temperatures are given in Tables 19-3 and 19-4.

Fluid properties are evaluated at the mean temperature of the substance. For many gases, including air, the Prandtl number to the  $n$ th power is close to  $0.9$ . So, for *gases only*, equation 19-22a reduces to

$$\frac{\bar{h}D}{\lambda} = 0.0207 \left( \frac{DG}{\mu} \right)^{0.8} \quad (19-22b)$$

*Example 19-2.*—Water flows through a condenser tube of  $\frac{3}{4}$  in. O.D. with an average velocity of 10 ft per sec. The average temperature of the water is 60 F. The tube has a wall of 18 BWG (Birmingham wire gage) metal, its thickness being 0.049 in. and its I.D. being 0.652 in. What is the value of the heat-transfer film coefficient  $\bar{h}$ ?

*Solution.*—Tables of tube sizes for various BWG are listed in engineering handbooks. From Table 19-4,  $\mu = 2.71$ ,  $\lambda = 0.344$ , and  $N_{Pr} = 7.9$ . Also,

$$G = 10 \times 3600 \times 62.4 = 2,250,000 \frac{\text{lb}}{\text{hr sq ft}}$$

and

$$N_{Re} = \frac{DG}{\mu} = \frac{0.652}{12} \times \frac{2,250,000}{2.71} = 45,200$$

TABLE 19-4  
VALUES FOR WATER

Deg F	$\mu$ $\frac{\text{lb}}{\text{hr ft}}$	$\lambda$ $\frac{\text{Btu}}{\text{sq ft hr } \frac{\text{deg}}{\text{ft}}}$	Prandtl Number
32	4.33	0.327	13.4
40	3.75	0.332	11.3
50	3.17	0.338	9.4
60	2.71	0.344	7.9
68	2.42	0.348	6.9
70	2.37	0.349	6.8
80	2.08	0.355	5.8
90	1.85	0.360	5.2
100	1.65	0.364	4.5
110	1.49	0.368	4.0
120	1.36	0.372	3.6
130	1.24	0.375	3.3
140	1.14	0.378	3.0
150	1.04	0.381	2.7
160	0.97	0.384	2.5
170	0.90	0.386	2.3
180	0.84	0.389	2.2
190	0.79	0.390	2.1
200	0.74	0.392	1.9
220	0.65	0.395	1.7
240	0.59	0.396	1.5
260	0.53	0.396	1.4
280	0.48	0.396	1.3
300	0.45	0.395	1.2
350	0.38	0.391	1.0
400	0.33	0.384	1.0
450	0.29	0.373	0.9
500	0.26	0.256	0.8

By equation 19-22a,

$$\frac{\bar{h}D}{\lambda} = 0.023(45,200)^{0.4}(7.9)^{0.4} = 0.023(5260)(2.29) = 277$$

Hence,

$$\bar{h} = \frac{0.344 \times 12}{0.652} \times 277 = 1750 \frac{\text{Btu}}{\text{sq ft hr deg}} \quad \text{Ans}$$

*Example 19-3*—If the tube in example 19-2 is 14 ft long and its average temperature is 70 F, how many Btu per hour are transferred?

*Solution*—The heat transferred per hour for each square foot of surface is

$$\bar{h}\Delta T = 1750(70 - 60) = 17,500 \text{ Btu}$$

The surface area, which is that of a cylinder, is

$$\pi DL = \pi \left( \frac{0.652}{12} \right) (14) = 2.39 \text{ sq ft}$$

Then

$$q = (2.39)(17,500) = 41,800 \frac{\text{Btu}}{\text{hr}} \quad \text{Ans.}$$

*Example 19-4.*—If saturated steam at an average temperature of 80 F condenses on the tube in example 19-2, how many pounds per hour of dry saturated steam may be thus condensed?

*Solution.*—The enthalpy of evaporation of steam at 80 F is 1048.6 Btu per lb. Hence, the amount condensed is

$$\frac{41,800}{1048.6} = 39.8 \text{ lb per hr} \quad \text{Ans.}$$

The temperature rise of the water in example 19-2 due to its receiving heat from the steam is 8 deg. The calculation is left as an exercise. Discuss the number of pounds of water required per pound of steam condensed.

**19-14. Fluid-to-Fluid Heat Transfer.**—Refer to Figs. 19-1 and 19-2. In a heat exchanger two fluids are separated by a metallic-tube wall. The fluids may be heating, cooling, condensing, or boiling. As heat is transferred from one fluid to the other, it must pass through a number of resistances: (1) a fluid film on the hot side, (2) a metallic wall, and (3) a fluid film on the cold side. These individual resistances, each of which is the reciprocal of the heat-transfer film coefficient, are properly combined to give the over-all coefficient of heat transfer which is used as follows:

$$q = UA(\text{MTD}) \quad (19-23)$$

where  $q$  is the heat transferred, in Btu per hr;

$U$  is the over-all coefficient of heat transfer, in  $\frac{\text{Btu}}{\text{sq ft hr deg}};$

$A$  is the effective heat-transfer area, in sq ft;

MTD is the mean temperature difference between the two fluids, in deg F.

If  $\bar{h}_a$  is the heat-transfer film coefficient for the hot fluid and  $\bar{h}_b$  is the coefficient for the cold fluid,

$$U = \frac{1}{\Sigma \text{Res}} = \frac{1}{\frac{l}{\bar{h}_a} + \frac{x}{\lambda} + \frac{l}{\bar{h}_b}} \quad (19-24)$$

All resistances must be based on the same area.

*Example 19-5.*—A steel plate,  $\frac{1}{2}$  in. thick and having a conductivity of 26, has a liquid on one side and a gas on the other side. The liquid is at 150 F and has an  $\bar{h}$  of 200; the gas is at 500 F and has an  $\bar{h}$  of 2. How many Btu per hour are transferred by each square foot of plate surface?

*Solution.*—The resistance of the plate is  $\frac{0.25}{12 \times 26} = 0.0008$ ; the resistance of the liquid film is  $\frac{1}{200} = 0.0050$ ; and the resistance of the gas film is  $\frac{1}{2} = 0.5000$ . The total resistance is then 0.5058, and  $U = \frac{1}{0.5058} = 1.977$ .

The over all temperature difference is  $500 - 150 = 350$  deg, and

$$q = 1977 \times 350 = 692 \frac{\text{Btu}}{\text{hr sq ft}} \quad \text{Ans}$$

**Example 19-6**—What is the temperature of the plate in example 19-5 at the surface in contact with the liquid?

**Solution**—The ratio of the resistance of the liquid film to the total resistance is  $\frac{0.0050}{0.5058} = 0.00988$ , or 0.988%. Hence, according to the principle of Ohm's Law, the temperature drop across the liquid film will be  $350 \times 0.00988 = 3.46$  deg. Then the temperature of the plate will be  $150 + 3.46 = 153$  F. Ans

Note the significance of the liquid film in controlling the plate temperature. Can you see why boiler tubes burn if the water level is not properly maintained?

If the plate in example 19-5 were made of copper instead of steel the amount of heat transferred would not be appreciably different. The discussion of this is left as an exercise.

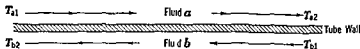


Fig 19-6 Fluids in Counter Flow Separated by Tube Wall

**19-15 Mean Temperature Difference for a Fluid Cooling and a Fluid Heating**—Consider a single tube selected from a bank making up a heat exchanger, as indicated in Fig 19-6. The two fluids are flowing in opposite directions, this type of flow is termed counter flow. Fluid a flows into the tube at temperature  $t_{a1}$  and flows out at temperature  $t_{a2}$ , having been cooled. Fluid b flows parallel to the tube axis on the outside, and its temperature rises from  $t_{b1}$  to  $t_{b2}$ . A typical cross-section will give the temperature distribution shown in Fig 19-7. Note how this chart resembles that in Fig 19-2.

There is resistance to the flow of fluid through the buffer zones and the zones of laminar flow. Gases, even in turbulent flow, offer a high resistance

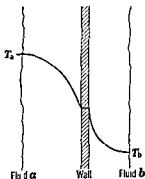


Fig 19-7 Temperature Drop Through Fluid Films

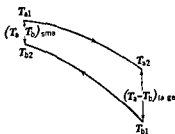


Fig 19-8 Counter Flow Temperature Distribution



to heat flow, while condensing vapors and boiling liquids offer little resistance. For gases the resistance of the metallic wall may be neglected, while for condensing vapors it must be considered.

The temperature distributions along the axis are usually plotted, as indicated in Fig. 19-8, for the purpose of obtaining the mean temperature difference. This difference may be found by either of the following relations:

$$\text{Arithmetic mean temp. diff. (MTD)} = \frac{TD_{\text{large}} + TD_{\text{small}}}{2} \quad (19-25)$$

$$\text{Logarithmic mean temp. diff. (LMTD)} = \frac{TD_{\text{large}} - TD_{\text{small}}}{\ln \left( \frac{TD_{\text{large}}}{TD_{\text{small}}} \right)} \quad (19-26)$$

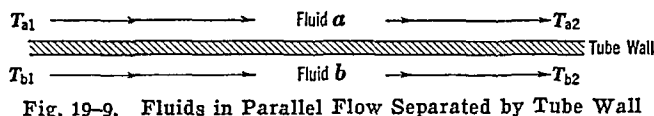


Fig. 19-9. Fluids in Parallel Flow Separated by Tube Wall

It is safer not to use the arithmetic mean difference, since this difference often yields results which depart from the true difference by an appreciable amount. The logarithmic mean difference is the better expression and should, in general, be used for all calculations. This expression can be used where each fluid exists in only *one* state in the heat exchanger. If a fluid should boil and then superheat, only boiling is considered and the superheating is neglected or compensated.

When parallel flow occurs, the two fluids on opposite sides of the tube flow in the same direction, as indicated in Fig. 19-9.

The logarithmic mean temperature difference is calculated as in counter-flow, as indicated in Fig. 19-10. Of the two types of flow, parallel flow gives less efficient use of the heat-transfer area, since for equal areas

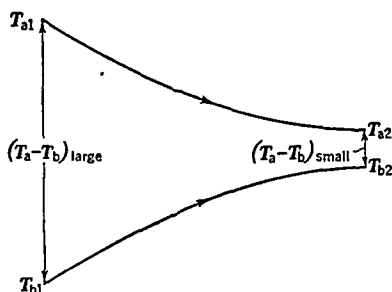


Fig. 19-10. Parallel-Flow Temperature Distribution

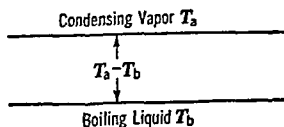


Fig. 19-11. Constant Temperature Difference in Evaporator

parallel flow allows a smaller quantity of heat to be exchanged per hour. Parallel flow is used where requirements other than efficient heat transfer are paramount, such as temperature control.

Economizers, air heaters, and convection superheaters fall into the class of heat exchangers in which both fluids change temperature. The first two types of exchangers practically always involve counterflow, while superheaters may employ parallel flow of fluids to reduce metal temperature below a maximum upper working limit.

**19-16. Mean Temperature Differences for Other Conditions.**—In some heat exchangers, such as evaporators and condenser-boilers of mercury-steam binary vapor cycles, the fluids are a condensing vapor and a boiling liquid. Since the temperatures of both fluids remain constant, as indicated in Fig. 19-11, the MTD is merely the difference between these temperatures.

In other heat exchangers, such as condensers and feed-water heaters, the fluids are a condensing vapor and a heating fluid. The distribution of temperatures is then as indicated in Fig. 19-12, and the logarithmic mean temperature difference should be found by applying equation 19-20.

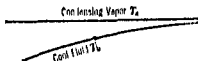


Fig. 19-12. Condenser Temperature Difference



Fig. 19-13. Cooling Hot Fluid by Evaporating Cool Fluid

Still other heat exchangers, such as steam boilers and evaporators in refrigerating systems, use a cooling fluid and an evaporating liquid. In this case the distribution of temperatures is as indicated in Fig. 19-13, and the logarithmic mean temperature difference can be computed by equation 19-20.

Note the important distinction between the change in temperature of a fluid and the temperature difference between two fluids.

**Example 19-7.**—A power-plant lube-oil cooler receives oil at 160 F and discharges it at 120 F. Cooling water enters at 60 F and leaves at 70 F. Calculate the MTD and the logarithmic mean temperature difference for both parallel flow and counter-flow conditions.

**Solution**—For parallel flow, represented in Fig. 19-10, the large temperature difference is  $160 - 60 = 100$  deg, and the small difference is  $120 - 70 = 50$  deg. The arithmetic mean temperature difference is obviously 75 deg. The logarithmic mean is

$$\frac{100 - 50}{\ln\left(\frac{100}{50}\right)} = \frac{50}{0.693} = 72.15 \text{ deg}$$

For counter-flow, represented in Fig. 19-8, the large temperature difference is  $160 - 70 = 90$  deg and the small difference is  $120 - 60 = 60$ . The arithmetic mean temperature difference is the same as before, or 75 deg. The logarithmic mean is

$$\frac{90 - 60}{\ln \left( \frac{90}{60} \right)} = \frac{30}{0.405} = 74.5 \text{ deg}$$

*Example 19-8.*—If the oil in the preceding example has a specific heat of 0.5, how many pounds of water are circulated per pound of oil?

*Solution.*—Each pound of oil loses  $0.5 \times 40 = 20$  Btu, while each pound of water rises 10 deg in temperature and gains 10 Btu. Hence, 2 lb of water must here be circulated per pound of oil.

*Example 19-9.*—In the preceding chapter, Fig. 18-11 and the accompanying text were concerned with the effectiveness of a regenerator in a gas-turbine plant. Consider that a gas-turbine plant takes in 50 lb of air per second, and that 50.6 lb of exhaust gas issue from the turbine in the same time. A regenerator raises the temperature of the air from 440 F to 740 F, while the temperature of the exhaust gases drops from 940 F to 650 F as a result of this heat transfer. Each gas has a heat-transfer film coefficient  $\bar{h}$  of 5, based on the same area. How many square feet of heat-transfer surface are required?

*Solution.*—Since gases are the controlling resistances, the resistance of the metal may be disregarded. Then

$$U = \frac{1}{\frac{1}{5} + \frac{1}{5}} = 2.5$$

The temperature data indicate that counter-flow is used. The logarithmic mean temperature difference is 205. (This should be checked as an exercise.) As air in this range has a mean specific heat of 0.25, the heat gained by the air is 75 Btu per lb; this value may also be computed by using the Keenan and Kaye Gas Tables. The heat transfer on an hourly basis is then

$$50 \times 3600 \times 75 = 13,500,000 \text{ Btu}$$

Hence, the area required is

$$\frac{13,500,000}{2.5 \times 205} = 26,300 \text{ sq ft}$$

It may also be noted that this regenerator has an effectiveness of 60%, based on temperatures.

*Example 19-10.*—A power-plant air heater is composed of steel tubes with 2 50-in. O. D. and 2.31-in. I. D., and the air flows across the tubes with  $\bar{h}$  of 6 while being heated from 50 F to 500 F. The flue gas flows inside the tubes with  $\bar{h}$  of 4 while cooling from 670 F to 300 F. The plant burns 5 tons of coal per hour and the air-fuel ratio is 14 to 1. How much surface area, based on the outside of the tubes, is needed?

*Solution.*—Since the outside surface area is required, the over-all heat-transfer coefficient  $U$  should be computed for outside tube conditions. Since the gas flows inside the tubes, its  $\bar{h}$  must be converted to an outside basis. The resistance is then

$$\frac{1}{4} \times \frac{2.50}{2.31} = 0.270$$

The resistance of the air film, which is outside the tubes, is  $\frac{1}{6} = 0.167$ . With steel tubes, the resistance of the tube metal may be checked for significant figures as follows: Using conductivity of 25 and taking the tube wall thickness as 0.095 in., we find that the resistance, converted to outside basis, is

$$\left( \frac{0.095}{25 \times 12} \right) \left( \frac{2.500}{2.405} \right) = 0.000387$$

The figures are not significant in view of the magnitude of the other resistances. However, if different fluids had been used, the relative resistance of the tube metal might have been significant. For instance, if the film coefficient of either fluid had been 2000 (as for condensing steam), the film resistance would have been down to 0.0005, and the tube metal resistance would have thus been relatively large.

An essential part of the art of engineering is to weigh the relative importance of the factors involved in a situation. To continue the solution, one notes that the total significant resistance, based on outside tube area, is 0.437, giving  $U = 2.29$ . The hourly heat transfer will be

$$14 \times 5 \times 2000 \times 0.24 \times 450 = 15,100,000 \text{ Btu}$$

The logarithmic mean temperature difference will be

$$\frac{250 - 170}{\ln\left(\frac{250}{170}\right)} = \frac{80}{0.385} = 207$$

Hence, the surface area will be

$$\frac{15,100,000}{2.29 \times 207} = 31,800 \text{ sq ft}$$

## PROBLEMS

NOTE: Unless otherwise stated, take atmospheric conditions as 14.7 psia and 60 F in all problems in this text.

### CHAPTER 1

1-1. Calculate the kinetic energy possessed by an automobile weighing 4000 lb and traveling at (a) 25 mph, (b) 50 mph, (c) 75 mph.

1-2. How much energy is transferred in bringing the automobile in Problem 1-1 to a stop over a distance of 300 ft at each speed? What is the heat equivalent of this energy? What is the average equivalent power during this time?

1-3. The automobile in Problem 1-1 has eight cylinders, each 3 in. in diameter. When a certain power is being developed, the average gage pressure in the cylinders is 110 psi. What is the average force in each cylinder? How much work is done by a piston moving 4 in.?

1-4. A boiler pressure gage reads 600 psi when the barometric pressure is 29.50 in. of mercury. In the same power plant, a condenser gage shows a vacuum of 28 in. of mercury. Calculate the absolute pressures (psia) in the boiler and in the condenser.

1-5. In a power plant, an electric generator that requires the equivalent of 30,000 kw input at its shaft coupling is driven by a direct-connected steam turbine. What is the horsepower applied at the shaft coupling? To how many Btu per min is this equivalent?

1-6. A low-pressure air line is equipped with a gage that reads 10 psi. It is proposed to equip the line with a U-tube manometer with a liquid to indicate the pressure. Determine the number of inches of each of the following liquids which would indicate the same pressure (a) water; (b) mercury; (c) oil having a specific gravity of 0.85.

1-7. An automobile weighing 3500 lb travels for 3 hr at an average speed of 30 mph, during which period its elevation is increased by 1000 ft. The average force due to wind and rolling resistance is 200 lb. What is the average horsepower of the automobile?

1-8. The automobile in Problem 1-7 consumes gasoline which has a heating value of 130,000 Btu per gallon. Owing to losses (thermodynamic and otherwise), only 20 per cent of the energy in the gasoline is effectively used to propel the car. How many gallons of gasoline are consumed on this trip?

1-9. How much gasoline would be consumed by the automobile in Problems 1-7 and 1-8 on the return trip, if speed and wind and rolling resistance remain unchanged?

1-10. The internal energy of saturated steam at 200 psia is 1113.7 Btu per lb greater than that of water at 32 F. Of this, 354.68 Btu per lb is the kinetic portion. How many Btu constitute the potential portion?

1-11. An electric water heater has a 2000-watt heating element. If losses are neglected, how many gallons of water per hour will it heat from 40 F to 150 F?

1-12. What is the cost of heating a gallon of water from 40 F to 150 F in an electric water heater, if losses are neglected and the "power" cost is \$0.01 per kwhr?

1-13. If 20 per cent of the power delivered by a power-plant generator is lost in transmission to the consumer's premises, and if the efficiency of the generator is 93%, how many horsepower must be supplied by the prime mover in order to operate the water heater of Problem 1-11?

1-14. How much per gallon would it cost to heat water from 40 F to 150 F by means of an oil-burning heater, if the fuel has a heating value of 20,000 Btu per lb, its

specific gravity is 0.80, and it costs \$0.13 per gallon? Assume that the efficiency of the heater is 50% ( $\epsilon$ , half of the heating value of the fuel is utilized in heating the water).

1-15 If a typical steam power plant consumes 1 lb of coal having a heating value of 13,000 Btu per lb, for each kilowatt-hour delivered to the transmission line, what is the over-all thermal efficiency of the station ( $\epsilon$ , the ratio of the net electrical output to the energy supplied in the form of fuel, both being expressed in consistent units)?

1-16 A glass tube consists of two sections. The lower one has an internal diameter of 0.5 in. and a height of 8 in., and the upper one has an internal diameter of 0.3 in. and a height of 14 in. The upper part is open to the atmosphere and is filled with water. The lower part is filled with mercury. Using the densities as those at 32°F, find the total force at the bottom of the mercury column and also the pressure.

1-17 If the internal diameter of the lower part of the tube in Problem 1-16 were changed to 0.3 in., what would be the total force and the pressure at the bottom of the mercury column?

1-18 What is the absolute pressure at a depth of 40 ft below the surface of the ocean? Take the specific gravity of salt water as 1.03.

1-19 A vacuum gage attached to a condenser, reads 28.9 in. Hg. If the barometric pressure is 29.86 in. Hg, determine the absolute pressure in the condenser, in pounds per square inch.

1-20 It is desired to maintain a pressure of 16.2 psia in the evaporator of a refrigeration machine. If the barometric pressure is 28.92 in. Hg, determine the number of inches of mercury to be held on a mercury manometer having one end open to the atmosphere.

1-21 An oil column is used to measure a pressure difference. The oil column is 38 in. high and the specific gravity of the oil is 0.85. Calculate the pressure difference, in pounds per square inch.

1-22 A waterfall is 324 ft high. Compare the temperature of the water at the bottom (after it has come to rest) with that at the top.

1-23 A d.c. motor uses 24 amp at 150 volts. The motor drives machinery located within a room. Determine the heat added to the room per hour as a result of the operation of the motor.

1-24 A drop of a forge hammer weighs 2 tons. Its drop is 6 ft. Assuming a free drop, calculate its velocity at the bottom of its drop (just before it strikes). Calculate also the heating effect produced, in Btu.

1-25 A tank of brine is to be heated electrically. The brine weighs 4200 lb. It takes 0.75 Btu to raise the temperature of 1 lb of brine 1 deg F (this is called the specific heat of the brine). Determine the amount of electrical energy, in kilowatt-hours, needed to heat the brine 20 deg F.

1-26 A falling weight of 220 lb turns paddles in a tank containing 20 lb of water. If the weight falls 16 ft, determine the temperature rise of the water in the tank. (It takes 1 Btu to heat 1 lb of water 1 deg F.)

1-27 Coal has a heating value of 14,000 Btu per lb. If the over-all efficiency of a power plant is 28% ( $\epsilon$ , 28 per cent of the energy of the coal is transformed into electrical energy) and the average load on the plant is 50,000 kw, determine the number of tons of coal required per year (8,760 hr).

1-28 An automobile has an over-all efficiency of 15% at 40 mph ( $\epsilon$ , 15 per cent of the energy of the fuel is delivered at the tires to propel the car). The car travels 120 miles in 3 hr and uses 6.5 gal of gasoline having a heating value of 140,000 Btu per gal. What is the average horsepower used?

1-29 A household refrigerator runs 35 per cent of the time and uses 0.3 kw when running. If the heat flow into the refrigerator box is 1100 Btu per hr, how much heat

must be removed from the refrigerator per hour? How much heat, net, is added to the kitchen per hour?

1-30. A lathe is driven by a  $\frac{1}{2}$ -hp motor. If 20 per cent of the rated horsepower is used in machining a part that weighs 2.2 lb, what is the temperature rise of the part at the end of 5 min of operation? Assume that no heat is lost from the part and that it requires 0.1 Btu to increase the temperature of 1 lb of the part 1 deg F.

## CHAPTER 2

2-1. Calculate the temperature at which both Fahrenheit and centigrade thermometers read alike. Then express this temperature as degrees Rankine and degrees Kelvin.

2-2. What is the weight of air in a tank having a volume of 40 cu ft, if the contents are at 150 F under a gage pressure of 300 psi? If the air is replaced by helium, how much lighter will the contents of the tank be?

2-3. Air, initially at 14.7 psia and 60 F, is compressed isothermally until the pressure is 147 psia. It is then heated until the temperature (absolute) is doubled at constant volume, and is afterward cooled at constant pressure until the original temperature is reached. Calculate the initial and final specific volumes and the final pressure; and plot the paths of the processes by use of  $P$ - $V$  coordinates.

2-4. The volume of a perfect gas is increased from 10 cu ft to 40 cu ft from an initial pressure of 100 psia to each of the following final pressures: (a) 10 psia, (b) 25 psia, (c) 50 psia, (d) 100 psia, (e) 200 psia, (f) 400 psia, (g) 1000 psia. Calculate the value of  $n$  for each path, and plot each path by use of  $P$ - $V$  coordinates.

2-5. Solve Problem 2-4 by plotting each line by use of logarithmic coordinates.

2-6. The initial temperature in Problem 2-4 is 500 R. Determine the final absolute temperature in each case.

2-7. A diesel truck engine has a compression ratio of 16 to 1. When operated in lowlands, the engine has a pressure at the beginning of compression of 14 psia, and  $n$  is 1.32. When operated in mountainous country, the cylinder pressure at the beginning of compression is 10 psia, and  $n$  is 1.30. Calculate the cylinder pressure at the end of compression in each case.

2-8. An aviation engine has a compression ratio of 8 to 1. If  $n$  for compression is 1.30 for all cases, determine the pressure at the end of compression in each of the following cases: (a) the cylinder pressure before compression is standard atmospheric; (b) the pressure at high altitudes is reduced to 6 psia; (c) the pressure of standard atmospheric is augmented by supercharging 25 inches of mercury, gage.

2-9. The compressor of a gas turbine raises the air pressure from standard atmospheric to 45 psi, gage, while the absolute temperature rises 15 per cent more than it would have if  $n$  were 1.40. If the initial temperature is 40 F, what is the final temperature?

2-10. A gas turbine receives gases at a gage pressure of 43 psi and 1300 F, and exhausts at standard atmospheric pressure. The absolute temperature drops 13 per cent less than it would have if  $n$  were 1.35. What is the exit temperature?

2-11. (a) Convert a temperature of 20 C to degrees R. (b) Convert 400 R to degrees C. (c) Convert 300 K to degrees R.

2-12. At 14.7 psia and 15 C, air has a density of 0.07651 lb per cu ft. At an altitude of 2000 ft, the standard density is 0.07213 lb per cu ft. For this small change in altitude assume that the density varies as a straight line with the altitude (*i.e.*, a graph representing the relation between the density and the altitude is a straight line). Determine the barometric pressure, in inches of mercury, at an altitude of 2000 ft.

2-13. A rubber inner tube is inflated with air at 50 F to a gage pressure of 5 psi. Its internal volume is then 2 cu ft. After lying in a hot sun its temperature rises to

110 F Assume that its volume increases 5 per cent as a result of the increase in pressure, and that the barometric pressure is 14.0 psia (a) Determine the gage pressure of the air after it is heated to 110 F (b) Find the value of the exponent  $n$  based on the initial and final conditions (c) Compute the percentage of the air (on a weight basis) which would have to be let out of the tube to bring the gage pressure back to 5 psi, assuming no change in temperature

2-14 Determine the weight of air present in a room 20 ft  $\times$  10 ft  $\times$  8 ft, if the barometric pressure is 28.9 in. Hg and the temperature is 75 F.

2-15 Air at 14.7 psia and 60 F is heated at constant pressure until its volume is doubled. It is then compressed isothermally until the pressure is 14.7 psia, and it is finally cooled at constant volume to 60 F. Calculate the initial and final specific volumes and the final pressure.

2-16 Air that is initially at 14 psia and 80 F is to be compressed to a temperature of 400 F. If the value of  $n$  for the compression is 1.36, to what pressure must the air be compressed?

2-17 (a) What compression ratio is necessary to obtain a pressure of 400 psig at the end of compression, if the initial pressure is 13.5 psia and the value of  $n$  for the compression is 1.35? Assume standard atmospheric pressure. (b) What would be the compression ratio if the initial pressure were changed to 10 psia?

2-18 The N.A.C.A. standard temperature at an altitude of 20,000 ft is 24.6 C, and the standard pressure at that elevation is 13.75 in. Hg (a) Determine the standard air density at an altitude of 20,000 ft (b) Calculate the pressure at an altitude of 20,000 ft, using the density calculated in (a), taking the standard density at sea level as 0.07651 lb per cu ft, and assuming that the density varies as a straight line with the altitude.

2-19 An automatic home water system has a storage tank with a total volume of 30 gal. The pressure-actuated control switch is set to stop the pump at a gage pressure of 50 psi and to start it when the gage pressure falls below 25 psi. Neglect the volume of any piping in the system (a) Compute the percentage of the tank volume which is filled with water when the pump stops after being put in operation for the first time. Assume that the tank is filled initially with air at 60 F and atmospheric pressure, and that this air is compressed polytropically to a final temperature of 100 F. (b) Find the number of gallons of water which can then be withdrawn from the tank before the pump starts again, assuming that  $n$  for the expansion process is 1.3 and that the water is reused. (c) Determine the value of  $n$  for part (a).

2-20 The quantity of compressed air delivered by a compressor, or used by an air-operated tool or other device, is usually expressed in terms of cu ft per min (cfm) of free air, which is the volume converted to atmospheric temperature and pressure. The capacity of a certain service-station compressor is 10 cfm of free air, and the volume of the receiver (storage tank) is 10 cu ft. Atmospheric conditions are 14.7 psia and 70 F. (a) If the capacity of the compressor is assumed to be unaffected by variations in discharge pressure and the final air temperature is to be 120 F, how long will it take to raise the gage pressure in the tank from 0 to 80 psi? (b) What will be the pressure in the receiver after the temperature of the air drops to atmospheric, if it is assumed that no air is added or withdrawn?

2-21 Determine the number of cubic feet of free air that can be withdrawn from the receiver in Problem 2-20 before the compressor will start again, if the air is withdrawn so slowly that the temperature remains constant at 70 F and the gage pressure corresponding to the low limit setting of the switch is (a) 70 psi, (b) 60 psi, (c) 50 psi.

2-22 The gross weight of a certain blimp is 20,000 lb, exclusive of the contained gas. It is filled with gas at 14.8 psia and atmospheric temperature (a) What must be its volume if it is to have zero net buoyancy at sea level conditions of 14.7 psia and 59 F when filled with helium? (b) What must be its volume when filled with hydrogen? (c) When the blimp is filled with the quantity of helium determined in part (a), how much



force (up or down) would have to be exerted by the elevators to maintain a constant altitude of 5000 ft, where the atmospheric pressure and temperature are 25 in. Hg and 40 F? Assume that the gas envelope did not stretch, that the temperature of the helium is the same as that of the ambient air, and that the volume of air displaced by the structure, fuel, crew, etc. is negligible compared to the displacement of the gas bag.

2-23. A tank with a volume of 20 cu ft contains air at a gage pressure of 100 psi and 100 F. A valve on the tank is suddenly opened, and the pressure in the tank is permitted to drop to atmospheric in a very short period of time. Calculate the weight of air left in the tank.

2-24. Convert the following temperatures: (a)  $-40^{\circ}\text{C}$  to degrees F; (b)  $290^{\circ}\text{K}$  to degrees F; (c)  $800^{\circ}\text{R}$  to degrees C; (d)  $200^{\circ}\text{F}$  to degrees K; (e)  $600^{\circ}\text{C}$  to degrees R.

2-25. Flue gases from a power plant contain 15%  $\text{CO}_2$  by volume. If all of the  $\text{CO}_2$  in the gases can be converted into dry ice, determine the total volume of gases required per ton of dry ice produced. The gases are at 14.7 psia and 300 F. Take  $R$  for  $\text{CO}_2$  as 34.9.

2-26. Air leaving a compressor is metered and found to be flowing at the rate of 142 cfm. Metering conditions are 120 psig and 220 F. Determine the volume of air entering the compressor per minute at 14.7 psia and 80 F.

2-27. A steam jet ejector pumps 0.3 lb of air per lb of steam when the air is at 0.5 psia and 85 F. Determine the weight of steam required per hour for 350 cfm of air.

2-28. A tank having a volume of 80 cu ft contains air at 14.5 psia and 80 F. The tank is immersed in water and reaches the water temperature of 40 F. The air is then pumped out of the tank until the pressure reaches 0.2 psia. Determine the difference between the final buoyant effect of the air in the tank and that when the tank was first immersed in the water.

2-29. A natural-gas pipe line of steel is being laid under water. The pipe line is 20 in. in diameter (outside) and is 0.25 in. thick (steel weighs 0.2833 lb per cu in.). If the pipe is filled with air at 14.5 psia and 50 F, determine the net buoyant effect per foot of length.

2-30. Solve problem 2-29, assuming that the pipe is filled with natural gas ( $R = 95.5$ ) at 500 psia, and 50 F.

2-31. Air at 40 psig and 20 F is heated until the pressure is 94 psig and the temperature is 500 F. Calculate the value of  $n$  for the process, if atmospheric pressure is 14.0 psia.

2-32. Air at 14.7 psia and 40 F is compressed to 147 psia according to the law  $PV^{1.4} = C$ . It is then heated at constant pressure until its temperature becomes 4200 F. It then expands according to the law  $PV^{1.4} = C$  until the pressure is 14.7 psia. If the weight of air involved is 0.4 lb, determine the final volume and temperature.

2-33. Compressed air at 80 F is to be expanded to atmospheric pressure. If the temperature after expansion is to be  $-150^{\circ}\text{F}$ , what must be the initial pressure of the air? Assume that the value of  $n$  for the expansion process is 1.4.

2-34. Air at 14.7 psia and 40 F is compressed to a pressure of 185 psia according to the law  $PV^{1.4} = C$ . It is then heated to 6860 R at constant volume. It is next expanded according to the law  $PV^{1.4} = C$  until its volume is 6.1 times the volume before expansion. It is finally cooled at constant volume until the pressure is 14.7 psia. Determine the final temperature. Consider 1 lb of the air.

2-35. In an air compressor the compression process obeys the law  $PV^{1.31} = C$ . The compressor receives 300 cfm of air at 14.5 psia and 70 F. If the compressor discharges at 145 psia, determine the volume of air discharged per minute and its temperature.

2-36. A flow nozzle is used to measure air flow. The pressure in the nozzle drops 4.2 in. of water. The original pressure is 16.2 psia, and the temperature is 80 F. Assume

that expansion obeys the law  $PV^{1.4} = C$  Can the density of the air be assumed to remain constant?

2-37 Determine the force exerted by the wind on the wall of a building if the wall is 60 ft  $\times$  80 ft and the wind velocity is 75 mph Temperature is 60 F

2-38 Air weighing 2 lb is heated from 140 psia and 40 F until the volume becomes 52.8 cu ft If the final temperature is 540 F determine the final pressure

2-39 Heat is added to 3 lb of air at a constant temperature of 540 F until the pressure drops to 200 psia The air then expands according to the law  $PV^{1.4} = C$  until the pressure is 20 psia If the original pressure is 400 psia determine the total change in volume

2-40 The volume of a clearance space of an air compressor is 4 per cent of the piston displacement The compressor has a 10 in bore and a 12 in stroke Before expansion the air is at 120 psia After expansion to 14.5 psia the air temperature is 40 F If the expansion follows the law  $PV^{1.25} = C$  calculate (a) the volume of the expanded air and (b) the air temperature before expansion

2-41 Air exists in a closed pipe line at 20 psia and 80 F The pipe line contains a little oil having a flash point of 350 F A valve to the pipe line is suddenly opened the pressure being increased to 200 psia Assume that this action compresses the air in the line according to the law  $PV^{1.4} = C$  Is there any danger of the oil exploding?

### CHAPTER 3

3-1 During the process of compression of air in a cylinder the heat transferred from the air is 4 Btu If the work required for the compression is 30 000 ft lb calculate the change in intrinsic energy

3-2 When a pound of water under a constant pressure of 300 psia is heated from 50 F to 400 F 356.9 Btu are required Calculate the mean specific heat of water over this range

3-3 A pound of air is compressed its temperature rising from 40 F to 400 F Determine its change of intrinsic energy if compression takes place under the following conditions (a) Absolutely no heat is transferred (b) compression follows the law  $PV^{1.2} = C$  (c) compression follows the law  $PV^{1.2} = C$

3-4 Calculate the work done in each part of Problem 3-3

3-5 Calculate the heat transferred in each part of Problem 3-3.

3-6 A quantity of air weighing 0.1 lb is compressed at a constant temperature of 40 F until the pressure is tripled Calculate the heat transferred

3-7 Air is heated at a constant pressure of 40 psia the volume increasing from 2 to 5 cu ft How much heat must be added if the original temperature is 90 F?

3-8 Calculate the change in intrinsic energy of the air in Problem 3-7 by two methods

3-9 A quantity of air having a volume of 500 cu ft at 20 psia and 120 F is compressed to 40 cu ft If the final pressure is 380 psia calculate the heat removed during compression

3-10 Assume that the combustion process occurring in a diesel engine after dead center obeys the law  $PV^{1.3} = C$  At dead center the pressure is 450 psia the temperature is 1100 F and the volume is 0.08 cu ft Combustion continues until the volume is 0.12 cu ft Calculate the work done

3-11 An indicator card of a diesel engine with a 15 to 1 compression ratio shows that when the piston has moved 20 per cent of the expansion stroke the pressure is 185 psi gage At 70 per cent of the expansion stroke the pressure is 36 psi gage Calculate the value of  $n$  for this part of the expansion and the work performed between these limits if the engine has a 6 in cylinder bore and a 9 in stroke

3-12. A diesel engine, 12 in.  $\times$  18 in., has a 15 to 1 compression ratio. The conditions at start of compression are 14 psia and 140 F. The compression exponent is 1.35. Assuming that the cylinder contents may be treated as air, calculate: (a) the pressure at the end of compression; (b) the temperature at the end of compression; (c) the total cylinder volume; (d) the weight of air present; (e) the work of compression; (f) the change of intrinsic energy during compression; (g) the heat transferred during compression.

3-13. (a) Under constant pressure conditions the specific heat for a certain gaseous mixture is 0.32. How much heat is required to raise the temperature of 8 lb of this gas from 500 R to 1500 R at constant pressure? (b) How much heat is required at constant volume if the specific heat is 0.22?

3-14. When 2.2 lb of air are compressed at a constant temperature of 40 F, 40 Btu are removed. What is the specific heat of air for these conditions?

3-15. Calculate the amount of heat which must be added isothermally to 4 lb of air at 240 F in order to double its volume.

3-16. Air is cooled at a constant pressure of 40 psia, with 48 Btu being removed from 2 lb of air. If the original temperature is 200 F, what is the final volume?

3-17. A quantity of a perfect gas weighing 8 lb, for which  $R=38.7$  and  $c_p=0.1269$ , is compressed polytropically (non-flow) from a gage pressure of 5.3 psi and 80 F to a gage pressure of 85.3 psi and a volume of 16.85 cu ft. The work required is 324 Btu. Calculate the heat added or removed by finding  $c_v$ ,  $n$ ,  $T_2$ , and  $U_2-U_1$ . Check your answer by also determining  $c_v$ .

3-18. Under certain conditions, 2000 Btu (in the form of heat) are required to raise the temperature of 15 cu ft of air, originally at a gage pressure of 15 psi, from 100 F to 240 F (non-flow). Calculate the work done: (a) from the general energy equation and (b) from  $\int P dV$ .

3-19. A cylinder contains 5.5 cu ft of air at 90 F and a gage pressure of 30 psi. The air is subjected to a non-flow process which increases its intrinsic energy by 100 Btu and causes its volume to triple. Calculate the heat exchange by two different methods.

3-20. A quantity of a certain perfect gas undergoes a non-flow polytropic process which decreases its internal energy by 100 Btu while the gas does 70,000 ft-lb of work. For this gas  $k=1.286$  and  $R=34.9$ . The initial conditions are 100 psi and 1000 F, and the final temperature is 100 F. Determine the heat exchange and the final gage pressure.

3-21. Air is compressed according to the law  $PV^{1.35}=C$ . The work required for 5 lb of air is 30,000 ft-lb. If the original temperature is 100 F, what is the final temperature?

3-22. During a partially irreversible expansion of 1 lb of air, the volume increases to 4 times its original value. Heat added during the expansion is 20 Btu. The air is originally at 60 psia and 140 F. The final air temperature is 140 F. Determine the work done, in foot-pounds.

3-23. Air in a vertical cylinder occupies 0.5 cu ft at 20 psia and 40 F. The air is heated by the addition of 2.6 Btu. The increase of internal energy is 34.3 Btu per lb. Calculate the work done.

3-24. It requires 53,000 ft-lb of work to compress some air in a cylinder. If the intrinsic energy increases 62 Btu during the compression process, is heat added or removed? How much?

3-25. The gases in a diesel engine expand, doing 1,800 ft-lb of work. During the expansion process, the heat removed from the gases is 0.5 Btu. Calculate the change of internal energy during the expansion process.

3-26. A gaseous mixture in a tank is at 200 psia and 80 F. The tank volume is 50 cu ft. When the tank is heated by the addition of 2310 Btu, the pressure is increased to 300 psia. Determine the specific heat  $c_v$  of the mixture if  $R$  for the mixture is 60.

3-27 When 4 lb of a gas are compressed the work required is 58 000 ft lb. The temperature increases 100 deg F and there is an increase of 68.6 Btu in the internal energy of the gas. Determine the specific heat of the gas for the compression process.

3-28 A gas expands in a cylinder the pressure dropping from 185.3 psig to 85.3 psig and the volume increasing from 2.0 to 4.0 cu ft. Determine the work of expansion.

3-29 The work required to compress 2 lb of air from 40 psia and 40 F to 540 F is 53 350 ft lb. Determine the value of  $n$  for the compression process.

3-30 Calculate the work of compression in a natural gas compressor if the gas is compressed from a volume of 2.2 cu ft to a volume of 1.39 cu ft when the pressure is increased from 500 psia to 900 psia.

3-31 Air expands according to the law  $PV^{1.35} = C$ . The air originally occupies 1.2 cu ft at 200 psia and 600 F. The final temperature is 200 F. Calculate the work of expansion.

3-32 Calculate the amount of heat which must be removed if 0.2 lb of air is compressed at a constant temperature of 80 F from 20 psia to 60 psia.

3-33 For the conditions in Problem 3-23 calculate (a) the temperature change and (b) the specific heat of air for the process.

3-34 Air is compressed in an air compressor from 14 psia to 84 psia its volume decreasing from 1.2 cu ft to 0.318 cu ft. The original temperature is 100 F. Calculate (a) the final temperature (b) the value of  $n$  for the compression process (c) the work of compression (non flow) (d) the heat added and (e) the specific heat of air for the process.

3-35 In an adiabatic expansion of 2 lb of air the work done is 26 000 ft lb. If the initial temperature is 140 F what is the final temperature?

3-36 Determine the net heat added, the net work done and the net change in internal energy for Problem 2-34.

3-37 Air in a vertical cylinder at 200 psia and 200 F occupies a volume of 1.1 cu ft. The piston is suddenly released moving upward to a stop where it comes to rest. At this point the air volume is 2.2 cu ft. The piston weighs 28 lb and travels upward 2.4 ft. Neglect energy given up by the piston when it hits its stop and also the air pressure on top of the piston. Determine the final temperature. (NOTE: The action may be assumed to be adiabatic but not reversible.)

3-38 Determine the work required (non flow) to compress 0.4 lb of air if 4 Btu are removed during the compression process. The specific heat for the process is  $-0.05$ .

3-39 In a steam engine steam expands from 125.3 psig and 353 F to 55.3 psig and 303 F. If the volume increases from 0.6 cu ft to 1.2 cu ft calculate the work done.

3-40 As 4 lb of air expand at constant temperature 20 Btu are added. If the air originally occupies 6 cu ft at 200 psia what is the final pressure?

## CHAPTER 4

4-1 Calculate the number of pounds of hydrogen gas per minute that must be circulated through a hydrogen cooled electric generator if the gas temperature rises 10 deg F while it absorbs heat. The generator is rated at 100 000 kw of which 2 per cent is given off as heat to the hydrogen.

4-2 Air flows through a compressor each pound having its enthalpy increased by 48 Btu while rejecting 10 Btu to the cylinder jacket. If the rate of flow is 12 lb per min determine the horsepower needed to compress the air.

4-3 Determine the temperature of the air leaving the compressor in Problem 4-2 if the air enters at 60 F.

4-4 Gases leave a combustion chamber and enter a gas turbine nozzle with a velocity of 224 ft per sec with an enthalpy of 350 Btu per lb. The gases leave the

nozzle with a velocity of 1200 ft per sec. What is the enthalpy of the gases leaving the nozzle?

4-5. Ammonia vapor enters a compressor with an enthalpy of 628 Btu per lb and leaves with an enthalpy of 716 Btu per lb. Each pound of ammonia gives up 24 Btu to the water jacket while being compressed. The shaft input is 10 per cent greater than the work done in the cylinder on the ammonia. If 50 lb of ammonia are circulated per hour, calculate the shaft horsepower required to drive the compressor.

4-6. An automobile engine uses 2 gal of gasoline per hour. Gasoline has a specific gravity of 0.73 and a chemical energy of 19,300 Btu per lb. The engine takes in 16 lb of air per pound of gasoline. The air and gasoline enter the engine with an enthalpy of 40 Btu per pound of mixture. The exhaust gases leave the engine with an enthalpy of 380 Btu per pound of gases, and also contain 100 Btu of chemical energy per pound of gases. The engine's water jackets and the air streaming around it remove 80,000 Btu per hr. Calculate the engine's horsepower.

4-7. A jet-propelled airplane takes in air at 700 ft per sec with a temperature of 0 F. For each pound of air, there is supplied 0.015 lb of fuel which has a chemical energy of 19,000 Btu per lb. Neglecting the enthalpy of the fuel, and assuming that 15 per cent of the chemical energy is still unburned at exhaust, calculate the velocity of the jet if its temperature is 800 F. (Consider that the jet has the properties of air, that air is a perfect gas, and that the plane's combustion chamber loses 14 Btu per pound of air.)

4-8. A power-plant air preheater of the recuperative type circulates flue gas through the insides of tubes while the air to be heated is forced around the outsides of the tubes. Data on the flue gas are as follows: 15,000 lb per hr enters the tubes at 700 F and leaves at 400 F, and  $c_p = 0.26$ . Air data are as follows: 3,060 cu ft per min enters at atmospheric pressure and 60 F. At what temperature does the air leave the heater?

4-9. A water heater mixes steam (1) with cold water (2), producing hot water (3). Kinetic energy and heat loss are negligible. We require 10,000 lb per hr of hot water at 210 F. The enthalpy of this water is 178 Btu per lb. If the enthalpy of the cold water is 10 Btu per lb and that of the steam is 1200 Btu per lb, how much steam is required per hour?

4-10. A turbine uses 100,000 lb per hr of steam which enters with an enthalpy of 1300 Btu per lb and negligible velocity. The turbine develops 10,000 hp. The steam leaves the turbine with a velocity of 500 ft per sec. Heat loss is negligible. What is the enthalpy of the exhaust steam, in Btu per pound?

4-11. Air at a gage pressure of 100 psi and 80 F flows with negligible velocity to a throttle valve. Some distance below the valve, the velocity is again negligible and the gage pressure is 10 psi. Find the temperature and specific volume of the low-pressure air. Is any work done in this process? How do you know?

4-12. Air at a gage pressure of 100 psi and 80 F flows with a negligible velocity into a perfect nozzle in which it is expanded adiabatically and reversibly to a gage pressure of 10 psi. Find the temperature, velocity, and specific volume of the air leaving the nozzle.

4-13. Any fluid flowing through a duct or pipe of uniform cross-sectional area undergoes a drop in static pressure because of fluid friction. Air at a gage pressure of 100 psi and 80 F flows with a velocity of 100 ft per sec into a pipe line which has sufficient friction to cause the gage pressure at the end of the line to be 10 psi. The flow is adiabatic (but not reversible) and the expansion causes the temperature to drop to 64 F at the exit.

(a) Calculate the velocity and specific volume of the air leaving the line.

(b) Check the validity of these results by applying the equation for "continuity of mass flow" (Equation 9-3 in the text).

(c) By comparing the results of Problems 4-11, 4-12, and 4-13, explain the differences between (1) reversible adiabatic expansion, (2) irreversible expansion in a constant-area channel, and (3) throttling.

4-14 A diffuser is a diverging (for subsonic velocities) tube designed to increase the pressure of a high velocity fluid stream at the expense of its kinetic energy. Air flows through a diffuser which reduces its velocity from 800 ft per sec to 250 ft per sec.

(a) If the flow is adiabatic, what is the increase in the specific enthalpy of the air and the increase in its temperature?

(b) If the process is also reversible, what is the ratio of the pressure at the exit to that at the entrance? Assume that  $t_1 = 60^\circ\text{F}$ .

4-15 A pump requires an input of 125 hp when delivering 1000 gpm of water at a gage pressure of 100 psi. Suction pressure is atmospheric. The internal cross sectional areas of the suction and discharge pipes are 7.39 and 3.36 sq in respectively. Assume that the specific weight of the water at both inlet and outlet is 62.40 lb per cu ft, that its specific heat is 1.000 and that 10 per cent of the mechanical energy supplied to the pump is lost externally, largely because of friction in the bearings and packing glands.

4-16 A test on an air compressor shows that it delivers 250 cfm of air (measured under inlet conditions) when using 25 shafts hp. The flow of water through the jacket is 11 lb per min and its temperature rise is  $8.5^\circ\text{F}$ . Air enters the compressor at  $83^\circ\text{F}$  and leaves at  $292^\circ\text{F}$ . Determine the heat lost from the compressor per minute (other than to the cooling water).

4-17 Steam enters an engine with an enthalpy of 1205 Btu per lb and leaves with an enthalpy of 1058 Btu per lb. The engine delivers 171 hp when using 3050 lb of steam per hour. Determine the heat lost from the engine per hour assuming no net change in kinetic energy of the steam.

4-18 Steam enters a turbine at the rate of 166,000 lb per hr with an enthalpy of 1430 Btu per lb. After partial expansion 42,000 lb of steam per hour are extracted, the remainder (124,000 lb per hr) completes the expansion and leaves the turbine with an enthalpy of 1020 Btu per lb. The enthalpy of the extracted steam is 1200 Btu per lb. Neglecting heat losses and net changes in kinetic energy, calculate the turbine horsepower.

4-19 How many pounds of air per pound of fuel must be supplied to the combustion chamber of a gas turbine to maintain the temperature of the gases at exit of the combustion chamber (and at entrance to the turbine) at  $1500^\circ\text{F}$ ? Air enters the combustion chamber at  $240^\circ\text{F}$  with an enthalpy of 72 Btu per lb. At  $1500^\circ\text{F}$  the gases have an enthalpy of 402 Btu per lb. The sum of the enthalpy and chemical energy of the entering fuel is 18,400 Btu per lb.

4-20 A Freon refrigerating machine is used to produce chilled brine. Freon enters the brine chiller with an enthalpy of 26 Btu per lb and leaves with an enthalpy of 86 Btu per lb. The flow of Freon is 2,200 lb per hr. The brine flow is 260 lb per min with a temperature drop of  $10^\circ\text{F}$ . The specific heat of the brine is 0.8. Calculate the heat flow into the brine chiller from the atmosphere.

4-21 Steam enters a turbine at the rate of 142,000 lb per hr with an enthalpy of 1400 Btu per lb and it leaves with an enthalpy of 1020 Btu per lb and with a velocity of 24,000 ft per min. The entering velocity may be neglected. Heat lost from the turbine is 78,000 Btu per hr. Calculate the horsepower output of the turbine.

4-22 Steam enters a steam engine with an internal energy of 1109.0 Btu per lb, a specific volume of 3.357 cu ft, and a pressure of 1.34 psia. The engine produces 174 hp when the steam flow is 2820 lb per hr. The heat lost from the engine is 3500 Btu per hr. Calculate the enthalpy in Btu per pound at engine exit.

4-23 Air enters a compressor at 14.0 psia and  $80^\circ\text{F}$  and is discharged at 88 psia. Compression follows the law  $PV^{1.4} = C$ . Assume that the heat lost from the air is lost during the compression process. The air flow to the compressor is 450 cfm. Calculate (a) the air temperature at compressor exit, (b) the heat lost from the air per minute (find  $c_p$ ), and (c) the compressor horsepower.

4-24. Gases enter a gas turbine at 1500 F and leave at 1200 F. The value of  $c_p$  for the gases is 0.27. The rate of gas flow is 2,400 lb per min. Assume that the action in the turbine is adiabatic but not isentropic. Calculate the turbine horsepower.

4-25. An air-vapor mixture enters a dehumidifier at 90 F and leaves at 50 F. There is 0.01 lb of vapor per lb of air at entrance and there is 0.00765 lb of vapor per lb of air at exit. The difference in the amount of vapor at entrance and exit leaves as condensate at 54 F, with an enthalpy of 22 Btu per lb. The enthalpies of the vapor at entrance and exit are 1101 and 1084 Btu per lb, respectively. If the air flow is 2100 lb per min, calculate the heat which must be removed from the dehumidifier per minute. (NOTE: Treat the air as a perfect gas in getting its enthalpy.)

4-26. Compressed air enters a pneumatic drill at 90 psia and 100 F and leaves at 14 psia and 40 F. The air flow to the drill is 2.6 cfm. Assuming adiabatic action, compute the horsepower developed.

4-27. An air stream, having a velocity of 1,000 ft per sec, a flow of 10 lb per sec, and a temperature of 80 F, mixes with a second air stream which has a velocity of 400 ft per sec, a flow of 12 lb per sec, and a temperature of 80 F. After mixing, the mean velocity is found to be 735 ft per sec. What is the temperature after mixing?

4-28. A household warm air furnace burns natural gas at the rate of 65 cu ft per hr. It uses 20 lb of air per lb of gas. The air and gas enter at 80 F, the gas having a pressure of 14.6 psia. Take the value of  $R$  for the natural gas as 96 and its chemical energy as 24,000 Btu per lb. The flue gases leave at 600 F. Take the value of  $c_p$  for the flue gases as 0.27. Assume complete combustion and no heat losses from the furnace. Calculate the amount of heat given up to the warm air by the combustion process. (HINT: Take 80 F as a datum plane for evaluating enthalpies. The air and fuel will have zero enthalpy at the datum.)

4-29. A nozzle is to deliver air at a velocity of 800 ft per sec at atmospheric pressure. Air is cooled to 100 F before entering the nozzle. The velocity entering the nozzle is estimated to be 100 ft per sec. What is the minimum initial pressure necessary to produce the given velocity? (HINT: Minimum pressure is needed for an isentropic expansion.)

4-30. A unit heater receives steam with an enthalpy of 1200 Btu per lb, and condensate leaves with an enthalpy of 80 Btu per lb. Air enters the heater at 75 F and 14.5 psia and leaves at 135 F. The air flow to the heater is 2,000 cfm. Determine the weight of steam required per hour if the heat lost from the heater equals 1 per cent of the heat given up by the steam.

4-31. Steam flow to a turbine is 180,000 lb per hr with an enthalpy of 1400 Btu per lb. Steam exhausts from the turbine with an enthalpy of 980 Btu per lb; 19,000 lb per hr of steam are extracted from the turbine with an enthalpy of 1240 Btu per lb and 21,000 lb per hr of steam are extracted with an enthalpy of 1110 Btu per lb. Neglect kinetic energies and heat losses. Find the turbine horsepower, assuming that mechanical losses are 350 hp.

4-32. A gaseous mixture enters a nozzle with no appreciable velocity at a pressure of 100 psia. It has a specific volume of 4 cu ft per lb and an intrinsic energy of 185 Btu per lb. It leaves at 20 psia with a specific volume of 12.6 cu ft and an intrinsic energy of 117 Btu per lb. Calculate the exit velocity.

4-33. A diesel engine delivers 2000 hp when using 780 lb of fuel per hour. Air enters the engine with an enthalpy of 20 Btu per lb. Gases leave with an enthalpy of 280 Btu per lb. There are 20 lb of air per lb of fuel. The fuel has a chemical energy of 18,500 Btu per lb. The water pump circulates 2600 lb of water per minute through the jackets, with a temperature rise of 26 deg F in the water. Neglect chemical energy in the exhaust and the enthalpy of the fuel. Calculate the summation of the heat lost from the engine and the unaccounted-for losses.

4-34 Freon enters a refrigerating compressor with an enthalpy of 82 Btu per lb and leaves with an enthalpy of 96 Btu per lb. The heat lost from the compressor is 9 per cent of the shaft input. The freon flow is 940 lb per hr. Determine the shaft horsepower of the compressor.

4-35 An unknown gaseous mixture enters a compressor at 80 F and is compressed to 380 F. It is then cooled to 80 F in an aftercooler. The water in the water jacket removes 450 Btu per min. This quantity is 80 per cent of the heat given up by the air in the compressor. The heat removed in the aftercooler is 2420 Btu per min. It is believed that the reported horsepower of 52 is in error. Calculate the horsepower using the above data.

## CHAPTER 5

5-1 An internal combustion engine has a maximum temperature in its cylinder of 4000 F while a steam engine can use a maximum of 1050 F. If each could work down to an atmospheric thermal sink temperature of 60 F, what would be the maximum possible efficiency of each?

5-2 Suppose that the engines in Problem 5-1 had such actual efficiencies that the internal-combustion engine had an efficiency of 33% *relative* to an ideal engine operating between those temperature limits and the steam engine had an efficiency of 50% *relative* to an ideal engine operating between its temperature limits. What would the actual thermal efficiencies be?

5-3 A Carnot Cycle engine has a temperature *range* of 300 Fahrenheit degrees. The heat transformed into work is 30 per cent of that rejected. Determine the temperatures of supply and rejection of heat and the thermal efficiency.

5-4 In a Carnot Cycle engine the working substance consists of 0.1 lb of air and 3 Btu are added per cycle at a temperature of 1540 F. Heat is rejected at 40 F. The maximum allowable pressure is 500 psia. Calculate

- Pressures, temperatures and volumes for all points of the cycle
- Heat rejected per cycle
- Work of each process
- Cycle efficiency
- Mean effective pressure (ratio of cycle work to piston displacement)

5-5 The temperature of an infinite source of heat is 800 F and 1000 Btu are removed from the source and added to the engine at a temperature of 600 F. The engine rejects heat at 40 F. Calculate

- Available energy removed from the source.
- Maximum output of the engine
- Loss in available energy as a result of the heat transfer

5-6 A 200 hp diesel engine uses 0.4 lb of fuel per hp hr and takes in 20 lb of air per lb of fuel. The exhaust gases leave the engine at 600 F. The lowest available temperature is 60 F. In the ideal case these exhaust gases may be cooled from 600 F to 60 F giving up energy which may partly be turned into work. If the specific heat of the gases is 0.26, calculate the available energy per pound of gas and per hour that can be removed by cooling these gases. Calculate also the maximum horsepower that can be produced by cooling the exhaust gases.

5-7 Air at 100 F and 50 psia is throttled to atmospheric pressure. Calculate the change in entropy per lb of air. If the lowest available temperature is 40 F, calculate the increase in unavailable energy of air. What is the decrease in available energy?

5-8 A Carnot Cycle engine has a temperature *range* of 1000 Fahrenheit degrees. It receives 1000 Btu per min at 1000 F. Calculate its horsepower.

5-9 In an economizer flue gases are cooled from 700 F to 350 F. The economizer receives 50,000 lb of water per hour at a temperature of 200 F and there are 65,000 lb of flue gases per hour. The specific heat of the flue gases may be taken as 0.26 and that of the water as 1.0. Assume that the lowest available temperature is 140 F. Calculate



- (a) Temperature of the water leaving the economizer.
- (b) Unavailable energy removed from the gases per hour.
- (c) Unavailable energy added to the water per hour.
- (d) Loss of available energy per hour as a result of the heat transfer.

5-10. A refrigerating machine operates on the Carnot Cycle and receives 24,000 Btu per hr at 10 F. It discharges its heat at 110 F. What horsepower is needed to drive it?

5-11. An engine operating on the Carnot Cycle develops 160 hp. The temperature at which heat is rejected is 60 F, which is 500 deg lower than that at which the engine receives heat. The heat comes from combustion of oil which has a heating value of 140,000 Btu per gal. How many gallons of oil are needed per hour?

5-12. Refer to Problem 5-11. If the temperature at which heat is rejected should rise to 220 F and if the high temperature and the horsepower developed are the same as before, how much more oil would be burned?

5-13. Prove that the exponent  $n$  equals  $k$  for isentropic compression of a perfect gas by the following method: On the  $T$ - $S$  plane, sketch an isentropic line rising from  $T_1$  to  $T_2$ ; through  $T_1$  sketch a constant-volume line; and through  $T_2$  sketch a constant-pressure line. These lines will intersect. Equate the entropy change at constant volume to that at constant pressure, solve the equation, eliminating temperature ratios, and obtain finally the following:

$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^k$$

5-14. Solve Problem 5-13 by drawing an isothermal line through  $T_2$  instead of a constant pressure line, obtain an intersection with the constant-volume line through  $T_1$ , and eventually show that  $PV^k = C$ .

5-15. Solve Problem 5-13 by drawing an isothermal line through  $T_2$  and a constant-pressure line through  $T_1$ , obtain an intersection, and eventually show that  $PV^k = C$ .

5-16. (a) A steam-engine plant receives heat at a mean temperature of 450 F and rejects heat at a mean temperature of 120 F. Calculate the maximum possible efficiency.

(b) An internal-combustion engine "receives" heat at a mean temperature of 4000 F and rejects heat at a mean temperature of 600 F. Calculate the maximum possible efficiency.

5-17. The steam plant in Problem 5-16 uses coal with a heating value of 14,000 Btu per lb, and the internal-combustion engine uses gasoline with a heating value of 20,000 Btu per lb. If each engine has an actual thermal efficiency of 35 per cent of the maximum, calculate the number of pounds of fuel used per horsepower-hour.

5-18. A Carnot-Cycle engine rejects heat at 80 F. If the heat rejected is twice the work, at what temperature is heat supplied?

5-19. (a) In the heater of Problem 4-9, is there a loss of heat from the apparatus?

(b) If the steam has an entropy of 1.7631 units per lb upon entering the heater and has an entropy of 0.3089 unit per lb when leaving the heater (in the liquid form), and a minimum temperature of 42 F is assumed, what is its change in unavailable energy, in Btu per hour?

(c) The cold water has an entropy of 0.0219 unit per lb; and after being heated its entropy is 0.3089 unit per lb. What is the change in its unavailable energy, in Btu per hour?

(d) What is the net change in unavailable energy in the heater? Does unavailable energy increase or decrease? Why?

5-20. A furnace burns 6200 lb of coal per hour. Gases enter the tubes at 2400 F and leave at 600 F. There are 14 lb of gases per pound of coal. Take the specific heat of the gases as 0.26. Water enters the boiler at its boiling temperature of 420 F, and steam leaves at the same temperature. The lowest available temperature is 100 F. Determine the loss of available energy (expressed in terms of horsepower) as a result of the heat transfer from the gases to the water.

5-21 The maximum temperature in a heat engine is 1040 F and the minimum temperature is 140 F. If the engine has an efficiency equal to 40 per cent of that of the Carnot Cycle for the same temperature limits, determine the power output when the heat supplied is 127 200 Btu per hr.

5-22 A Carnot Cycle engine using air has a cold body temperature of 140 F. During isentropic compression the pressure increases to 12 times the value before the compression. How much heat must be added per minute to produce 125 hp?

5-23 Refrigeration is desired at 0 F. A hot body is available at 100 F. Determine the refrigerating effect in Btu per minute per horsepower input to a reversed Carnot Cycle engine.

5-24 In a Carnot Cycle the pressure decreases during addition of heat from 600 psia to 300 psia. If the weight of the working substance (air) is 0.2 lb and if heat is added at 600 F and rejected at 100 F, determine the work output per cycle.

5-25 A new type of gas turbine is developed having a maximum temperature of 1200 F when exhausting at 600 F. The turbine is said to deliver 4000 hp when using 180 gal of fuel per hr with a heating value of 145 000 Btu per gal. Make calculations to show if this performance is reasonable.

5-26 A new carburetor is developed which is said to give 60 miles per gal at 50 mph. The known values for losses in the transmission system, the wind resistance of the car, bearing losses in the car, and frictional losses in the engine total 44 hp at this speed. Make calculations to show whether or not the reported 60 miles per gal is reasonable. Take the heating value of gasoline as 140 000 Btu per gal.

5-27 A newly developed refrigeration machine picks up heat at 0 F and delivers it at 90 F. When the horsepower input is 32, the reported refrigeration produced is 400 000 Btu per hr. Is this performance possible? Is it reasonable?

5-28 In a Carnot Cycle heat is to be rejected at 80 F. The lowest pressure is to be 14.7 psia. The maximum cylinder volume is to be 1.5 cu ft. The maximum allowable pressure is 1500 psia and the maximum allowable temperature is 820 F. Determine the work output per cycle and the mean effective pressure.

5-29 A Carnot Cycle heat engine drives a heat pump which operates on the reversed Carnot Cycle. The heat engine receives heat at 600 F and exhausts to a building at 100 F. The heat pump picks up heat at 30 F and delivers it to the building at 100 F. How much heat is supplied to the building when the heat input to the heat engine is 5 000 Btu per hr?

5-30 A heat engine is said to have an efficiency of 68% when rejecting heat at a mean temperature of 300 F. If this is true, what is the minimum temperature at which heat is supplied?

5-31 Air is compressed (non flow) from 14.0 psia and 80 F to 140 psia and 280 F. Calculate the change in entropy per pound of air. Do you have to know if the process is reversible?

5-32 Air is compressed isentropically (non flow) from 14.0 psia and 80 F to 140 psia. Calculate per pound (a) the increase in available energy of the air, (b) the increase in unavailable energy of the air.

5-33 A quantity of air weighing 1 lb expands in a cylinder from 140 psia and 540 F to 14 psia according to the law  $PV^{1.35} = C$ . Calculate the change in entropy.

5-34 Air weighing 3 lb expands reversibly in a cylinder doing 32 000 ft lb of work. The heat added during the expansion is 6 Btu. Initial pressure and temperature are 140 psia and 540 F. Calculate the change in entropy.

5-35 Solve Problem 5-34 for a completely irreversible expansion to 14 psia.

5-36 Air weighing 0.3 lb is compressed at a constant temperature of 40 F. The original pressure is 20 psia. If the entropy decreases 0.05 unit during the process, calculate (a) the work required and (b) the final pressure.

5-37. When 6 lb of air at 80 F are compressed at constant pressure, the entropy increases 1.0 unit. What is the final temperature?

5-38. In a water heater, water is heated from 80 F to 220 F by steam which is condensing at 260 F. The amount of water heated per hour is 4000 lb. The lowest available temperature is 80 F. How much available energy is lost per hour as a result of the heat transfer?

5-39. In a regenerator of a gas-turbine unit, the gases are cooled from 900 F to 400 F. Air enters the regenerator at 250 F. The gas turbine uses 1200 lb of fuel per hour and 50 lb of air per lb of fuel. Take the mean specific heat of the air as 0.25 and that of the gases as 0.26. The lowest available temperature is 80 F. Assuming no heat losses, calculate:

- (a) Temperature of the exit air.
- (b) Heat transferred per hour.
- (c) Unavailable energy removed from the gases per hour.
- (d) Unavailable energy added to the air per hour.
- (e) Percentage of available energy removed from the gases which is lost in heat transfer.

5-40. An automotive engine develops 80 hp when using 6.4 gal of fuel per hour. The heating value of the fuel is 140,000 Btu per gal. It is proposed to use vapor-cooling of the engine by producing steam in the jackets at 230 F. If 30 per cent of the heating value of the fuel goes into the jacket water and the lowest available temperature is 80 F, calculate the maximum horsepower which can be produced by using steam formed in the jackets to produce power.

5-41. Steam leaves a steam generator with an entropy of 1.6257 units per lb. A proposed small steam line causes such a loss of pressure that the entropy increases to 1.6422 units per lb. By use of a larger line, the entropy entering the turbine is estimated to be 1.6328 units. The steam flow is 300,000 lb per hr. If the lowest available temperature is 70 F, calculate the saving in horsepower by use of the larger pipe.

5-42. At part load, the steam flow to a turbine is controlled by throttling the steam. This throttling increases the specific entropy from 1.5878 units to 1.7027 units. At part load, the steam flow is 12,800 lb per hr. If the lowest available temperature is 90 F, calculate the horsepower lost by throttling.

5-43. Air leaves a nozzle at 80 F with a velocity of 1000 ft per sec. This velocity is reduced to an insignificant value by friction and turbulence, without a measurable change in pressure. If the lowest available temperature is 80 F, calculate the loss in available energy by destroying the velocity.

5-44. A Carnot-Cycle engine receives 100,000 Btu per hr at 600 F. Calculate the percentage increase in power output that may be obtained by lowering the cold-body temperature from 80 F to 60 F.

5-45. Solve Problem 5-44 if the heat were received at 200 F.

5-46. A steam turbine uses 400,000 lb of steam per hr to produce 65,000 hp. The steam gives up 950 Btu per lb in the condenser at a constant temperature of 90 F. If the lowest available temperature is 70 F, calculate the maximum horsepower lost as a result of heat transfer.

5-47. In a compression of air (non-flow), 4.2 Btu are removed from 0.6 lb of air as its temperature increases from 80 F to 320 F. Calculate the change in entropy.

## CHAPTER 6

6-1. At 100 F, the change in entropy during vaporization is 1.8531 units per pound of steam. How does the product  $Ts_{fg}$  compare with the value of  $h_{fg}$  at this temperature in the steam tables?

- 6-2 A constant pressure of 20 psia is maintained while 1 lb of water is heated from 80 F to 1000 F. Make a  $T$ - $S$  plot of this constant pressure heating process to scale.
- 6-3 Repeat Problem 6-2 but use a constant pressure (psia) of (a) 0.5069, (b) 14.7, (c) 100, (d) 2000, (e) 3206.2, (f) 5500.
- 6-4 Calculate the heat supplied in each case in Problems 6-2 and 6-3.
- 6-5 Calculate the change of intrinsic energy in each case in Problems 6-2 and 6-3.
- 6-6 A pound of steam at 600 psia has an enthalpy of 1200 Btu. Determine  $t$ ,  $s$ ,  $v$ , and  $u$ .
- 6-7 Repeat Problem 6-6 with an enthalpy of 1500 Btu.
- 6-8 Steam in a tank is at 200 psia and of 90% quality. Heat is added until the pressure is 250 psia. Calculate per pound (a) heat added and (b) change in enthalpy.
- 6-9 Dry saturated steam at 400 F is heated at constant temperature by the addition of 100 Btu per lb. Specify the final condition of the steam and calculate the work done and the change in intrinsic energy per pound.
- 6-10 Steam expands in a cylinder isentropically without flow from 5500 psia and 1600 F to 325 psia. Specify the final temperature and calculate the work done per pound.
- 6-11 Calculate the exponent  $n$  for the expansion of Problem 6-10.
- 6-12 Steam at 300 psia and 500 F expands isentropically without flow to 100 psia. Calculate the work done if the original volume is 3.535 cu ft.
- 6-13 A pound of steam expands isentropically without flow from 132 psia and 420 F and does 31,200 ft lb of work. Specify the final conditions.
- 6-14 A boiler receives feed water at 50 F and turns it into steam of 98% quality at 120 psia. How much heat is supplied per pound?
- 6-15 Solve Problem 6-14 for a boiler that receives feed water at 250 F and turns it into steam which issues at 600 psia and 850 F.
- 6-16 Solve Problem 6-14 for a boiler that receives feed water at 400 F under a pressure of 1500 psia and turns it into steam which issues at 1300 psia and 1050 F.
- 6-17 A quantity of water weighing 100 lb is initially at 60 F and is cooled at constant atmospheric pressure until the temperature is 0 F. Calculate the heat removed and the change in volume and in entropy.
- 6-18 Steam at 205 psia and of 99% quality enters a superheater. It leaves at 200 psia and 500 F. Calculate the heat added per pound in the superheater.
- 6-19 Steam in a cylinder is at 150 psia and of 90% quality. If 200 Btu per lb are added at constant pressure, calculate the work done per pound.
- 6-20 Steam flows through an engine expanding isentropically from the critical point to 3 psia. Calculate the work done per pound.
- 6-21 Steam flows through a turbine expanding isentropically from 1300 psia and 1050 F to 1 psia. Calculate the work done per pound.
- 6-22 Consider that the expansions of Problems 6-20 and 6-21 are cases of throttling and specify the final conditions in each case.
- 6-23 Dry saturated steam at 1400 psia flows through a series of throttling devices which successively lower its pressure to atmospheric. Specify its condition at 500, 100, 48, and 14.7 psia.
- 6-24 Steam of unknown quality at 450 psia enters a throttling calorimeter at various times and emerges at 19 psia with the following temperatures: (a) 240 F, (b) 340 F, (c) 225.24 F. In each case, what is the quality of the high pressure steam?

6-25. Make a Mollier-type ( $H$ - $S$ ) plot of the constant-pressure processes in Problems 6-2 and 6-3.

6-26. Steam is isothermally expanded from 11 psia to 1 psia. Calculate the change in specific enthalpy for each of the following temperatures at which expansion takes place: 200, 400, 800, 1600 F. What conclusions do you draw from these results?

6-27. Steam entering a surface condenser from a turbine has a moisture content of 6% at a temperature of 104 F. The condensate leaves the condenser at 98 F. The cooling medium is water, which flows through the condenser tubes; heat is transferred through the tube walls from the steam to the cooling water, which enters at 60 F and leaves at 70 F. How many pounds of cooling water are required per pound of steam condensed?

6-28. Water under a pressure of 100 psia and at a temperature of 294 F enters a high-pressure feed pump, from which it leaves at 1500 psia and 300 F. If the pumping is performed without heat loss through the pump casing, calculate the work per pound of water pumped.

6-29. Considering the universal gas constant to be 1545, show whether the perfect-gas laws are applicable to steam under each of the following conditions of pressure and temperature: (a) 1 psia and 120 F; (b) 1 psia and 500 F; (c) 14.7 psia and 220 F; (d) 14.7 psia and 500 F; (e) 450 psia and 460 F; (f) 450 psia and 1000 F; (g) 3000 psia and 700 F; (h) 3000 psia and 1600 F.

6-30. Water, initially at 6000 psia and 300 F, is throttled until it is a saturated liquid. Determine the change in  $P$ ,  $t$ ,  $v$ , and  $s$ . The saturated liquid is then further throttled until the pressure is atmospheric. Again determine the change in  $P$ ,  $t$ ,  $v$ , and  $s$ . Then make a  $T$ - $S$  plot of the three state points connected by the enthalpy line concerned.

6-31. From Keenan and Keyes Steam Tables, determine the pressure  $P$ , temperature  $T$ , weight  $W$ , enthalpy  $H$ , entropy  $S$ , volume  $V$ , quality  $x$  or degrees superheat (sup. ht.), as applicable, and the internal energy  $U$  of water in the states defined by the following data (note that all values are total quantities for the specified weights):

- (a)  $W=1$ ,  $V=1.0993$ ,  $S=1.4793$ .
- (b)  $P=700$ ,  $T=700$  F,  $S=0.3133$ .
- (c)  $P=700$ ,  $S=1.4800$ ,  $W=1.03$ .
- (d)  $U_{fg}=0$ ,  $V=5.03$ .
- (e)  $W=5$ ,  $S=8.00$ ,  $V=13.900$ , superheated 66.04 deg F.
- (f)  $W=1$ ,  $x=100\%$ ,  $S=1.7566$ .
- (g)  $T=1600$  F,  $V=1227.0$ ,  $W=0.5$ .
- (h)  $W=1$ ,  $T=0$  F,  $H=1061.8$ .
- (i)  $W=1$ ,  $S_g=2.5433=S$ .
- (j)  $P=5000$ ,  $T=700$  F,  $W=1$ .
- (k)  $P=0.08854$ ,  $V=3306$ ,  $W=3$ .

6-32. The specific entropy of saturated liquid ammonia at 80 psia and 44.40 F is 0.1982, and for the dry and saturated vapor at the same pressure it is 1.2545. The specific volume of the saturated liquid is 0.02548, and the change in internal energy during vaporization is 478.6 Btu per lb. Calculate: (a) the heat required to vaporize 1 lb of this substance at 80 psia; (b) the change in  $\frac{PV}{J}$ ; (c) the specific volume of the dry saturated vapor at 80 psia.

6-33. The equation  $PV^n=C$  usually can be applied without appreciable error to the expansion of steam in an engine cylinder. Assume that  $n=1$  for steam which has an initial pressure and temperature of 300 psia and 600 F. The total volume at the end of expansion is 3.0075 cu ft and the pressure is 100 psia. Determine: (a) the weight of steam in the cylinder; (b) the temperature at the end of expansion; (c) the work of the process in Btu per lb. (Remember that the fundamental expression for the work done in any reversible non-flow process is  $\int P dV$ .)

6-34 Suppose that the initial and final states as determined in Problem 6-33 represent the conditions of steam entering and leaving respectively a steady flow device which is perfectly insulated and in which there is no net change in velocity. Calculate the work done in Btu per lb.

6-35 When valves are closed at both ends of a pipe line 2 lb of steam at 400 F are trapped in the line. The internal volume of the pipe is 15.40 cu ft. After the pipe has cooled to the ambient temperature a gage connected to it indicates a vacuum therein of 29.00 in Hg. The barometer reading is 29.76 in. Find (a) the initial pressure of the steam (b) the final temperature of the steam (c) the amount of heat lost by the steam.

6-36 Suppose that the steam in Problem 6-21 were throttled from the specified initial conditions to 305 psia at the entrance to the turbine.

(a) How much work would not be done in the turbine by 1 lb of steam?

(b) Assuming a minimum allowable temperature of 101.74 F determine the increase in unavailable energy resulting from the throttling.

(c) Is the loss in the available energy of the steam equal to the reduction in the theoretical work of the turbine? Why?

6-37 (a) Boiling water at 20 psia is heated to 600 F at constant pressure. Determine the heat added per pound.

(b) Determine the heat added at a pressure of 400 psia.

(c) Determine the heat added at a pressure of 1400 psia.

6-38 Water enters a steam generator at 1500 psia and 300 F. Steam leaves at 1400 psia. The heat added is 1160.8 Btu per lb. Specify the steam conditions at exit. Do not neglect the effect of pressure on the enthalpy of the liquid.

6-39 Steam is to expand isentropically to 1 psia. If the maximum moisture content after expansion is not to exceed 10% and if the maximum initial temperature is not to exceed 950 F specify the maximum initial pressure which may be used with maximum moisture content after expansion.

6-40 Water enters a pump at 20 psia and 140 F. It leaves at 1500 psia. Calculate the theoretical work per pound of water by use of Table IV in Keenan and Keves and compare with the work obtained by assuming no change in volume in the pump.

6-41 Is the following steam wet, dry saturated, or superheated?

(a)  $p=100$  psia  $v=4.801$  cu ft per lb

(b)  $t=450$  F  $h=1150$  Btu per lb

(c)  $p=120$  psia  $u=1200$  Btu per lb

(d)  $t=312$  F  $u=1102.1$  Btu per lb

(e)  $p=400$  psia  $t=680$  F

(f)  $t=310$  F  $s=0.4504$  unit per lb

(g)  $p=3206.2$  psia  $t=705.40$  F

(h)  $p=80$  psia  $s=1.6207$  units per lb

6-42 Specify the conditions of steam (pressure and temperature if superheated or pressure and quality if saturated) for the following data.

(a)  $t=400$  F  $h=1239.0$  Btu per lb

(b)  $t=300$  F  $s=1.6000$  units per lb

(c)  $t=447$  F  $u=1118.6$  Btu per lb

(d)  $v=2.061$  cu ft per lb  $s=1.4451$  units per lb

6-43 Determine the internal energy of a pound of steam having a volume of 9.54 cu ft and an enthalpy of 1244.4 Btu.

6-44 Determine the values of  $v$ ,  $h$ , and  $s$  per pound of water for the following conditions.

(a) 200 psia 50 F

(b) 3000 psia 660 F

(c) 1500 psia 250 F

6-45. Water is heated from 100 F to 300 F at a pressure of 200 psia. If the heat added is calculated by neglecting the effect of pressure on enthalpy, what error is caused?

6-46. At the start of the admission process in a steam engine, the clearance space of 0.1 cu ft contains dry saturated steam at 140 psia. Dry saturated steam is supplied at 140 psia. At the end of the admission, the cylinder pressure is 140 psia, the volume is 0.4 cu ft, and the quality is 70%. Calculate the work done and the heat lost during admission.

6-47. Freon, F-12, enters an expansion valve at 80 F. It then passes into an evaporator which it leaves at 20 psia and 30 F. If the freon flow is 380 lb per hr, calculate the heat picked up by the freon per hour.

6-48. Freon, F-12, enters a condenser at 140 psia and 150 F. Freon leaves the condenser at 139 psia and 98 F. The Freon flow is 170 lb per hr. Calculate the heat removed from the Freon per hour. Do you think the effect of pressure on enthalpy of the liquid can be neglected in this case?

6-49. (a) Freon, F-12, enters a compressor at 16 psia and 0 F and leaves at 140 psia and 160 F. If the heat lost per minute is 30 Btu and the Freon flow is 35 lb per min, calculate the compressor horsepower.

(b) How is it possible for heat to be removed during the compression process when the entropy increases?

6-50. (a) Water enters a steam generator at 200 psia and 220 F, and steam leaves at 140 psia and 400 F. Determine the error caused in the calculated heat added by neglecting the effect of pressure on the enthalpy of the liquid.

(b) Solve the problem if water enters at 2000 psia and 400 F and steam leaves at 1400 psia and 900 F.

6-51. How much heat must be removed per pound, if water at 80 F is turned into ice at -20 F?

6-52. Moist air enters a heater at 10 F and leaves at 130 F. The air flow is 82,000 lb per hr. There is 0.0012 lb of vapor per lb of air. Calculate the heat added to the vapor per hour. (NOTE: As the vapor pressure is very low, neglect its effect on enthalpy. Take the enthalpy at a given temperature equal to that of saturated vapor at the same temperature.)

6-53. (a) Water is heated at atmospheric pressure from 80 F to 180 F. Calculate the error in the calculated heat added by assuming a specific heat of unity.

(b) Solve the problem for water heated from 300 F to 400 F at 800 psia.

6-54. Steam enters a turbine at 200 psia and 400 F and leaves at 16 psia to enter a second turbine. It leaves the second turbine at 1 in. Hg, abs. Assume isentropic expansions and no losses between the turbines. Compare the works of the two turbines.

6-55. In an evaporator, raw water enters at 70 F. After vaporizing, it leaves at 50 psia with 99.8% quality. Heat is supplied by steam which enters at 130 psia and 350 F. Condensate leaves at 340 F. If raw water is supplied at the rate of 4,200 lb per hr, calculate the amount of steam required.

6-56. A building, which is steam heated, loses 400,000 Btu per hr. Steam enters the building at 12 psig with 98% quality. Condensate leaves at 130 F. Machines and building occupants give off 85,000 Btu per hr. Calculate the number of pounds of steam required per hour.

6-57. To obtain reliable results, steam leaving a throttling calorimeter should possess at least 10 deg of superheat. It is estimated that the quality of steam at 160 psia is about 94%. Determine the minimum vacuum, in in. Hg, which must be obtained for the calorimeter exhaust to make it possible to determine the steam quality.

6-58. Steam at 190 psia and 640 F is throttled to 110 psia and 620 F. State two reasons why the final enthalpy is less than the original enthalpy.

6-59 (a) Steam at 400 psia and 700 F expands in a turbine to 1 psia. If the actual work is 75 per cent of the isentropic work, calculate the enthalpy per pound at turbine exit. Neglect heat losses and kinetic energy changes.

(b) Compare the change in unavailable energy in the turbine (use a lowest available temperature of 101.74 F) with the difference between the actual and theoretical turbine works.

6-60 The centrifugal compressor of a water vapor refrigeration system receives vapor at 40 F with 98% quality and compresses it to 1 psia. The vapor flow is 40 lb per min. The work of isentropic compression is 75 per cent of the actual work. Calculate (a) the compressor horsepower and (b) the volume of vapor to the compressor, in cubic feet per minute.

## CHAPTER 7

7-1 A boiler delivers dry saturated steam at 200 psia. The condenser pressure is 2 psia. For the Carnot Cycle and a saturated liquid entering the boiler, calculate (a) the heat supplied per pound, (b) the net work per pound, (c) the cycle efficiency.

7-2 Repeat Problem 7-1 for the Rankine Cycle.

7-3 A boiler delivers steam to a Rankine Cycle engine at 200 psia and 500 F. The exhaust pressure is atmospheric. Calculate

- Heat supplied per pound
- Work per pound
- Heat rejected per pound
- Pounds of steam per horsepower hour
- Thermal efficiency

7-4 A steam engine receives steam at 200 psia and 500 F. The exhaust pressure is atmospheric. The engine uses 20 lb of steam per hp hr. Calculate

- Heat chargeable per pound
- Work per pound
- Heat rejected per pound
- Waste energy per pound
- Actual thermal efficiency of engine
- Engine efficiency

7-5 Calculate the weight of steam that must be supplied per hour to a steam turbine which is to deliver 5000 hp. The turbine receives steam at 300 psia and 700 F and exhausts at 1 psia. Assume that the engine efficiency is 72%.

7-6 A steam turbine receives steam at 1400 psia and 1050 F and exhausts at 1 psia. Calculate the Rankine-Cycle efficiency and the specific steam rate including and neglecting the work of the boiler feed pump.

7-7 In a Rankine Cycle the absolute exhaust pressure is 1 in. Hg. Steam leaves the superheater at 1000 F. Calculate and plot the efficiencies (neglecting the work of the feed pump) for the following initial pressures in psia: 200, 500, 800, 1200, 2000, 3200, 5500.

7-8 A steam turbine operates on the reheating cycle. Steam enters at 1200 psia and 1000 F and expands isentropically to 180 psia where it is reheated to 1000 F. Expansion then takes place isentropically to an absolute pressure of 1 in. Hg. Calculate the thermal efficiency of the cycle and compare it with that of a Rankine Cycle.

7-9 Consider that during each part of the expansion for the reheating cycle in Problem 7-8 the engine efficiency is 75% and that for the straight Rankine Cycle conditions a turbine actually would have an engine efficiency of only 70%. Calculate the actual thermal efficiency in each case.

7-10 A steam turbine power plant operates on the extraction cycle. Steam enters the turbine at 1200 psia and 1000 F and expands isentropically to 180 psia where extraction takes place. The steam not extracted continues isentropic expansion to



an absolute pressure of 1 in. *Hg*. Calculate the thermal efficiency of the cycle and compare it with that of a Rankine Cycle.

7-11. Consider that, during the high-pressure part of the expansion in Problem 7-10, the engine efficiency is 75%; and that, during the low-pressure part, it is 68%. Calculate the thermal efficiency of the cycle.

7-12. Consider that, during both parts of the expansion in Problem 7-10, the engine efficiency is 75%; and that the steam not extracted at 180 psia is reheated at that pressure to 950 F before continuing its expansion to condenser pressure. Calculate the thermal efficiency of the cycle.

7-13. Consider that the output of the turbine in Problem 7-10 is to be 20,000 hp. Calculate the following items for both the Rankine Cycle and the extraction cycle:

(a) Weight of steam required per hour.

(b) Volume of steam per second at turbine exhaust.

(c) Amount of cooling water per second, assuming a temperature rise of 15 deg F for the cooling water and no subcooling of the condensate.

7-14. Steam enters a turbo-generator at 1400 psia and 1000 F, and leaves at 1 in. *Hg*, abs. The actual steam rate is 7.1 lb per kw-hr. Determine (a) the actual thermal efficiency and (b) the engine efficiency (R.C.R.).

7-15. Steam enters a turbine at 400 psia and 700 F, and leaves at 1 psia. The steam flow is 88,200 lb per hr. Mechanical losses in the turbine total 180 hp. The engine efficiency (R.C.R.) based on the shaft horsepower is 74%. Determine: (a) the shaft horsepower; (b) the enthalpy per pound of exhaust steam, neglecting heat losses from the steam and assuming no net change in kinetic energy.

7-16. (a) A steam boiler is capable of delivering 60,000 lb of steam per hour at 200 psia and 600 F. Determine the horsepower output of a turbine receiving this steam, if it exhausts at 1 psia and has an engine efficiency (R.C.R.) of 75%.

(b) Determine the thermal efficiency of the turbine.

7-17. A steam turbine receives steam at 330 psia and 740 F, and exhausts at 1 psia. Steam is extracted at 39 psia for feed-water heating. If the throttle flow is 133,500 lb per hr, determine for the theoretical case: (a) the turbine horsepower and (b) the thermal efficiency.

7-18. The reheating steam turbine (actual) receives 288,000 lb of steam per hour at 600 psia and 800 F, and exhausts at 1 in. *Hg*, abs. After expansion to 30 psia, the steam is reheated to 700 F. The high-pressure turbine has an engine efficiency (R.C.R.) of 78% and the low-pressure turbine has an engine efficiency of 74%.

(a) Determine the theoretical and actual enthalpy drops per pound in both turbines.

(b) Compute the actual enthalpy per pound at exit for both turbines.

(c) Determine the heat added per pound in the reheater.

(d) Find the over-all thermal efficiency.

(e) Compute the turbine horsepower.

7-19. The enthalpy of steam leaving a turbine is to be 1050 Btu per lb. The actual turbine work desired per pound of steam is 150 Btu. The exhaust pressure is 20 psia.

(a) Determine the enthalpy per pound of supply steam, the thermal efficiency, and the steam rate.

(b) Can engine efficiency be determined?

7-20. A steam turbine receives steam at 600 psia and 800 F, and exhausts at 1 in. *Hg*, abs. Steam is extracted for feed-water heating at 134 psia and 16 psia. Determine the thermal efficiency for the theoretical case. (HINT: Work with the high-pressure heater first. Assume that 1 lb of steam enters the turbine, with  $X$  lb of steam going to the high-pressure heater; and that  $(1-X)$  lb of saturated water at 16 psia leaves the low-pressure heater and goes to the high-pressure heater. Make an energy balance on the high-pressure heater and solve for  $X$ . Now make an energy balance on the low-pressure heater and solve for the steam flow to it.)

7-21 A steam turbine is to furnish dry saturated steam at its exhaust at 20 psia. The maximum allowable temperature entering the turbine is 700 F. Assume an engine efficiency of 66%. Determine (a) the maximum pressure entering the turbine and (b) the steam flow for a turbine output of 2000 hp.

7-22 A steam turbine receives steam at 2000 psia and 1100 F and exhausts at 1 psia. For the Rankine Cycle calculate

- Work per pound in the turbine
- Work per pound in the pump (use Fig. 3 in Keenan and Keyes)
- Heat supplied per pound
- Thermal efficiency

7-23 Repeat Problem 7-22 neglecting the change in enthalpy in the pump.

7-24 Specify the capacity in pounds per hour of a steam generating unit which delivers steam to a 50 000 kw turbo generator at 800 psia and 900 F. The turbine exhausts at 1 psia and has an engine efficiency based on the generator output of 78%.

7-25 Steam enters a turbine at 400 psia and 700 F and leaves at 1 psia with 90% quality. Neglecting heat losses calculate (a) the waste energy per pound and (b) the engine efficiency.

7-26 Steam leaves a steam generator at 900 psia and 900 F and enters a steam turbine at 850 psia and 880 F. The turbine has an engine efficiency of 78% when exhausting at 1.5 in Hg abs. Condensate leaves the condenser at 86 F and enters the steam generator. Calculate the thermal efficiency of the cycle.

7-27 A steam turbo generator has a heat rate of 10 600 Btu per kw-hr when supplied with steam at 1200 psia and 900 F and exhausting at 1 psia. Calculate the steam rate.

7-28 A steam turbo generator has a steam rate of 17 lb per kw-hr at rated load when receiving steam at 600 psig and 740 F and exhausting at 10 psig. The efficiency of the generator is 97% at the rated load of 3000 kw. Mechanical losses of the turbine (which are converted to heat) total 60 hp. Calculate the enthalpy of the exhaust steam.

7-29 Calculate the engine efficiencies on both the brake basis and the internal basis for Problem 7-28.

7-30 A steam turbine receives steam at 400 psia and 700 F and exhausts at 20 psia. It produces 3500 shaft hp when using 43 400 lb of steam per hour. Mechanical losses total 50 hp. Determine the internal engine efficiency.

7-31 Steam enters the high pressure unit of a reheating turbine at 1400 psia and 900 F and leaves at 300 psia to be reheated. Exhaust pressure of the low pressure unit is 1 psia. In the theoretical case it requires 192.4 Btu per lb to reheat the steam. Calculate the theoretical thermal efficiency.

7-32 A reheating steam turbine receives steam at 850 psia and 900 F and exhausts at 1 psia. It produces 33 000 hp when using 190 000 lb of steam per hour. Reheating requires 120 Btu per pound of steam. Neglecting mechanical losses calculate the enthalpy of the exhaust steam.

7-33 Solve Problem 7-8 for a reheat pressure of 72 psia. Why is the thermal efficiency slightly higher with the higher reheat pressure? Why not use a very high reheat pressure?

7-34 A steam turbine receives steam at 400 psia and 700 F and exhausts at 1 psia. It has one extraction at 44 psia for feed water heating. Assume that the internal engine efficiency of the turbine up to point of extraction is 78% and that the efficiency for the remainder of the turbine cycle is 74%. Assume the theoretical case for the extraction feed water heater. Determine the percentage of the steam entering the turbine which must be extracted for feed water heating.

7-35 If the steam flow to the turbine in Problem 7-34 is 180 000 lb per hr and mechanical losses total 1 per cent of the input to the shaft calculate the shaft horsepower of the turbine.

7-36. A steam turbine receives steam at 300 psia and 700 F and exhausts at 2 in. Hg, abs. After expansion to 35 psia, steam is removed from the turbine and reheated to 700 F. A sufficient quantity of the 35 psia steam, before being reheated, is used for feed-water heating. Assume the theoretical case for both the turbine and the heater. Calculate the thermal efficiency of the cycle.

7-37. In a binary vapor cycle, dry saturated mercury is produced at 180 psia. The exhaust pressure for the mercury turbine is 2 in. Hg, abs. The condensing mercury produces dry saturated steam at the condensing mercury temperature. The steam turbine exhausts at 2 in. Hg, abs. Determine the thermal efficiency of the theoretical cycle, using the following properties of mercury:

$P$	$t$	$h_f$	$h_g$	$S_f$	$S_g$
2 in. Hg	456.4	13.92	140.65	0.0204	0.1587
180 psia	999.5	31.02	124.29	0.0349	0.1201

7-38. Repeat Problem 7-37, using an engine efficiency of 75% for each turbine. Allow a temperature difference of 26.4 F between the condensing mercury and the boiling water by reducing the boiling temperature of the water.

7-39. A steam turbo-generator receives steam at 400 psia and 700 F and exhausts at 1 psia; and 18 per cent of the throttle steam is extracted for feed-water heating at 44 psia. The turbo-generator produces 20,000 kw when using 228,000 lbs of steam per hour. The efficiency of the generator is 97%. Calculate the brake engine efficiency of the turbine.

## CHAPTER 8

8-1. A steam engine developing 211 ihp at 225 rpm has a head-end mep of 96 psi, a crank-end mep of 98 psi, a piston rod 2 in. in diameter, a mechanical efficiency of 87%, and a single cylinder the bore of which equals the stroke. Calculate: (a) the torque at the shaft coupling and (b) the bore of the cylinder.

8-2. The engine in Problem 8-1 is supplied with steam at 225 psia and 480 F; and exhaust is atmospheric. If the engine efficiency based on ihp is 0.44, calculate: (a) the actual thermal efficiency based on bhp, and (b) the total amount of steam consumed, in pounds per hour.

8-3. A steam engine has 8% clearance and is 12" X 18". At 200 rpm it uses 2882 lb of steam per hour. Cut-off occurs at 25 per cent of the stroke, at which point the pressure is 107 psia. Compression starts 22 per cent before the end of the return stroke at a pressure of 16 psia, with the steam dry saturated. Neglecting rod area and leakages, and assuming that the same amount of steam is admitted to each side, calculate the steam quality at cut-off.

8-4. If, for the engine in Problem 8-3, release occurs at 90 per cent of the stroke, at which point the pressure is 37 psia, calculate the quality at this point.

8-5. The engine in Problem 8-3 receives steam at 120 psia and of 99% quality and has an engine efficiency of 0.60. Determine the quality at atmospheric exhaust pressure, assuming a radiation loss of 2 per cent of available energy. Show on a  $T$ - $S$  plane how the condition of the steam varies as it passes through the engine.

8-6. An engine receives dry saturated steam at 150 psia. Cut-off is at 25 per cent of the stroke. Calculate the work per pound and the thermal efficiency, assuming isentropic expansion to the end of the stroke and neglecting clearance for the following back pressures: 2, 4, 7, 10, 14.7, and 20 psia. Compare with the Rankine Cycle for the same conditions.

8-7. An engine receives dry saturated steam at 150 psia, and exhausts at 14.7 psia. Calculate the work per pound, assuming isentropic expansion to the end of the stroke and neglecting clearance, for the following cut-off points in per cent of stroke: 20, 50, 75, 100%. Determine also the per cent of loss in available energy due to incomplete expansion.

8-8 An engine receiving dry saturated steam at 150 psia has the following amounts of the steam condensed by initial condensation at the various cut-offs

% Cut-off	% Steam Condensed
5	54
10	42
15	34
20	28
30	21
40	16

Keeping in mind the results of Problem 8-7 discuss what determines the proper cut-off at rated load. Should the per cent at cut-off at rated load be the same for an engine which is to carry part load most of the time as for an engine which is normally to run near rated load?

8-9 A steam engine is to be designed for 28% cut-off at rated load. The engine is to carry 750 bhp at 200 rpm. The engine is to have twin cylinders with a stroke 1.3 times the bore and a diagram factor of 0.8. Calculate the bore and the stroke neglecting the rod diameter if the engine is to receive steam at 160 psia and 450 F and exhaust is to be at 16 psia. Mechanical efficiency is to be 92%.

8-10 A compound engine receives steam at 300 psia and 550 F and exhausts at 14.7 psia. The intermediate receiver pressure is 70 psia. The engine efficiency for the high pressure cylinder is 0.78 and that for the low pressure cylinder is 0.65. The mechanical efficiency is 91%. Calculate the specific steam rate and engine efficiency for the whole engine, based on the bhp. Why is the engine efficiency for the low pressure cylinder much lower than that for the high pressure cylinder?

8-11 A simple steam engine when supplied with dry saturated steam at 100 psia, and exhausting at 14.7 psia has the following data.

ihp	Steam lb per hr	Mechanical Eff %
25	900	72
50	1500	85
75	2100	88
100	2700	90
125	3400	91
150	4200	92

- Plot the following curves versus ihp: Willans' line, friction hp, engine efficiency based on ihp, and specific steam rate based on ihp.
- Why doesn't the steam consumption become zero at zero ihp?
- At what ihp does the bhp equal zero?
- Why does Willans' line deviate from a straight line?
- As the engine is overloaded, what happens to the steam rate and to the mechanical efficiency?

8-12 The engine of Problem 8-10 is to be a triple-expansion engine instead of a compound one. The engine efficiency for the high pressure cylinder then is 79% that for the intermediate-pressure cylinder is 74% and that for the low pressure cylinder is 69%. Select the two intermediate receiver pressures by dividing the available energy of the corresponding Rankine Cycle into three equal parts. Then calculate the specific steam rate and the engine efficiency for the whole engine based on the bhp. Finally, calculate the ratio of the sum of the individual available energies of the three stages to the available energy of the Rankine Cycle.

8-13 A poppet valve uniflow single-cylinder steam engine is to deliver 200 bhp with cut-off at 20 per cent of its stroke and at 200 rpm. It is to receive steam at 180 psia and to exhaust at 2 psia. The stroke is 1.5 times the bore, and the mechanical efficiency is 92%. Determine the bore and the stroke.

- 8-14 Determine the torque for Problem 8-13.

8-15. A simple D, slide-valve, two-cylinder engine is to produce 400 ihp at 300 rpm when exhausting at 20 psia. The engine is 13 in.  $\times$  16 in. and has 30% cut-off at rated load. Determine the initial pressure.

8-16. Using a normal diagram factor, determine the maximum possible brake horsepower of a simple D, slide-valve, two-cylinder engine receiving steam at 200 psia and exhausting at 20 psia. The engine is 12 in.  $\times$  16 in. and operates at 300 rpm. The mechanical efficiency is 92%.

8-17. A small throttle-governed engine has a no-load steam consumption equal to 10 per cent of that for rated load (ihp basis). At rated load of 60 ihp, the indicated engine efficiency (R.C.R.) is 48% when receiving dry saturated steam at 120 psia and exhausting at 16 psia. Determine: (a) the steam flow for 20 ihp; (b) the indicated engine efficiency (R.C.R.) at 20 ihp; (c) the brake thermal efficiency at 20 ihp if the mechanical efficiency is 60% at that load.

8-18. A steam engine receives steam at 20 psia and 400 F and allows it to expand until the pressure is 30 psia. Exhaust pressure is 4 psia. Assume isentropic expansion and no heat transfer to cylinder walls, and neglect clearance. Calculate the thermal efficiency. Compare it with that of the Rankine Cycle.

8-19. Solve Problem 8-18, for an exhaust pressure of 1 in. Hg, abs.

8-20. A steam engine, receiving steam at 200 psia and exhausting at 20 psia, has its cut-off at 25 per cent of the stroke. The engine is single cylinder and double acting; and when operating at 300 rpm, it produces 65 bhp. Mechanical efficiency is 90%. The engine is 10 in. by 15 in. Calculate the diagram factor.

8-21. A steam engine has an indicated mep of 60 psi on both head and crank ends. The engine is 10 in. by 15 in. and operates at 300 rpm. The piston-rod diameter is 2 in. Calculate the error caused in calculation of the indicated horsepower by neglecting the area of the piston rod.

8-22. A steam engine has a brake engine efficiency of 50%, when receiving steam at 200 psia and 400 F and exhausting at 3 psia. If the mechanical efficiency is 92%, calculate (a) the indicated thermal efficiency and (b) the number of pounds of steam per hour for 200 bhp.

8-23. A throttle-governed steam engine has a steam rate of 400 lb per ihp at rated load of 200 ihp and a mechanical efficiency of 92% at this load. The no-load steam consumption is 12 per cent of that at rated load. Determine the steam consumption, in pounds per hour, at 60 bhp if the mechanical efficiency at this load is 79%.

8-24. A steam engine, 20 in. by 24 in., operates at 150 rpm and develops 225 ihp. The engine receives steam at 200 psia and exhausts at 16 psia. If the diagram factor is 80%, calculate the per cent cut-off.

8-25. A simple D, slide-valve engine receives steam at 150 psia, dry saturated, and exhausts at 16 psia. The cut-off occurs at 30 per cent of the stroke. The engine is 8 in. by 12 in. and operates at 360 rpm. If the indicated engine efficiency is 40%, calculate the weight of the steam used per hour.

## CHAPTER 9

9-1. Air is moving in a pipe with a velocity of 600 ft per sec. It undergoes isentropic expansion between sections 1 and 2. The temperatures at 1 and 2 are, respectively, 140 F and 100 F. Calculate the velocity at section 2.

9-2. Solve Problem 9-1 if the velocity at point 1 is 60 ft per sec.

9-3. The pressure of the air at section 1 in Problem 9-1 is 100 psia. Calculate the areas at sections 1 and 2 if the air flow is 20,000 lb per hr.

9-4. Calculate the throat and exit areas of an air nozzle to expand from 80 psia and 90 F to atmospheric pressure isentropically, if the flow is to be 5000 lb per hr.

9-5 Solve Problem 9-4 if the nozzle efficiency is 98% up to the throat and is 95% over all

9-6 The first stage of a steam turbine receives steam at 600 psia and 800 F this stage discharges at 400 psia. The nozzles have an efficiency of 97% and each nozzle must pass 2000 lb of steam per hour. What is the throat area of each nozzle?

9-7 The turbine in Problem 9-6 is mistakenly supplied with dry saturated steam at 600 psia. If the nozzle discharge pressure is still 400 psia but the nozzle efficiency is only 95% what will be the flow in pounds per hour, calculated by equilibrium expansion conditions?

9-8 Solve Problem 9-7 for 100% nozzle efficiency by (a) equilibrium flow and (b) supersaturated flow

9-9 Calculate the velocity of sound in the following gases at 32 F and atmospheric pressure: air, hydrogen, nitrogen, helium, oxygen

9-10 Starting with equation 9-2b prove that acoustic velocity is expressed by equation 9-5

9-11 For isentropic expansion compute cross sectional areas for a nozzle to discharge 5 lb of air per second from initial conditions of 300 psia and 1000 F to 30 psia taking pressure increments of 30 psi. Plot area, velocity and specific volume against pressure

9-12 Repeat Problem 9-11 for steam instead of air

9-13 Compare the acoustic velocities in Problems 9-11 and 9-12

9-14 Calculate the force exerted by an air jet operating under the conditions of Problem 9-11 if the jet discharges into a medium at 30 psi

9-15 Solve Problem 9-14 if the jet after leaving the nozzle at 30 psia discharges into the atmosphere at 14.7 psia

9-16 Calculate throat and exit areas for a nozzle to discharge 5000 lb of steam per hour from 250 psia and 460 F to 14.7 psia. Neglect friction to the throat and take the over all efficiency as 92%. Assume that supersaturation is present but that equilibrium is restored by the time the exit is reached

9-17 Air expands in a nozzle from 100 psia and 140 F to 60 psia. If the exit temperature is 62 F, what is the nozzle efficiency?

9-18 A nozzle having a throat area of 0.72 sq in. and a mouth area of 1.2 sq in. receives steam at 200 psia and 600 F and exhausts at 20 psia. The over all nozzle efficiency is 90%. Friction up to the throat may be neglected. (a) Calculate the flow in pounds per second. (b) Is the mouth area correct?

9-19 The velocity of steam approaching a row of nozzles is 400 ft per sec. The nozzles receive the steam at 300 psia and 600 F. The back pressure is 180 psia and the nozzle efficiency is 92%. Determine the total exit area required for a steam flow of 112,000 lb per hr.

9-20 (a) A nozzle receives air at 200 psia and 200 F and exhausts at 40 psia. The throat area is 1.5 sq in. Determine the flow in pounds per hour, friction being neglected.

(b) Determine the flow if the initial pressure is changed to 300 psia.

(c) Determine the flow for an initial pressure of 200 psia and an initial temperature of 300 F.

(d) Explain the reason for the change in flows in parts (b) and (c).

9-21 A nozzle receives 1 lb of steam per second of 96% quality at 200 psia. The exhaust pressure is 120 psia. Neglect the effects of friction and supersaturation.

(a) Calculate the required exit area treating the wet steam as a homogeneous mixture.

(b) Calculate the required exit area assuming the wet steam to be a non homogeneous mixture. Find the area required for the 0.96 lb of dry steam and also the area required for the 0.04 lb of water.

9-22. Steam at 200 psia and 400 F expands to 110 psia. The exit area of the nozzle is 2.0 sq in. Neglect the effect of friction.

(a) Calculate the flow, assuming equilibrium expansion.

(b) Calculate the flow, assuming supersaturated expansion.

9-23. (a) Repeat Problem 9-22, using a nozzle efficiency of 97%. (NOTE: Because of the relatively small pressure drop, the effect of friction on specific volume may be neglected.)

(b) How may the effect of friction on specific volume be taken into account?

9-24. Gases enter a gas turbine at 80 psia and 1200 F, and expand to 50 psia in the first row of nozzles. The gas flow is 140,000 lb per hr. Calculate the total required nozzle exit area if the nozzle velocity coefficient is 0.98. The gases may be assumed to behave like air. No serious error will be caused by treating air as a perfect gas. See Chapter 18 for a more accurate treatment.

9-25. The desired steam velocity at exit of a row of nozzles is 1400 ft per sec. The nozzle efficiency is 92%. If the steam leaves the nozzles at 20 psia and 300 F, specify the conditions at nozzle entrance. (HINT: This will be a trial-and-error solution unless a Mollier diagram or a similar diagram is used.)

9-26. The desired air velocity at exit of a row of nozzles is 1000 ft per sec. The nozzle efficiency is 92%. If the air leaves the nozzles at 16 psia and 140 F, specify the conditions at nozzle entrance.

9-27. The air flow through a nozzle having an exit area of 2 sq in. is 2.9 lb per sec. At exit, the air is at 50 psia and 158 F. If the nozzle efficiency is 92%, specify the conditions at nozzle entrance.

9-28. The desired exit velocity of a group of steam nozzles is 1400 ft per sec. The steam enters the nozzles at 50 psia and 340 F with a velocity of 500 ft per sec. If the nozzle efficiency is 94%, specify the conditions at nozzle exit.

9-29. Dry saturated steam expands isentropically from 10 psia to 6 psia. Determine the error caused by using the suggested value of 1.3 for  $k$  when solving for the specific volume and velocity at exit instead of the true value of  $k$  given on page 82 of Keenan and Keyes, *Thermodynamic Properties of Steam*.

9-30. Gases enter a row of gas-turbine nozzles at 1500 F and 80 psia, and expand to 50 psia. Determine the error caused in the exit velocity by assuming that the gases have a value of  $k$  of 1.4 and a value of  $c_p$  of 0.24 when the true value of  $k$  is 1.33 and that of  $c_p$  is 0.27.

9-31. Assuming that steam is initially dry saturated at 80 psia and recovery will start when the expansion reaches a point where the expected moisture content is 3.5%, determine the degree of supersaturation existing just before condensation starts. Assume isentropic expansion.

9-32. A convergent-divergent steam-turbine nozzle is to have an angle of nozzle wall divergence (angle between the wall and the axis) of 6°. The nozzle receives steam at 400 psia and 700 F, and the steam expands to 85 psia. The steam flow is 4 lb per sec. The nozzle efficiency is 93.2%. If the nozzle is round in cross section, determine the length of the nozzle between the throat and the mouth. Neglect friction to the throat.

9-33. A steam-turbine nozzle is tested for efficiency by measuring the reactive force of the nozzle as the steam is accelerated in the nozzle from a negligible velocity to the exit velocity. Steam enters the nozzle at 120 psia and 500 F and leaves at 40 psia. The steam flow is 3.22 lb per sec and the reactive force is 215 lb. Calculate the nozzle efficiency. (HINT:  $F = ma$ .)

9-34. A steam-turbine nozzle receives steam at 400 psia and 600 F and has a back pressure of 80 psia. The nozzle has a throat area of 0.6 sq in. and a mouth area of 0.7 sq in. Determine the pressure right at the exit of the nozzle, neglecting the effects of friction. (HINT: This is a trial-and-error problem.)

9-35 In a gas turbine the gases enter the nozzle at 60 psia and 1400 F and exhaust at 16 psia. Treating the gases as cold air ( $c_p$  at room temperatures as far as specific heats are concerned) determine the temperature at exit if the nozzle velocity coefficient is 96%.

9-36 Dry saturated steam enters a nozzle at 200 psia and expands to 120 psia. Neglect friction. The nozzle exit area is 1.44 sq in. Calculate the flow (a) by assuming equilibrium expansion (b) by treating the expansion as supersaturated.

9-37 A nozzle receives steam at 200 psia and 400 F and exhausts at 30 psia. The steam flow is 8000 lb per hr. Neglect friction to the throat. The over all nozzle efficiency including the effects of supersaturation is 92%. Determine the throat and mouth areas.

9-38 A nozzle receives dry saturated steam at 120 psia and exhausts at 70 psia. The velocity approaching the nozzle is 400 ft per sec. The exit area is 2.0 sq in. The nozzle efficiency is 97%. Neglecting the effects of friction on specific volume calculate the rate of steam flow.

9-39 A nozzle delivers 0.8 lb of air per sec and the exit pressure is 20 psia and the exit temperature is 140 F. The exit area is 1.44 sq in. The theoretical exit velocity is 950 ft per sec. Calculate the nozzle efficiency.

9-40 A steam turbine nozzle receives steam at 400 psia and 600 F. The back pressure is 100 psia. The throat area is 1.44 sq in. and the mouth area is 1.9 sq in. Neglect friction to the throat. If the over all nozzle efficiency is 90% calculate the flow in pounds per second.

## CHAPTER 10

NOTE Unless otherwise stated the problems are for a simple impulse stage.

10-1 A simple steam turbine having one row of nozzles and one row of blades receives steam at 200 psia and 500 F. Exhaust pressure is 17 psia. The steam flow is 6000 lb per hr. The nozzle angle is  $16^\circ$ . The nozzle efficiency is 85%. The blades are equiangular. The blade velocity is 800 ft per sec. and the blade velocity coefficient is 0.88. Calculate (a) the actual velocity at the nozzle exit (b) the blade angles (c) the blade horsepower (d) the blade efficiency.

10-2 Repeat Problem 10-1 using a blade outlet angle that is 5 deg less than the blade inlet angle.

10-3 Steam leaves a row of nozzles with a velocity of 1600 ft per sec at a  $14^\circ$  angle. For equal blade angles calculate (a) the proper blade speed for maximum efficiency (b) the proper blade entrance angle and (c) the maximum blade work per pound of steam (d) What is the axial thrust?

10-4 The steam flow to the blades of a two row Curtis stage is 82000 lb per hr. The steam enters the blades with an absolute velocity of 2600 ft per sec and leaves with an absolute velocity of 500 ft per sec. If the blade horsepower is 3500 calculate the blade reheat per pound of steam.

10-5 For a particular nozzle the nozzle velocity coefficients vary with velocity as follows

Theoretical Velocity	Velocity Coefficients
500	0.974
1000	0.970
1500	0.965
2000	0.957
2500	0.937
3000	0.915
3500	0.903
4000	0.895



Calculate and compare the theoretical enthalpy drops necessary, using a blade speed of 600 ft per sec and a nozzle angle of  $12^\circ$ , for each of the following: (a) Rateau stage; (b) two-row Curtis stage; (c) three-row Curtis stage.

10-6. The blade efficiencies for the stages in Problem 10-5 are as follows: Rateau, 85%; two-row Curtis stage, 75%; three-row Curtis stage, 67%. Calculate: (a) the blade output in Btu per pound of steam and (b) the combined nozzle and blade efficiency for each of the three cases.

10-7. Steam leaves the last row of nozzles of a steam turbine at 1 psia with 87% quality and with a velocity of 2000 ft per sec. The steam flow is 500,000 lb per hr. The nozzle angle is  $20^\circ$ . There is a full admission (*i.e.*, admission of steam around the entire  $360^\circ$ ). The mean blade diameter is 8.5 ft. The blade velocity coefficient is 0.88, and the wheel speed is 1800 rpm. The blade exit angle is  $30^\circ$ . Calculate the blade exit height, assuming that the blade passage is completely filled with steam. The mean blade pitch is 1.0 in., and the blades at exit have a thickness of 0.03 in.

10-8. A 12-stage steam turbine receives steam at 300 psia and 700 F, and exhausts at 1 psia. The average stage efficiency is 74%. Assume a reheat factor of 1.07. The steam flow to the turbine is 95,000 lb per hr. Mechanical losses total 65 hp. Calculate: (a) the internal horsepower; (b) the shaft horsepower; (c) the engine efficiency based on shaft horsepower; (d) the mechanical efficiency.

Is it reasonable to have an engine efficiency greater than the average stage efficiency?

10-9. A steam turbine receives steam at 200 psia and 500 F and exhausts at 20 psia. The turbine is made up entirely of Rateau stages, all having the same mean diameter. The mean blade speed is 500 ft per sec. The nozzle angle is  $15^\circ$  and the nozzle velocity coefficient is 0.92. If the proper blade speed is to be used, calculate: (a) the actual nozzle exit velocity; (b) the theoretical enthalpy drop per row of nozzles; (c) the theoretical number of stages required if the reheat factor is 1.05.

Would the required number of stages be changed if the effects of the nozzle angle and the nozzle velocity coefficient are neglected?

10-10. A steam turbine consists of one two-row Curtis stage and two Rateau stages. The turbine receives steam at 400 psia and 700 F and exhausts at 40 psia. The efficiency of the Curtis stage is 60% and that of each Rateau stage is 70%. Assume that all three stages have the same blade speeds and that each stage takes its proper proportion of the available energy. Calculate the internal work per pound of steam and the proper exit pressure from each stage.

10-11. A reaction turbine has a mean blade speed of 600 ft per sec. The blade outlet angles are  $20^\circ$ . Use the proper steam speed for maximum efficiency. Calculate the work done per pound of steam.

10-12. For Problem 10-11, the carry-over efficiency is 52% and the nozzle efficiency is 90%. Calculate: (a) the actual enthalpy drop per row due to nozzle action; (b) the theoretical enthalpy drop per row due to nozzle action; (c) the combined nozzle and blade efficiency.

10-13. Repeat Problems 10-11 and 10-12 but use a ratio of blade speed to steam speed of 0.8.

10-14. A 50,000-kw steam turbine has the following steam consumption:

Load, kw	lb per hr
10,000	135,000
25,000	275,000
35,000	370,000
40,000	455,000
50,000	625,000

(a) Plot the Willans' line (lb per hr vs kw).

(b) Calculate and plot the steam rate, lb per kw-hr vs load.

(c) What does the steam flow at zero kw represent?

10-15 The mean blade speed of a simple steam turbine is 1 000 ft per sec. The ratio of blade speed to steam speed is 0.395. The nozzle efficiency is 88%. The exhaust pressure is to be 10 psi gage. The maximum allowable moisture in the exhaust is 7%. The engine efficiency of the turbine based on energy delivered to the shaft is 70%. Specify the initial steam conditions.

10-16 A steam turbine stage has blades 30 in. in length. The blades are mounted on a wheel having a diameter of 60 in. The turbine speed is 1800 rpm. Steam leaves the nozzles at a  $16^\circ$  angle with a velocity of 2 000 ft per sec. Calculate the proper blade inlet angle at the blade tip and also the proper angle at the blade root ( $\therefore$  base of the blade).

10-17 A steam turbine receives steam at 400 psia and 700 F. The turbine consists of two two row Curtis stages each producing approximately the same work. The average stage efficiency is 60%. The mechanical efficiency of the turbine is 98.5%. The turbine produces 8600 shaft hp with a steam flow of 122 000 lb per hr. Specify the exit pressure.

10-18 A 1000 kw steam turbine receives steam at 150 psi gage and 450 F and exhausts at 5 psi gage. Assume standard barometer. At the various loads the steam flows are as follows:

kw	lb per hr
225	9 500
325	12 000
475	16 000
700	22 200
910	27 600

Plot the following quantities vs kw:

- Pounds of steam per hour
- Steam rate
- Engine efficiency
- Thermal efficiency

10-19 Steam leaves a row of nozzles with a velocity of 1500 ft per sec. The nozzle angle is  $15^\circ$ . The ratio of blade speed to steam speed (absolute) is 0.4. Blades are symmetrical. Steam enters the blades without shock ( $\therefore$  the blade inlet angle equals that determined by the diagram). The blade velocity coefficient is 92%. Determine the blade horsepower for a steam flow of 218 000 lb per hr.

10-20 (a) Determine the total axial thrust for Problem 10-19.

(b) If the number of active blades ( $\therefore$  blades in front of nozzles at any instant) is 60, determine the steam force exerted on a blade parallel to the blade motion.

10-21 Steam leaves a row of nozzles with a velocity of 1600 ft per sec. With a steam flow of 60 000 lb per hr, the blade horsepower is 1000 and the increase in enthalpy in the blade passages is 5.0 Btu per lb. Calculate the absolute velocity at blade exit.

10-22 Steam enters the first row of a two row Curtis stage with a velocity of 2600 ft per sec. The nozzle angle is  $16^\circ$  and the blade speed is 500 ft per sec. Make blades symmetrical and make all blade inlet angles equal to the theoretical. Neglecting friction, determine (a) all blade angles, (b) the blade work per pound of steam, (c) the blade efficiency.

10-23 A steam turbine receives steam at 280 psia and 660 F and exhausts at 66 psia. For the particular type of turbine the reheat factor is 1.05 and the average stage efficiency is 70%. Neglecting mechanical losses, calculate (a) the turbine horsepower for a steam flow of 54 000 lb per hr, (b) the enthalpy per pound at exhaust.

10-24 A steam turbine is to consist of two Curtis stages each with two rows. It is to receive steam at 300 psia and 600 F and to exhaust at 40 psia. The average stage efficiency is 60%. Determine the reheat factor.

10-25. The average nozzle efficiency for the nozzles in Problem 10-24 is 88%. The ratio of the blade speed to the steam speed is 0.2. The turbine operates at 3600 rpm. Determine the mean blade diameter.

10-26. The steam flow to the blades of a stage of a reaction turbine is 83,000 lb per hr. The mean blade diameter of the stage is 36 in. and the turbine operates at 3600 rpm. The ratio of blade speed to steam speed is 0.75, and the blade outlet angle is  $20^\circ$ . The nozzle efficiency is 90% and the carry-over efficiency is 60%. Determine: (a) the blade horsepower and (b) the isentropic drop in the stage per pound.

10-27. Steam leaves a row of nozzles with a velocity of 1800 ft per sec. The nozzle angle is  $16^\circ$ . Use the proper blade speed for maximum efficiency. Blades are equiangular. Neglect friction. Calculate (a) the blade work per pound; (b) the blade efficiency; and (c) the blade angles.

10-28. Solve Problem 10-27, but use a blade speed of 200 ft per sec lower than that in Problem 10-27.

10-29. Solve Problem 10-27 but use a blade speed of 200 ft per sec higher than that in Problem 10-27.

10-30. (a) Why would a lower value of blade speed, say that given in Problem 10-28, be selected for design conditions rather than the higher values as given in Problems 10-27 and 10-29?

(b) Solve Problem 10-28, but use a blade exit angle 5 deg less than that in Problem 10-28.

10-31. Solve Problem 10-30, but use a blade velocity coefficient of 0.95.

10-32. Steam enters a simple turbine at 118 psia and 400 F and exhausts at 20 psia. The nozzle efficiency is 92% and the nozzle exit angle is  $15^\circ$ . The mean diameter of the blades is 30 in. The turbine operates at 3600 rpm. The blade velocity coefficient is 90%. The blades are equiangular. If the steam flow is 3,000 lb per hr, calculate the blade horsepower.

10-33. Steam enters a row of blades with a relative velocity of 1200 ft per sec and without shock. The blade inlet angle is  $24^\circ$  and the outlet angle is  $20^\circ$ . The blade speed is 500 ft per sec. The blade velocity coefficient is 0.92. Calculate the blade efficiency.

10-34. A row of blades produces 1900 hp with a steam flow of 72,000 lb per hr. The blade speed is 700 ft per sec and the nozzle exit angle is  $16^\circ$ . The blades are equiangular. Neglecting friction, calculate the blade efficiency.

10-35. Steam leaves a row of nozzles at 200 psia and 400 F with a velocity of 1900 ft per sec. Nozzle exit angle is  $15^\circ$ . Blade speed is 800 ft per sec. Blade velocity coefficient is 0.90. Blade exit angle is 5 deg less than the inlet angle. Determine the enthalpy at blade exit, and make an energy balance on the blades.

10-36. The steam in Problem 10-35, after leaving the blades, enters another row of nozzles with velocity of 300 ft per sec. What is the enthalpy of the steam at this point?

10-37. Steam enters the first row of blades of a two-row Curtis stage with a velocity of 2600 ft per sec. The nozzle angle is  $16^\circ$  and the blade speed is 500 ft per sec. Make all blade exit angles 4 deg less than the calculated blade inlet angle. The blade velocity coefficients in the first moving, stationary, and second moving rows of blades are 0.88, 0.91, and 0.94, respectively. Determine the work per pound and the blade efficiency. Compare your answer with that of Problem 10-22.

10-38. The steam supply in Problem 10-32 is throttled for part-load operation, the pressure being reduced to 53 psia. Assuming that the nozzle efficiency and the blade speed do not change, calculate the angle at which the steam strikes the blade. Compare with that of Problem 10-32.

10-39. Steam leaves a row of nozzles at an angle of  $14^\circ$  and with a velocity of 2000 ft per sec. The blades are 20 in. in length and are mounted on a wheel which has a

diameter of 3 ft and operates at 3600 rpm. Calculate the proper blade inlet angle at the tip and root (base) of the blades.

10-40 A group of four reaction stages receives steam at 150 psia and 400 F. The mean blade speed is 400 ft per sec. The ratio of blade speed to steam speed is 0.8. Efficiency due to nozzle action is 90% and the carry-over efficiency is 60%. The blade outlet angles are 20°. Determine the pressure at exit of the group. (HINT: Use the Mollier diagram to determine the pressure.)

10-41 The steam velocity at exit of the stationary blades of a reaction turbine is 700 ft per sec. The blade velocity is 550 ft per sec. Blade exit angles are 20°. The efficiency due to nozzle action is 90% and the carry-over efficiency is 55%. Calculate the combined nozzle and blade efficiency.

10-42 Steam enters a turbine at 400 psia and 700 F and exhausts at 1 psia. The reheat factor is 1.06. The average stage efficiency is 74%. The steam flow is 65 000 lb per hr. Mechanical losses total 120 hp. Neglecting the exit kinetic energy, determine (a) the shaft horsepower and (b) the enthalpy per pound at turbine exit.

10-43 A two-stage turbine has each stage made up of two-row Curtis wheels. The steam leaves the first stage at 100 psia and 460 F. The stage reheat of the first stage is 46 Btu. If the exhaust pressure of the turbine is 20 psia, how much of the reheat lost in the first stage is recoverable in the second stage? The efficiency of the second stage is 70%. (HINT: Find the work done in the second stage with and without reheat in the first stage.)

10-44 A two-stage turbine consists of two two-row Curtis stages. The turbine receives steam at 400 psia and 700 F and exhausts at 20 psia. The internal engine efficiency of the turbine is 78%. The intermediate pressure in the turbine is 100 psia. The efficiency of the first stage is 77%. Calculate (a) the efficiency of the second stage and (b) the reheat factor.

10-45 The exhaust steam from a turbine is to be used for heating purposes. The steam is desired at 30 psia, dry saturated. The steam flow is to be 30 000 lb per hr. The turbine generator is to produce 2000 kw. Mechanical and electrical losses total 110 kw. Assume an internal engine efficiency of 75%. Specify the conditions at turbine entrance. (HINT: Use the Mollier diagram.)

## CHAPTER 11

11-1 An air compressor receives 400 cu ft of free air per minute and discharges it at 147.0 psia. If the value of  $n$  for the compression line is 1.33 and clearance is to be neglected, calculate (a) the volume discharged in cu ft per min, (b) the temperature of the air leaving the compressor, (c) the theoretical horsepower required, (d) the heat removed by the jacket water per minute.

11-2 An air compressor has a piston displacement of 1.2 cu ft. The clearance is 5%. Atmospheric pressure is 14.2 psia, the discharge pressure is 120 psi gage, and  $n$  is 1.34. There is a pressure drop of 0.5 psi through the suction valves and a pressure drop of 2.8 psi through the discharge valves. Calculate (a) the partial volume of the expanded clearance air at the end of suction, (b) the volume of incoming air under cylinder conditions, (c) the volumetric efficiency (theoretical).

11-3 A two-cylinder double-acting compressor 10 in  $\times$  12 in runs at 400 rpm. Clearance is 4%. Atmospheric pressure is 14.0 psia and  $n$  is 1.35. The pressure drops through the suction and discharge valves are 0.6 psi and 3.0 psi, respectively. The discharge pressure is 135 psi gage. Calculate the theoretical volumetric efficiency.

11-4 Calculate the theoretical horsepower required in Problem 11-3.

11-5 An air compressor receives air at 14.5 psia and discharges it at 145 psia. Clearance is 3% and  $n$  is 1.33. Neglect drops through the valves.

- (a) Calculate the theoretical volumetric efficiency.
- (b) Calculate the efficiency for a clearance of 8%.
- (c) Calculate the efficiency for a clearance of 3% and a discharge pressure of 72.5 psia.

11-6. If the compressor in Problem 11-1 has clearance of 4%, calculate the piston displacement required per minute.

11-7. A compound compressor receives 400 cu ft of free air per minute and discharges it at 147 psia. Assume that  $n$  is 1.33 for both stages, take the proper intercooler pressure, and assume perfect intercooling. Calculate the following:

- (a) the volume leaving the low-pressure cylinder, in cu ft per min;
- (b) the volume entering the high-pressure cylinder, in cu ft per min;
- (c) the volume leaving high-pressure cylinder, in cu ft per min;
- (d) heat removed from each cylinder by the jacket water, in Btu per min;
- (e) heat removed in the intercooler, in Btu per min;
- (f) total theoretical horsepower;
- (g) savings due to the use of jackets (compare the total horsepower with that required for isentropic compression with an intercooler in use);
- (h) savings due to the use of the intercooler (compare the total horsepower with that required for a single-stage jacketed compressor).

11-8. An air compressor requires 72 ihp when receiving 500 cu ft of free air per minute delivering it at 60 psi gage. Calculate the isothermal compression efficiency.

11-9. The mechanical efficiency for the compressor in Problem 11-8 is 88%. Calculate the isothermal compressor efficiency.

11-10. A compressor is to be driven by a 50-hp motor. It receives air at 14.0 psia and delivers it at 90 psia. The isothermal compressor efficiency is 70%. Calculate the maximum amount of free air that can be compressed per minute.

11-11. The air flow from a compressor is 220 cu ft per min, measured at 110 psia and 250 F. The atmospheric pressure is 14.5 psia. Free air is at 70 F. The discharge pressure from the compressor is 115 psia. The isothermal compressor efficiency is 68%. Calculate the horsepower required.

11-12. A double-acting, two-stage air compressor, 10 in.  $\times$  12 in., is to deliver air at 200 psia. Neglecting drops through valves, calculate the diameter of the high-pressure cylinder, if the stroke there is also 12 in. Neglect the rod diameter. Assume perfect intercooling and optimum intercooler pressure.

11-13. A single-cylinder, double-acting air compressor is to be designed to deliver 300 cu ft of free air per minute at 70 psia. The bore is 80 per cent of the stroke. The speed is 300 rpm. Pressure drops through the suction and discharge valves are 0.5 and 2.0 psi, respectively. The clearance is 5%, and  $n$  is 1.35. To take care of leakages, make the compressor 7 per cent larger than otherwise required. The mechanical efficiency is 90%, and the isothermal compression efficiency is 75% (based on *outside* pressures). Calculate the bore, stroke, and shaft horsepower.

11-14. A centrifugal compressor receives air at the atmospheric pressure of 13.8 psia and 80 F, and delivers it at 40 psia. The adiabatic compression efficiency is 82% and the mechanical efficiency is 98%. The compressor handles 20,000 cu ft of free air per minute. Calculate the shaft horsepower required.

11-15. Determine the exit temperature from the compressor in Problem 11-14. Assume that the compression is adiabatic but not isentropic. The mechanical losses do not build up the air temperature.

11-16. A gas-turbine unit burns 1200 lb of fuel per hour and uses 60 lb of air per lb of fuel. The compressor of the unit receives air at 14.2 psia and 80 F. The *pressure ratio* in the compressor is 5 to 1. The adiabatic compression efficiency is 85% and the mechanical efficiency is 98%. Calculate the compressor shaft horsepower.

11-17. A three stage air compressor receives air at 14.6 psia and discharges it at 900 psia. Assume that the value of  $n$  is 1.34. The isothermal compressor efficiency is 68%. The compressor handles 200 cu ft of free air per minute. Calculate the minimum shaft horsepower.

11-18. A four stage, single acting compressor is to deliver air at 3000 psi gage. The lowest pressure cylinder has a piston 3 in. in diameter. All cylinders have the same length of stroke. Neglecting clearance, leakages and pressure drops and assuming optimum conditions of intercooling, calculate the diameter of each of the cylinders.

11-19. A two cylinder, double acting air compressor, 12 in.  $\times$  18 in. has a volumetric efficiency of 80% at 200 rpm. It has an isothermal compression efficiency of 75% and a mechanical efficiency of 92%. Atmospheric pressure is 14 psia and delivery pressure is 84 psia. Determine (a) the number of cubic feet of free air delivered per minute and (b) the shaft horsepower.

11-20. A compressor, 12 in.  $\times$  18 in., has a clearance volume of 4%. The suction pressure in the compressor is 14 psia and the discharge pressure is 84 psia. Air delivery is controlled by the use of clearance pockets. Using a value 1.35 for  $n$ , determine the volume of clearance pockets, per side, to give zero delivery of air.

11-21. A Roots type compressor is to deliver 10,000 cfm of free air. It receives air at 14 psia and discharges at 20 psia. Determine the theoretical horsepower, and compare it with the isentropic horsepower.

11-22. A centrifugal compressor receives air at 14.2 psia and 75 F, and discharges it at 32 psia and 250 F. For a flow of 14,000 cfm, the shaft horsepower is 1006. Determine (a) the adiabatic compression efficiency and (b) the mechanical losses in horsepower.

11-23. Air enters a single stage reciprocating compressor at 14.0 psia and 72 F, and leaves at 82 psia and 400 F. Is the compression adiabatic, or is heat added to or removed from the air? Give a reason for your answer.

11-24. Air enters a single stage reciprocating compressor at 14.2 psia and 68 F, and leaves at 78 psia. With an air flow of 36 lb per min. the shaft horsepower is 52 and the mechanical efficiency is 86%. The jacket water picks up 150 Btu per min. Determine either the actual temperature of the air leaving or the maximum air temperature leaving, specifying which temperature you have determined.

11-25. An air compressor has a piston displacement of 1 unit of volume. The clearance is 3.5%. At start of expansion the clearance air is at a pressure of 105 psia. At the end of suction the cylinder air is at a pressure of 13.8 psia. Atmospheric pressure is 14.2 psia. Compression obeys the law  $PV^{1.35} = C$ . Assume that expansion of the clearance air also obeys this law. Neglect heating during suction. Calculate

- Volume of expanded clearance air
- Volume of air drawn in at cylinder conditions
- Volume of air drawn in at outside conditions
- Maximum volumetric efficiency

11-26. An air compressor has 4% clearance. Air enters the compressor (not the cylinder) at 14.5 psia and leaves the compressor (not the cylinder) at 90 psia. There is a pressure drop of 0.5 psi through the suction valves and a drop of 2.5 psi through the discharge valves. Compression obeys the law  $PV^{1.35} = C$ . Calculate the theoretical volumetric efficiency.

11-27. An air compressor without clearance receives air at 14.4 psia and delivers it at 70 psia. Neglect drops through valves. The compression obeys the law  $PV^{1.35} = C$ . The piston displacement is 1.0 cu ft. Calculate the work of each of the three processes and the net work. Compare the net work with the approximate value obtained by using

$$\frac{n}{n-1} (P_1 V_1 - P_2 V_2)$$

11-28. Solve Problem 11-27 for 4% clearance. What values must be used as  $V_1$  and  $V_2$  to obtain the work by use of a cycle work equation equal to the work of the four processes?

11-29. Calculate the net work for Problem 11-27 if the compression is: (a) isothermal; (b) isentropic.

11-30. (a) Calculate the heat removed from the air in the compressor of Problem 11-27 and also for an isothermal compression by assuming that all the heat is removed during the compression process (non-flow).

(b) Solve the problem by using the steady-flow concept to find the heat removed.

11-31. A natural-gas compressor with eight cylinders, double-acting, 18 in. by 24 in., operates at 240 rpm. It receives the gas at 500 psia and 100 F and has a volumetric efficiency of 80%. Determine the number of cubic feet of gas at 14.7 psia and 60 F which is handled per minute by the compressor.

11-32. A double-acting, single-cylinder compressor is 10 in. by 12 in. and operates at 360 rpm. It receives air at 14.4 psia and 70 F and discharges at 90 psia. Clearance is 3%. Compression obeys the law  $PV^{1.35} = C$ . The pressure drops through the suction and discharge valves are 0.5 psi and 2.0 psi, respectively. Because of leakages, the compressor delivers only 96 per cent of what it could without leakage. Calculate the volume of free air handled per minute.

11-33. A given compressor is now in use. It is proposed to increase its output by installing a new motor which will be belted to the compressor and will increase the compressor speed. The motor available is a 75-hp one. The desired amount of air to be handled is 450 cfm of free air (at 14.0 psia and 80 F). The discharge pressure is 95 psia. Assume an isothermal compression efficiency of 75% and a mechanical efficiency of 92%. Will the 75-hp motor be of ample size?

11-34. A two-stage air compressor handles 500 cfm of air at 14.5 psia and 60 F. Discharge pressure is 140 psia. Assuming a value of  $n$  of 1.35, calculate the theoretical power required for (a) proper intercooler pressure; (b) an intercooler pressure of 35 psia; (c) an intercooler pressure of 65 psia.

11-35. A two-stage air compressor receives air at 14.5 psia and discharges it at 140 psia. The length of the stroke is the same for the two cylinders. The diameter of the low-pressure cylinder is 15 in., and the diameter of the high-pressure cylinder is 9 in. Neglecting pressure drops through valves and assuming perfect intercooling, determine the intercooler pressure.

11-36. A centrifugal compressor handles 12,000 cfm of free air (at 14.2 psia and 75 F). The shaft horsepower is 1,400 and the mechanical losses total 30 hp. Calculate the air temperature at exit.

11-37. Determine the adiabatic compression efficiency for Problem 11-36 if the discharge pressure is 44 psia.

11-38. The shaft horsepower of an axial-flow compressor is 2300 when receiving 16,500 cfm of air at 14 psia and 75 F and discharging at 56 psia. If the temperature of the air leaving the compressor is 418 F, how much heat is lost from the compressor per minute? Is this heat lost from the air itself?

11-39. (a) Determine the adiabatic compression efficiency in Problem 11-38. State the assumption necessary.

(b) Find the mechanical efficiency in Problem 11-38.

11-40. (a) Make a plot of the isentropic compressor work per pound of air against the ratio of discharge pressure to suction pressure. Carry the plot up to a pressure ratio of 20 to 1. Initial conditions are 14.7 psia and 80 F.

(b) Make a plot for initial conditions at 1 psia and 80 F.

## CHAPTER 12

12-1 Determine the coefficient of performance of a refrigerating machine of 5 tons capacity which requires 4 hp for its operation

12-2 If the machine in Problem 12-1 were to be used as a heat pump for heating purposes what would be considered its c p ?

12-3 How many tons of ice can be made in 24 hours by a 10 ton refrigerating machine if the ice is cooled to 14 F and was made from water which was supplied at 54 F? The specific heat of ice is 0.50

12-4 An air refrigerating machine operating between pressures of 14.7 and 147 psia is to have a capacity of 4 tons. The low pressure air enters the compressor at 24 F, and the high pressure air enters the expander at 64 F. Compression and expansion are isentropic. Calculate the net horsepower required, the c p and the piston displacement required per minute for both compressor and expander.

12-5 Repeat Problem 12-4 changing the operating pressures to 50 and 300 psia.

12-6 In a theoretical Freon F-12 compression system having isentropic compression the condenser pressure is 100 psia and the evaporator pressure is 38.58 psia. For dry compression calculate (a) the weight of refrigerant circulated per minute for 5 tons, (b) the horsepower and c p, (c) the piston displacement per minute.

12-7 Repeat Problem 12-6 using wet compression.

12-8 In a Freon compression system the evaporator pressure is 28 psia and the condenser pressure is 140 psia. Freon leaves the evaporator at 20 F and leaves the condenser at 78 F. It enters the compressor at 30 F and enters the expansion valve at 76 F. There is a drop in temperature of 10 deg between the compressor and the condenser. Neglect pressure drop through the valves and assume isentropic compression. Calculate the horsepower required for 20 tons.

12-9 For the conditions in Problem 12-8 make a T-S diagram to scale.

12-10 Make an energy balance for Problem 12-8.

12-11 Freon enters an expansion valve at 68 F, leaves the evaporator at 20 psia and 20 F and enters the compressor at 30 F. The volumetric efficiency of the compressor is 80%. The compressor is single acting and has four cylinders each 10 in  $\times$  12 in. Calculate the tonnage with a compressor speed of 300 rpm.

12-12 Repeat Problem 12-11 using evaporator pressures of (a) 8 and (b) 28 psia with respective volumetric efficiencies of 78% and 82%.

12-13 Repeat Problem 12-11, using liquid temperatures of 50 F and 80 F at the expansion valve entrance.

12-14 Repeat Problem 12-11 assuming that the fluid leaves the evaporator at 0 F and enters the compressor at 10 F.

12-15 Given the following data for a Freon refrigerating machine:

Water flow through condenser = 420 lb per min

Temperature rise of condensing water = 14 F

Freon data are as follows:

Temperature entering expansion valve = 64 F

Temperature in evaporator coils = 7.7 F

Temperature entering compressor = 20 F

Temperature leaving compressor = 150 F

Temperature entering condenser = 140 F

Saturation temperature in condenser = 132.1 F

Temperature leaving condenser = 66 F

Freon leaves the evaporator dry saturated. The compressor cylinders have fins which help to dissipate heat to the atmosphere. Assume that this heat transfer is 10 per cent of the input work.



Determine the pressures in the evaporator and the condenser from the temperatures. As the pressure drops through the compressor valves are unknown, the compression line on the  $T$ - $S$  plane is also uncertain. Estimate the horsepower per ton, assuming that there is no external "radiation" from the condenser.

12-16. Compare the cost per hour of heating a house by the conventional oil-burning heating system, with that of heating by a Freon heat pump under each of the following sets of conditions: (a) When the outdoor temperature is 10 F, the house requires 60,000 Btu per hr; for this condition, use an evaporator pressure of 20 psia. (b) When the outdoor temperature is 48 F, the house requires 20,000 Btu per hr; for this condition, use an evaporator pressure of 52 psia. In each case, assume that the condenser pressure is to be 200 psia, the isentropic compressor efficiency is to be 80%, and electricity costs  $1\frac{1}{4}$  cents per kwhr. The electric motor has an efficiency of 92%. The oil-burning conventional heating plant has an efficiency of 75%, fuel oil costs 11 cents per gal, and the heating value of the oil is 140,000 Btu per gal.

12-17. (a) Refrigeration is desired at 30 F. The refrigerating machine can reject heat at 60 F. Determine the maximum possible c.p.

(b) Determine the maximum c.p. if refrigeration is desired at  $-200$  F and heat is to be rejected at 100 F.

12-18. A refrigerating machine has a condenser pressure of 100 psia and an evaporator pressure of 20 psia. The refrigerant is Freon, F-12. Freon enters the compressor at 20 F and enters the expansion valve at 80 F. The compressor has a volumetric efficiency of 75% and an adiabatic compression efficiency of 72%. The piston displacement is 200 cfm. Neglect line losses. Determine: (a) the weight of refrigerant circulated per minute; (b) the refrigeration per pound of refrigerant; (c) the tons of refrigeration; (d) the theoretical and actual work per pound; (e) the horsepower (actual); (f) the c.p. (actual).

12-19. Freon, F-12, enters a condenser at 100 psia and 150 F, and leaves at 68 F. It enters the expansion valve at 70 F and leaves the evaporator at 20 psia and 30 F. Owing to heat leakage into the evaporator, the net tonnage is 95 per cent of that produced by the Freon. Water circulates through the condenser at the rate of 1075 lb per min, and the temperature is increased by 9.5 deg F. The heat picked up by the water in the condenser is 98 per cent of that given up by the Freon. Calculate the net tonnage of refrigeration.

12-20. Determine the tonnage of refrigeration which may be produced per 100 shaft horsepower for the following conditions: Freon, F-12, enters the expansion valve at 74 F, leaves the evaporator at 20 psia and 20 F, and enters the compressor at 20 psia and 30 F. The compressor discharge pressure is 100 psia. Assume a mechanical efficiency of 92% and an adiabatic compression efficiency of 78%.

12-21. A water-vapor refrigerating system is to produce water at 45 F. Warm water enters the evaporator at 60 F, and vapor leaves the evaporator with a quality of 98%. The compressor has an adiabatic compression efficiency of 78% and a mechanical efficiency of 98%. It is desired to provide 1500 lb per min of chilled water. The condenser pressure is 1 psia.

(a) By making an energy balance on the evaporator, compute the number of pounds of vapor produced per minute.

(b) Determine the theoretical compressor work per pound of vapor.

(c) Compute the compressor shaft horsepower.

(d) Find the c.p.

12-22. (a) Assume that in Problem 12-16(a) heat is picked up from the ground instead of from the air, permitting an evaporator pressure of 44 psia. At the same time the condenser pressure is reduced to 140 psia. Compute the cost of heating the house by a Freon heat pump.

(b) Why should a condenser pressure of 200 psia be used in Problem 12-16?

12-23 Refrigeration is desired at 0 F Heat may be rejected at 100 F Calculate the minimum horsepower required to produce 50 tons of refrigeration

12-24 A heat pump receives heat at 30 F and delivers it at 120 F If the heat pump is to deliver 100 000 Btu per hr calculate the minimum horsepower required

12-25 A heat pump is operated by an electric motor The input to the motor is 5 kw The motor efficiency is 80% For conditions of operation the c p of the unit as a refrigerator based on shaft horsepower is 4.0 The heat pump is located in the building it is to heat Calculate the total heat added to the building per hour as a result of operation of the heat pump

12-26 A refrigerating machine is to produce 100 tons Freon F-12 enters the expansion valve at 100 psia and 76 F leaves the evaporator at 16 psia and 0 F and enters the compressor at 16 psia and 10 F The volumetric efficiency of the compressor is 80% Calculate the piston displacement required per minute

12-27 The shaft horsepower of a small refrigerating machine is 2.3 Freon F-12 enters the compressor at 20 psia and 10 F and leaves at 100 psia and 150 F If the flow of refrigerant is 4.85 lb per hr calculate (a) the heat lost from compression per hour and (b) the compressor efficiency

12-28 Freon F-12 enters an expansion valve at 78 F and leaves the evaporator at 28 psia and 20 F The net tonnage desired is 25 Heat leakage into the evaporator is 4 per cent of the gross tonnage The c p of the unit based on the gross tonnage is 4.2 Calculate the horsepower required

12-29 Freon, F-12 enters a compressor at 28 psia and 30 F and enters the expansion valve at 76 F The rate of flow of F-12 is 80 lb per min The horsepower input to the compressor is 38 Heat lost from the compressor equals 12 per cent of the work input Neglecting line losses calculate the heat removed in the condenser per hour

12-30 A liquid heat exchanger is placed in a Freon F-12 system producing 50 tons of refrigeration This heat exchanger uses the vapor from the evaporator to cool the liquid The F-12 leaves the condenser at 80 F and leaves the evaporator at 20 psia and 0 F In the heat exchanger the vapor is heated to a temperature of 60 F Neglect line losses Calculate the required amount of refrigerant in pounds per minute with and without the heat exchanger

12-31 The compressor in Problem 12-30 discharges at 100 psia If the compression is isentropic determine the horsepower required with and without the use of the heat exchanger

12-32 A theoretical F-12 system operates with an evaporator pressure of 28 psia and a condenser pressure of 140 psia Assume no subcooling in the condenser and take the vapor as dry saturated when leaving the evaporator Assume that an expanding cylinder (isentropic expansion) replaces the expansion valve Calculate the net input horsepower for 10 tons of refrigeration with and without the expansion cylinder

12-33 Assume that the value of  $n$  for the compression process in a F-12 refrigerating compressor is 1.06 Clearance volume is 4% The piston displacement is 100 cfm For a constant discharge pressure of 100 psia calculate the number of cubic feet of Freon handled per minute for evaporator pressures of 8, 20, and 44 psia neglecting leakage See Chapter 11 for discussion of volumetric efficiency of compressors.

12-34 Determine the weight flowing for each evaporator pressure in Problem 12-33 if the vapor leaving the evaporator is dry saturated

12-35 A water vapor refrigeration system is to deliver chilled water at 48 F when supplied with 2000 lb of water per minute at 60 F Vapor leaves the evaporator dry saturated Calculate the number of cubic feet of vapor leaving the evaporator per minute (HINT Make an energy balance around the evaporator and find the weight of vapor produced per minute)

12-36. A theoretical Freon, F-12, compression system has vapor going to the compressor at 19.2 psia with 99.02% quality. Discharge pressure is 140 psia. The Freon enters the expansion valve at 76 F. For 20 tons, calculate (a) the number of pounds of Freon per minute; (b) the horsepower.

12-37. A F-12 refrigerating machine has an evaporator pressure of 20 psia. The condenser pressure is 100 psia. Assume that there is no subcooling and that dry saturated vapor leaves the evaporator. The tonnage produced is 50. Owing to an increase in cooling-water temperature, the condenser pressure increases to 140 psia. Calculate the increase in isentropic horsepower caused by this increase in water temperature. (NOTE: It is assumed that the compressor speed may be increased to produce the desired 50 tons with the higher condenser pressure.)

12-38. Freon, F-12, enters a compressor at 16 psia and  $-10$  F. Discharge pressure is 140 psia. Isentropic compression efficiency is 84%. Mechanical efficiency is 92%. The rate of flow of Freon is 80 lb per min. The heat lost from the compressor is 8 per cent of the shaft input. Calculate (a) the shaft horsepower; (b) the temperature of the Freon at the compressor exit.

12-39. Freon, F-12, enters a compressor at 20 psia and 10 F and leaves at 140 psia and 160 F. The mechanical efficiency of the compressor is 93%. The shaft hp is 44. For a flow of Freon of 90 lb per min, calculate (a) the heat lost from the compressor per minute; (b) the isentropic compressor efficiency.

12-40. Freon, F-12, leaves a condenser at 84 F and 110 psia. The liquid line is long, having many fittings. In addition, the elevation of the expansion valve is greater than that of the condenser. These total effects cause a pressure drop between the condenser and the expansion valve of 17 psi. If there is no gain or loss of heat in the line, specify the quality entering the expansion valve.

12-41. How much heat must be removed per pound of Freon, and to what temperature must the Freon be cooled, to prevent any formation of vapor before the expansion valve of Problem 12-40 is reached?

12-42. Solve Problem 12-6, using an evaporating temperature of  $-155$  F.

12-43. A refrigerating machine is to maintain the temperature of a cold room at  $-20$  F when the total heat flow into the cold room is 40,000 Btu per hr. Allow a temperature difference of 10 deg to cause heat flow into the evaporator. Dry saturated vapor leaves the evaporator. Cooling-water temperature is 70 F. Allow a temperature difference of 10.9 deg to cause heat flow from the condenser. Liquid Freon leaves the condenser at 76 F. Compression efficiency is 80%; mechanical efficiency is 93%; and volumetric efficiency is 76%. Calculate (a) the shaft horsepower and (b) the piston displacement of the compressor.

12-44. A refrigerating machine is to chill brine from 20 F to 10 F. The brine is sodium chloride and has a specific heat of 0.8. The refrigerating machine produces a gross tonnage of 20. Heat leakage into the evaporator of the machine is 5 per cent of the gross refrigeration. If the refrigerating machine operates 60 per cent of the time, calculate the weight of brine cooled per day of 24 hours.

12-45. A heat pump picks up 600 Btu per min in its evaporator. Liquid F-12 enters the expansion valve at 74 F. The F-12 leaves the evaporator at 44 psia and 40 F. The work input to the compressor is 15 Btu per lb, with the heat lost from the compressor being 10 per cent of the work input. Neglect line losses and calculate the following:

- Enthalpy per pound at compressor exit.
- Heat given up in the condenser, Btu per minute.
- Compressor horsepower.

## CHAPTER 13

13-1 A carburetted water gas has the following constituents by volume  $H_2$ , 40%  $CO$ , 19%  $CH_4$ , 25%  $C_2H_4$ , 8.5%  $O_2$ , 0.5%  $CO_2$ , 3%  $N_2$ , 4%. Convert this to a weight basis.

13-2 A hydrocarbon fuel contains the following mixture by weight  $H_2$ , 8%  $CO$ , 43%  $CH_4$ , 5%  $C_2H_4$ , 7%  $CO_2$ , 14%  $N_2$ , 23%. Convert this to a volume basis.

13-3 Write the complete combustion equation for each of the following fuels with theoretical air and determine the number of mols of air per mol of fuel and the number of pounds of air per pound of fuel (a)  $H_2$ , (b)  $CO$ , (c)  $C$ , (d)  $CH_4$ , (e)  $C_2H_4$ , (f)  $C_3H_8$ , (g)  $CH_3OH$ , (h)  $C_2H_5OH$ .

13-4 Solve Problem 13-3 for 25 per cent of excess air.

13-5 Calculate the number of mols of products of combustion per mol of fuel resulting in each case in Problem 13-3 and determine also the number of pounds of products per pound of fuel.

13-6 Repeat Problem 13-5 for the conditions in Problem 13-4.

13-7 The carburetted water gas in Problem 13-1 is burned with 10 per cent of excess air. Calculate (a) the volume of air supplied per cubic foot of fuel, (b) the volume of products at 14.0 psia and 500 F per mol of fuel, and (c) the volumetric analysis of the dry products of combustion.

13-8 An automobile engine burns gasoline  $C_8H_{18}$  with a carburetor set so that 18 lb of air are supplied per pound of gasoline. Calculate (a) the per cent of excess air and (b) the specific volume of the products of combustion in the exhaust line at 15 psia and 1000 F.

13-9 A diesel engine operating at light load on fuel oil  $C_{12}H_{26}$  takes in 50 lb of air per pound of fuel. Calculate (a) the per cent of excess air and (b) the specific volume of the exhaust gases at 15 psia and 500 F.

13-10 Octane is burned with 95 per cent of theoretical air. Assuming that the hydrogen is completely burned, calculate the Orsat analysis.

13-11 Using the higher heating values at constant pressure of octane and of carbon monoxide from Table 13-1, calculate the percentage loss due to incomplete combustion for the conditions in Problem 13-10.

13-12 Calculate the lower heating value at constant pressure of each of the following gases at 77 F: (a) hydrogen, (b) carbon monoxide. Express the results in Btu per cu ft.

13-13 A gasoline engine has a specific fuel consumption of 0.55 lb of  $C_8H_{18}$  per hp-hr. Determine its thermal efficiency using (a) the higher heating value of the fuel and (b) the lower heating value. The engine receives the fuel as a liquid.

13-14 An Orsat analysis taken on a steam boiler plant gave the following results:  $CO_2$ , 13%  $O_2$ , 6%  $CO$ , 0.5% and  $N_2$ , 80.5%. For each pound of coal fired,  $\frac{1}{2}$  lb of carbon was burned. Determine the air-fuel ratio by weight.

13-15 For the conditions in Problem 13-14, determine the weight of dry flue gases per pound of coal.

13-16 A quantity of blast furnace gas is found to include 10.4 mols of  $H_2$ , 53.6 mols of  $CO$ , 3.2 mols of  $CH_4$ , 0.4 mols of  $O_2$ , 16.4 mols of  $CO_2$  and 116 mols of  $N_2$ . What is its analysis in per cents by weight?

13-17 An Orsat analysis taken on the products of combustion resulting from burning the carburetted water gas of Problem 13-1 gave the following results:  $CO_2$ , 11.3%  $O_2$ , 3.7%  $N_2$ , the remainder. Determine the volume of air supplied per cubic foot of fuel and the per cent of excess air.

13-18. The carburetted water gas of Problem 13-1 is reported to have a lower heating value at constant pressure of 528 Btu per cu ft at 62 F. Calculate its higher heating value under these conditions.

13-19. How many Btu per cubic foot would have been indicated for the gas in Problem 13-18 by a calorimeter which condensed 0.03 lb of water vapor per cubic foot of fuel?

13-20. From the values given in Table 13-1, calculate the higher heating values of liquid octane and of liquid dodecane on the basis of Btu per gallon, if the octane represents gasoline with a specific gravity of 0.68 and the dodecane represents fuel oil with a specific gravity of 0.82.

13-21. A gaseous mixture consists of 8 lb of  $H_2$ , 88 lb of  $CO_2$ , 14 lb of  $N_2$ , 48 lb of  $O_2$ , and 56 lb of  $CO$ . Determine the volume of this mixture at 20 psia and 160 F.

13-22. The following gases are forced into a cylinder: 20 cu ft of  $O_2$  at 15 psia and 90 F; 30 cu ft of  $N_2$  at 16 psia and 110 F; 28 cu ft of  $CO_2$  at 18 psia and 75 F; and 60 cu ft of  $H_2O$  at 1.2 psia and 120 F. After a period of time the pressure in the cylinder is found to be 32 psia and the temperature is 150 F. Determine the gas density in the cylinder.

13-23. A particular type of diesel engine requires 35 per cent of excess air at full load. The fuel contains 84% C and 14%  $H_2$  (the rest is inert). Air is supplied at 14.4 psia and 85 F. Determine the number of cfm of air required for a 1200-hp diesel engine using 0.4 lb of fuel per hp-hr.

13-24. (a) Assuming complete combustion, determine the Orsat analysis for Problem 13-23.

(b) If the exhaust gases are at 15 psia and 600 F, determine the number of cfm produced.

13-25. A coal contains 84% C, 5%  $H_2$ , and 6%  $O_2$  (the rest is inert). When 7200 lb of coal are burned per hour, the Orsat analysis shows:  $CO_2$ , 12.8%;  $O_2$ , 6.4%;  $CO$ , 0.4%. If air is supplied at 14.4 psia and 90 F, determine: (a) the per cent of excess air and (b) the number of cfm of air.

13-26. A blast-furnace gas has the following composition:  $CH_4$ , 1%;  $CO$ , 27%;  $H_2$ , 2%;  $N_2$ , 58%; and  $CO_2$ , 12%. Determine the number of cubic feet of air required per cubic foot of fuel.

13-27. Determine the higher heating value of the gas in Problem 13-26, in Btu per cubic foot at 14.7 psia and 77 F.

13-28. When  $C_5H_{12}$  is burned with 20 per cent of excess air, to what temperature can the products of combustion be cooled before condensation starts? Assume dry air at the start.

13-29. If  $C_5H_{12}$  is burned with 20 per cent of excess air, to what temperature must the products of combustion be cooled in order to condense 1 lb of water vapor per pound of  $C_5H_{12}$ . Assume dry air at the start.

13-30. Gaseous  $C_5H_{12}$  is burned at constant pressure, and the products are cooled to the original temperature of 77 F without condensation of water vapor.

(a) Determine the difference, in cubic feet per mol of fuel, between the volume of the products and the volume of the air-fuel mixture.

(b) Calculate the work done on the atmosphere, in Btu per mol of fuel.

(c) Which is larger, the lower heating value at constant pressure or the lower heating value at constant volume?

(d) Calculate the lower heating values at constant pressure and constant volume.

13-31. Carbon dioxide is flowing in a line at the rate of 20 lb per min at 30 psia and 140 F. Nitrogen is flowing in a second line at the rate of 10 lb per min at 50 psia and 180 F. The two lines come together, the carbon dioxide and nitrogen mixing at 30 psia.

Calculate the temperature after mixing. Assume perfect gases and take  $c_p$  for carbon dioxide as 0.202

13-32 Calculate the net change in entropy for Problem 13-31

13-33 Calculate the density of the mixture in Problem 13-31

13-34 (a) Air is composed of 21% oxygen and 79% atmospheric nitrogen by volume. Calculate the weight analysis and the weight per mol of air.

(b) Using the weight per mol calculated in part (a), determine the gas constant  $R$  of air by means of the universal gas constant.

13-35 Determine the number of cubic feet of air required to burn 35 lb of benzene  $C_6H_6$  when 25% excess air is used. The air is at 14.7 psia and 60 F.

13-36 Determine the number of cfm of air for a furnace burning 30 tons of coal per hour. The coal contains 82% C, 5.5%  $H_2$ , and 8%  $O_2$  (the remainder is inert) and 40% excess air is used.

13-37 An oil-fired furnace uses 2000 gal of oil per hour (7.5 lb per gal). Assume that the oil is equivalent to  $C_{12}H_{22}$ . Use 20% excess air. Determine the volume of gases leaving the furnace per minute at 14.0 psia and 500 F.

13-38 Air is supplied in the combustor of a gas turbine at the rate of 60 lb per pound of fuel. Treat the fuel as  $C_{12}H_{22}$ . Determine the volumetric analysis of the *wet* products. Determine also the weight per mol of wet products.

13-39 Calculate the specific heat at constant pressure of the products in Problem 13-38 using Table 2-1. Take the specific heat of carbon dioxide as 0.202.

13-40 Combustion of  $C_3H_{18}$  results in the following Orsat analysis:  $CO_2$ , 13.77%;  $O_2$ , 0.79%;  $CO$ , 0.17%. Determine the per cent excess air used.

13-41 Calculate the mols of each wet product in Problem 13-30 per 100 lb of fuel.

13-42 A coal has a higher heating value of 13,500 Btu per lb. The coal contains 78% C, 6%  $H_2$ , and 8%  $O_2$ , the rest being inert. Calculate the lower heating value of the coal. (NOTE: The oxygen in the coal is combined with some of the hydrogen, reducing the hydrogen which is free to burn.)

13-43 In Problem 13-20 determine the lower heating values.

13-44 Methane  $CH_4$  is burned in air which contains 0.02 lb of water vapor per pound of air. If 20% excess air is used, calculate the number of pounds of vapor in the products per cubic foot of  $CH_4$  at 14.7 psia and 77 F.

13-45 If  $C_3H_8$  is burned in air containing 0.02 lb of water vapor per pound of air, to what temperature can the products be cooled before condensation starts? The pressure is 15 psia and 10% excess air is used.

## CHAPTER 14

14-1 Assume that 16 lb of oxygen are heated from 100 F to 500 F at constant pressure. Calculate the heat supplied: (a) under perfect gas laws; (b) by using Table 14-1.

14-2 What is the mean specific heat in each part of Problem 14-1?

14-3 Find the values required in Problems 14-1 and 14-2 but consider the oxygen to be heated from 100 F to 4540 F.

14-4 Compare the results of Problems 14-1 and 14-3 with those achieved by the use of Table 14-2.

14-5 A chemical plant operates a reciprocating compressor which receives 800 lb of methane per hour at 80 F and discharges it at 380 F. The compressor water jackets absorb heat from the methane that is equivalent to 10 per cent of the enthalpy change. What is the ihp of the compressor?

14-22 Determine the specific heats  $c_p$  and  $c_v$  of the diatomic gases listed in Table 2-1 by assuming that the molecules have energies only of translation and rotation. Compare the calculated results with those given in the table

14-23 A diesel engine produces 1240 brake hp when using 478 lb of fuel per hour. Assume that the fuel is  $C_{12}H_{26}$  and that 50 per cent of excess air is used. Air enters the engine at 60 F and gases leave at 540 F. Water is circulated at the rate of 1600 lb per min and there is a rise in its temperature of 30 deg F. Determine the amount of heat lost from the engine and not accounted for.

14-24 Using the heating values given in Table 13-1, determine the chemical energies of benzene, octane, and dodecane. The internal energy of benzene at 77 F is 397 Btu per mol.

14-25 Determine the theoretical maximum temperature in Problem 14-11 if the mixture is burned at constant volume.

14-26 Using Table 14-2 determine the mean specific heat  $c_p$  (pound basis) of water vapor: (a) between 60 F and 140 F, (b) between 540 F and 1540 F, (c) between 1540 F and 3540 F, (d) between 60 F and 1540 F.

14-27 Starting with equation 14-29 write a general expression for  $ds$  in terms of  $c_p$ ,  $P$ ,  $V$  and  $T$ . Simplify this expression for a perfect gas.

14-28 Derive an equation for the change in intrinsic energy of a gas obeying Van der Waals' equation.

14-29 Table IV in Keenan and Keyes' Steam Tables shows that the entropy of compressed water at 32 F exceeds that of saturated water. Demonstrate that this must be true by using the general equations of thermodynamics. (HINT: Use one of Maxwell's relations.)

14-30 Calculate the gas constant  $a$  and  $b$  in Van der Waals' equation for water, using the critical volumes from the Steam Tables. Why do not these values agree with those in Table 14-3?

14-31 Steam is heated from 240 F to 1000 F at constant pressure. Using the specific heat equations in Table 14-1 calculate the change in entropy per pound. Compare the results with those obtained by using steam tables at 1 psia.

14-32 Determine the chemical energy of  $CH_4$  per mol.

14-33 An internal combustion engine operates on natural gas (assume it to be  $CH_4$ ). The engine receives the gas at 200 psia and 100 F at the rate of 20 cfm. Entering air is also at 100 F and 20% excess air is used in the engine. The gases leave the engine at 1040 F. Assume complete combustion. The engine produces 2000 hp. Calculate the total heat lost from the engine per minute.

14-34 Air is supplied at the rate of 60 lb per pound of fuel in the combustor of a gas turbine. Treat the fuel as  $C_{12}H_{26}$ . The gases enter the turbine at 2000 R and leave at 1500 R. Calculate the horsepower output of the turbine if fuel is burned at the rate of 8 lb per min. Use Table 14-2.

14-35 (a) What error would be caused in Problem 14-34 if the products were treated as air and variable specific heat (Table 14-2) were used?

(b) Determine the error caused in Problem 14-34 if the products were treated as air for which  $C_p$  is 0.24.

14-36 In an internal-combustion engine the temperature at start of combustion is 540 F. The fuel is  $C_8H_{18}$  and is supplied with 20% excess air. Determine the maximum possible temperature after combustion. Assume constant volume combustion. (HINT: Write an energy balance equation and solve for the maximum temperature by trial and error.)

14-37 Solve Problem 14-36 but treat the products as air with  $c_v$  equal to 0.1715. Assume that the products are to be heated by a quantity of heat equal to the chemical energy of the fuel.

14-38. A fuel oil contains 84% C and 14%  $H_2$  by weight. It is burned in a furnace with 20% excess air. Passing through over the boiler tubes, the gases are cooled from 2500 R to 2000 R. Determine the mean specific heat  $c_p$  of the gases for this range of temperature. Use Table 14-2. (NOTE: Neglect the 2 per cent of the fuel which is unknown.)

14-39. Air enters an axial-flow compressor at 77 F and leaves at 340 F. Determine the error caused in the calculated horsepower by assuming that  $c_p = 0.24$ .

14-40. Determine the error caused in the calculation of the volume of steam by use of equation 14-8 at the following conditions:

- (a) 1 psia and 120 F.
- (b) 200 psia and 400 F.
- (c) 1000 psia and 1000 F.

14-41. Natural gas (assume  $CH_4$ ) leaves a compressor station in a pipe line at 900 psia and 150 F. Calculate the error caused in the calculation of the density of the gas by treating it as a perfect gas. Assume that Fig. 14-6 gives means of getting true density.

14-42. Check the latent heat of steam at atmospheric pressure by means of the Clapeyron equation, using steam table values for other quantities in the equation.

14-43. Calculate the error caused by the use of Van der Waals' equation in Problem 14-41.

14-44. Air is heated at a constant pressure of 20 psia from 140 F to 1540 F. Determine whether the air can be treated as a perfect gas by considering the following questions:

- (a) Is  $PV$  equal to  $WRT$ ?
- (b) Is the specific heat constant?

14-45. Carbon dioxide, used for refrigeration, leaves a compressor at 900 psia and 200 F. Determine its density by use of: (a) perfect-gas laws; (b) Van der Waals' equation; (c) Fig. 14-16.

## CHAPTER 15

15-1. Air at a barometric pressure of 30.46 in. Hg and a temperature of 90 F has a dry-air partial pressure of 29.90 in. Determine the relative humidity and the dew point.

15-2. Air at standard barometric pressure and 90 F has a wet-bulb temperature of 80 F. Determine the relative humidity, the dew point, and the specific humidity.

15-3. Determine the absolute humidity in Problems 15-1 and 15-2.

15-4. Air at 90 F is saturated under a total pressure of 14 psia. Determine the density of the dry air, the specific humidity, and the absolute humidity.

15-5. Calculate the enthalpy of saturated air, in Btu per pound of dry air, for Problems 15-1, 15-2, 15-3, and 15-4.

15-6. Air at 13.5 psia and 40 F, having a wet-bulb temperature of 33 F, flows through a unit heater from which it emerges at 100 F. Calculate: (a) the relative humidity before and after heating, and (b) the heat supplied per pound of dry air.

15-7. Air at 20 F having a dew point of 10 F is heated at standard atmospheric pressure to 90 F. Calculate: (a) the relative humidity before and after heating, (b) the absolute humidity before and after heating, and (c) the heat supplied per pound of dry air.

15-8. Air at 12 psia and 92 F, having a relative humidity of 80%, is cooled to 60 F while passing refrigerating coils. Calculate: (a) the relative humidity before and after cooling, (b) the absolute humidity before and after cooling, and (c) the specific humidity before and after cooling.

15-9. If, in Problem 15-8, 1000 cfm enter at 92 F, determine the heat transferred, in Btu per minute.



15-10 Air at 16 psia and 70 F, with a relative humidity of 20%, is run through a spray of chilled water (40 F) and emerges with its temperature at 60 F. What are its final specific humidity and its final relative humidity? Assume that weight of spray water supplied equals weight evaporated.

15-11 Outside air at 5 F and 80% relative humidity enters a warm-air furnace, to which water is supplied at 50 F for humidification, and air leaves the furnace at 130 F, at which temperature it enters the upstairs of the house. Here it is cooled to room conditions of 70 F and 50% relative humidity. Neglect vaporization in the house and assume that heat losses transmitted from the house amount to 60 000 Btu per hr. Calculate (a) the number of pounds of dry air per minute required to maintain the house at the specified temperature, (b) the number of pounds of water consumed per 24 hours, and (c) the heat added per hour in the furnace.

15-12 Repeat Problem 15-11, assuming that half of the air entering the furnace is recirculated air at 70 F.

15-13 Air at 130 F, having a wet bulb temperature of 70 F, enters a drying tunnel containing wet products. If the air leaves adiabatically saturated, calculate (a) the dew point of the entering air, (b) the specific humidity of the entering air, (c) the specific humidity of the exit air, and (d) the number of pounds of moisture removed from products per 1000 cu ft of entering air.

15-14 Consider that the rate of flow of the air in Problem 15-13 is changed so that the air leaves at 80 F and has a wet bulb temperature of 70 F. Calculate the rems listed and also the dew point of the exit air.

15-15 The air in an auditorium of 2000 seats is to be conditioned to 74 F dry bulb and 60% R.H. Outside conditions are 92 F and 55% R.H. Air is to be supplied at the rate of 30 cu ft per min per person (auditorium conditions). Calculate (a) the tonnage required in the dehumidifier, and (b) the heat that must be supplied per minute to the heater.

15-16 Repeat Problem 15-15 using 30 per cent of make-up air (by weight of dry air) and the rest recirculated air. Recirculated air leaves the auditorium at 80 F and 54% R.H.

15-17 A certain house with air at 73 F and 28% R.H. has an infiltration of 15 000 cu ft of air per hour. Heat lost through the walls is 80 000 Btu per hour, measured at outside conditions of 10 F and 60% R.H. The house will be just as comfortable at 70 F and 55% R.H. Assume that the heat lost through the walls is directly proportional to the difference between the inside temperature and the outside temperature, and that the amount of infiltration does not change. Is there a saving or a loss if the house temperature is lowered to 70 F? (NOTE: Air that infiltrates must be heated up to house temperature and its humidity must be increased to the desired amount by vaporizing and heating water which comes in at 50 F.)

15-18 Solve Problem 15-17 if the outside conditions are 50 F and 60% R.H., and the heat lost through the walls is 22 200 Btu per hr for an inside temperature of 73 F.

15-19 (a) From a study of Problems 15-17 and 15-18, what can you say about the desirability from an economical standpoint of humidifying a house?

(b) Will the amount of infiltration change your answer?

15-20 A well insulated auditorium seating 2500 persons is supplied with 30 cfm per person of air at 70 F and 50% relative humidity. 60 per cent of this air is recirculated. Each hour a person gives off 384 Btu and adds 0.153 lb of moisture to the atmosphere. The outside air is at 0 F and is 100% saturated. This outside air is first run through a tempering heater from which it emerges at 40 F, and is then mixed with the recirculated air from the auditorium. The mixture is washed in a spray chamber and is given a final heating of 70 F. The temperature of the water supplied to the spray is 48 F. Calculate the following:

(a) The specific humidity, the weight of dry air per minute, and the enthalpy of the air entering the auditorium.

(b) The specific humidity and the enthalpy of the air leaving the auditorium.

(c) The weight of dry air recirculated per minute.

(d) The specific humidity and the enthalpy of the outside air.

(e) The relative humidity and the enthalpy of the air leaving the tempering heater.

(f) The specific humidity and the enthalpy of the air leaving the mixer.

(g) The number of pounds of moisture added per pound of dry air in the spray chamber.

(h) The Btu per minute supplied in the tempering coil.

(i) The Btu per minute supplied in the final coil heater.

15-21. Air is saturated at 14.5 psia and 80 F. Compute the density of the saturated air and compare it with that of dry air at the same temperature and pressure.

15-22. Air is desired with a relative humidity of 5% at 90 F. To what temperature must it be cooled if it is completely saturated at the low temperature?

15-23. Air at 85 F has a specific humidity of 0.014. How much water can be removed per pound of dry air if the air can be cooled to 35 F?

15-24. A refrigerating machine can cool air to 45 F, where the relative humidity is 95%. Determine the number of pounds per minute of dry air which may be cooled by a 20-ton refrigerating machine if the air enters at 90 F and 60% relative humidity.

15-25. (a) By use of Carrier's equation, determine the vapor pressure and specific humidity for 85 F dry bulb and 72 F wet bulb, for the following barometric pressures: 15 psia, 14 psia, 12 psia, and 10 psia.

(b) Draw conclusions from part (a) in regard to the error caused by neglecting the effect of barometric pressure when determining specific humidities by use of a psychrometric chart.

15-26. (a) Determine the specific volumes in Problem 15-25 (a).

(b) Answer Problem 15-25(b) for specific volumes instead of specific humidities.

15-27. (a) Determine the specific enthalpies in Problem 15-25 (a).

(b) Answer Problem 15-25(b) for specific enthalpies instead of specific humidities.

15-28. Air is supplied to a drying chamber at 140 F and 20% relative humidity. It leaves with a relative humidity of 95%. Water is to be vaporized in the drying chamber at the rate of 40 lb per min. Neglect the internal energy of the liquid water and, assuming adiabatic action, determine the number of cfm of air to be supplied.

15-29. Air has a dry-bulb temperature of 80 F and a wet-bulb temperature of 70 F. Atmospheric pressure is 14.2 psia. Determine: (a) the dew point, (b) the relative humidity, and (c) the specific humidity.

15-30. Air has a dry-bulb temperature of 85 F and a wet-bulb temperature of 60 F. Atmospheric pressure is 14.5 psia. Determine: (a) the dew point, (b) the relative humidity, and (c) the specific humidity.

15-31. In a certain part of a steam condenser, the measured temperature and pressure are 82 F and 1.175 in. Hg, abs., respectively. (NOTE: Because of condensation, the vapor is saturated.) Determine: (a) the vapor pressure, (b) the dry-air pressure, and (c) the number of pounds of air per pound of vapor.

15-32. Calculate the true enthalpy at standard atmospheric pressure, in Btu per pound of dry air, and compare it with that at the wet-bulb temperature for the following conditions:

(a) 40 F dry bulb and 34 F wet bulb.

(b) 80 F dry bulb and 70 F wet bulb.

(c) 95 F dry bulb and 78 F wet bulb.

15-33. Air has a dry-bulb temperature of 90 F and a reported dew-point temperature of 74 F. The correct dew point is 75 F. Atmospheric pressure is 14.3 psia. Calculate the per cent error caused in the specific humidity by the incorrect dew point.

15-34 Air at 14.6 psia has a dry bulb temperature of 90 F and a reported wet bulb temperature of 75 F. The true wet bulb temperature is 74 F. Calculate the per cent error in the specific humidity caused by the incorrect wet bulb temperature.

15-35 Air at 14.6 psia has a wet bulb temperature of 74 F and a reported dry bulb temperature of 91 F. The true dry bulb temperature is 90 F. Calculate the per cent error in specific humidity caused by the incorrect dry bulb temperature.

15-36 Compute the specific humidity of saturated air at 14.7 psia and the following temperatures: -40 F, 0 F, 40 F, 80 F, and 120 F.

15-37 The vapor pressure of air at standard atmospheric pressure and 90 F is increased from 0.3 psia to 0.6 psia. Calculate the percentage increase in specific humidity.

15-38 Air at standard atmospheric pressure is at 90 F and 50% R.H. It is cooled to 70 F. If the original volume is 3,500 cu ft, compute the final volume.

15-39 Air at standard atmospheric pressure is at 90 F and 50% R.H. It is cooled to 50 F. If the original volume is 3,500 cu ft, compute the final volume.

15-40 Air at standard atmospheric pressure is at 50 F and 60% R.H. It is heated to 90 F and 0.01 lb of moisture is added per pound of dry air. If the original volume is 2,500 cu ft, compute the final volume.

15-41 Air is to be heated from 40 F to 120 F at standard atmospheric pressure. At 120 F the air volume is to be 5,000 cu ft and the R.H. is to be 20%. If the original relative humidity is 40%, calculate the weight of water to be added.

15-42 It is desired to get 20,000 cu ft of air per minute at 74 F and 60% R.H. This is obtained by taking air at 90 F and 55% R.H., cooling it until sufficient dehumidification has been obtained, and then heating it to 74 F. Calculate the amount of water vapor condensed per minute.

15-43 A building having a volume of 60,000 cu ft is to have one air change per hour. If the building is to be maintained at 70 F and 45% R.H. and outside conditions are 0 F and 80% R.H., calculate the amount of water to be vaporized per hour to maintain the relative humidity in the building. Assume standard atmospheric pressure.

15-44 Air at standard atmospheric pressure, 90 F and 65% R.H. is cooled to 50 F and is then reheated to 75 F. Calculate the heat removed per pound of dry air, considering and neglecting the enthalpy of the condensate.

15-45 Calculate the final relative humidity and the heat added per pound of dry air in Problem 15-44.

15-46 Air at 70 and 14.7 psia has a relative humidity of 60%. Calculate the wet bulb temperature. (SUGGESTION: Use Carrier's equation with a trial and error solution.)

15-47 Air at standard atmospheric pressure is at 90 F and 50% R.H. It is cooled to 40 F (assume 100% R.H. at this temperature) and then is heated to 140 F and piped to a drying chamber. It leaves the drying chamber with the same wet bulb temperature it had at entrance and with a R.H. of 95%. Calculate the moisture picked up per minute in the drying chamber if the original volume of the air is 12,000 cfm.

15-48 A dehumidifier system uses recirculation. There are 10,000 cfm of recirculated air at 80 F, 60% R.H. and 14.7 psia, and there are 5,000 cu ft of outside air at 90 F and 65% R.H. Determine:

(a) Specific humidity of the mixed air entering the dehumidifier

(b) Enthalpy per pound of dry air entering the dehumidifier

15-49 Air at 14.2 psia, 50 F and 60% R.H. is heated to 90 F and 50% R.H. The water for vaporization is supplied at 60 F. Calculate the error caused in the heat added by neglecting the enthalpy of the water added.

15-50 If 40 per cent of the temperature drop from a room to the outside air through a window occurs between the room and the inner glass surface and there is condensation

on the glass, compute the minimum relative humidity in the room. The room is at 72 F, and the outside temperature is 0 F.

## CHAPTER 16

16-1. Calculate the thermal efficiency on the air-standard basis for an Otto Cycle having each of the following compression ratios: (a) 2, (b) 4, (c) 5.5, (d) 6.4, (e) 7.8, (f) 10, (g) 16.

16-2. For Problem 16-1, plot a curve of efficiency versus compression ratio.

16-3. In an air-standard Otto Cycle, with a compression ratio of 6.1 to 1, the heat added per pound of air is 1000 Btu. The conditions at the start of compression are 14.7 psia and 40 F. Calculate the following quantities:

- (a) The pressures, temperatures, and specific volumes for all points of the cycle.
- (b) The heat rejected per pound.
- (c) The work per pound for each process.
- (d) The thermal efficiency.
- (e) The mep.

16-4. Plot the cycle in Problem 16-3 to scale, using both  $P$ - $V$  and  $T$ - $S$  coordinates.

16-5. Solve Problem 16-3 if only 500 Btu per pound of air are added.

16-6. Superpose a plot of the cycle of Problem 16-5 on that of Problem 16-4, using  $P$ - $V$  and  $T$ - $S$  coordinates.

16-7. An air-standard Otto Cycle has the following temperatures: before compression, 100 F; after compression, 740 F; before expansion, 4000 F. Calculate the temperature at the end of expansion and the thermal efficiency.

16-8. Calculate the heat supplied and the change in entropy per pound of air in Problem 16-7.

16-9. Compare the theoretical horsepowers of two air-standard Otto-Cycle engines in each of which 10 lb of air per minute go through the cycle of operations. In both engines, the temperature before compression is 120 F and the compression ratio is 7, but one engine starts to reject heat at 1800 F while the other starts to reject heat at 2800 F.

16-10. Compare the theoretical horsepowers of two air-standard Otto-Cycle engines in each of which 10 lb of air per minute go through the cycle of operations. In both engines, the minimum temperature is 120 F and the maximum temperature is 5000 F, but one engine has a compression ratio of 4.5 and the other has a compression ratio of 7. Also calculate the temperature at which each engine starts to reject heat.

16-11. For each of the engines in Problems 16-9 and 16-10, the minimum cylinder pressure is 13 psia. Calculate the maximum pressure.

16-12. In an air-standard Diesel Cycle with a 15.8 compression ratio, 1000 Btu are added per pound of air. The conditions at the start of compression are 14.7 psia and 40 F. Calculate the following:

- (a) The pressures, temperatures, and specific volumes for all points of the cycle.
- (b) The heat rejected per pound of air.
- (c) The work per pound for each process.
- (d) The cycle efficiency.
- (e) The mep.

16-13. Plot the cycle in Problem 16-12 to scale, using both  $P$ - $V$  and  $T$ - $S$  coordinates.

16-14. Solve Problem 16-12 if only 500 Btu per pound of air are added.

16-15. Superpose a plot of the cycle of Problem 16-14 on that of Problem 16-13, using  $P$ - $V$  and  $T$ - $S$  coordinates.

16-16. Solve Problem 16-12 as a "limited pressure" or Dual Cycle. Add 500 Btu at constant volume and the rest at constant pressure.

16-17 Plot the cycle of Problem 16-16 to scale using  $P$ - $V$  and  $T$ - $S$  coordinates

16-18 An air standard Diesel Cycle has the following temperatures before compression 100 F after compression 740 F before expansion 4000 F Calculate the temperature at the end of expansion and the thermal efficiency

16-19 Solve Problem 16-3 for a Brayton Cycle instead of an Otto Cycle.

16-20 Plot the cycle of Problem 16-19 to scale using  $P$ - $V$  and  $T$ - $S$  coordinates

16-21 An engine 8 in  $\times$  10 in has a 6 to 1 compression ratio Using the air standard Otto Cycle with conditions of 14 psia and 100 F at the start of compression and a heat added of 1100 Btu per pound of air determine (a) the maximum cycle work of the engine and (b) the maximum mep

16-22 An air standard Diesel Cycle has a 15 to 1 compression ratio and an expansion ratio of 5 to 1 At the start of compression the temperature is 60 F and the pressure is unknown Determine the cycle efficiency

16-23 Repeat Problem 16-22 for an initial temperature of 120 F

16-24 For the cycle specified in Problem 16-3 but with an initial temperature of 120 F determine the thermal efficiency and the mep

16-25 (a) At the start of compression in an air standard Brayton Cycle the pressure is 14.0 psia and the temperature is 120 F The isentropic compression ratio is 3 to 1 and the heat added per pound of air is 1300 Btu Determine the cycle efficiency and the mep

(b) What effect does the compression ratio have on the efficiency and the mep?

16-26 An air standard Otto Cycle has an mep of 250 psi and a cycle efficiency of 54% Determine the heat added per pound of air if conditions at the start of compression are 14 psia and 60 F

16-27 In Problem 16-16 assume that 800 Btu are added at constant volume and 200 Btu are added at constant pressure Determine the cycle efficiency

16-28 In an air standard Brayton Cycle isentropic compression starts at 14.0 psia and 60 F The maximum pressure is 70 psia and the maximum temperature is 2000 F Determine (a) the cycle efficiency (b) the cycle work per pound of air (c) the mep

16-29 An air standard Otto Cycle has a 7 to 1 compression ratio At start of compression conditions are 14 psia and 70 F The maximum allowable pressure is 1000 psia Calculate (a) the cycle thermal efficiency and (b) the mep

16-30 An air standard Otto Cycle has a 7 to 1 compression ratio At start of compression conditions are 14 psia and 70 F The maximum allowable temperature is 6000 F Calculate (a) the cycle thermal efficiency and (b) the mep

16-31 An air standard Diesel Cycle has a 15 to 1 compression ratio At start of compression conditions are 14 psia and 70 F The maximum allowable temperature is 4500 F Calculate (a) the cycle thermal efficiency and (b) the mep

16-32 An air standard Diesel Cycle has an 18 to 1 compression ratio and a 6 to 1 expansion ratio (ratio of the volume after to the volume before isentropic expansion) If conditions at start of compression are 14.7 psia and 80 F calculate (a) the net cycle work per pound of air and (b) the cycle thermal efficiency

16-33 An air standard Otto Cycle has a 6.5 to 1 compression ratio and a mep of 220 psi If conditions at start of compression are 14.5 psia and 140 F calculate the maximum temperature

16-34 An air standard Dual Cycle has a compression ratio of 13 to 1 The conditions at start of compression are 14 psia and 70 F Maximum allowable pressure is 1000 psia and maximum allowable temperature is 5000 F Calculate (a) the cycle thermal efficiency and (b) the mep

16-35. An air-standard Otto Cycle is to be designed for a thermal efficiency of 51.2% and a mep of 240 psia. If conditions at start of compression are 14.2 psia and 75 F, calculate the maximum cycle temperature.

16-36. Solve Problem 16-3 for an initial temperature of 140 F. What effect does initial temperature have (a) on thermal efficiency and (b) on mep?

16-37. An air-standard Diesel Cycle has a 14 to 1 compression ratio and a mep of 190 psia. If conditions at start of compression are 14.5 psia and 100 F, calculate the cycle thermal efficiency.

16-38. An air-standard Otto Cycle has the following temperatures: start of compression, 80 F; maximum temperature, 6200 F; temperature at end of expansion, 2200 F. If the initial pressure is 14 psia, compute the cycle efficiency and the mep.

16-39. During addition of heat in an air-standard Diesel Cycle, the volume is tripled. The compression ratio is 15 to 1. Conditions at start of compression are 14.3 psia and 85 F. The weight of air present is 0.2 lb. Calculate (a) the cycle work and (b) the cycle thermal efficiency.

16-40. An internal-combustion engine has a piston displacement of 0.4 cu ft and a compression ratio of 6 to 1. The engine makes 2000 cycles per min. Conditions at start of compression are 13.8 psia and 120 F. Assume that the equivalent heat added per pound of air is 800 Btu. Using the air-standard Otto Cycle, determine the horsepower.

16-41. An air-standard Dual Cycle has a 5 to 1 expansion ratio and a 15 to 1 compression ratio. Assume that the heat added at constant pressure equals the heat added at constant volume. If the temperature at start of isentropic compression is 110 F, calculate the cycle thermal efficiency.

16-42. Solve Problem 16-19 for an initial pressure of 100 psia instead of 14.7 psia. This is done to minimize the size of both the compressor and the power cylinders. Do you recommend this procedure?

16-43. Work Problem 16-28 but increase the maximum temperature to 4000 R. What effect does maximum temperature have on cycle thermal efficiency and mep?

16-44. An air-standard Diesel Cycle has a 16 to 1 compression ratio. The temperature at start of compression is 120 F. The temperature at end of expansion is 2000 F. Calculate the cycle thermal efficiency.

16-45. An air-standard Otto Cycle and an air-standard Diesel Cycle each reject 300 Btu per pound of air. The conditions at start of compression for both are 14 psia and 80 F. Both have the same maximum temperature of 6000 F. Calculate and compare the cycle works per pound of air and the cycle thermal efficiencies.

## CHAPTER 17

17-1. An eight-cylinder automobile engine having the dimensions  $3\frac{1}{4}$  in.  $\times$   $4\frac{7}{8}$  in. develops 135 bhp at 3400 rpm. Calculate the torque and the bmep.

17-2. The engine in Problem 17-1 is used in an automobile in which the rear-axle ratio is such that the engine makes 3.35 revolutions to 1 revolution of the wheels when the car is in high gear. If the outer diameter of the tire is 29 in., what is the car's speed in miles per hour when the engine makes 3400 rpm?

17-3. The engine in the preceding problems has a compression ratio of 6.70 to 1, and consumes gasoline having a higher heating value of 20,000 Btu per lb. If, under the conditions in Problem 17-1, the brake relative efficiency is 70% with theoretical air-fuel ratio, determine the brake specific fuel consumption.

17-4. Refer to Problem 17-3 and determine the volumetric efficiency, based on air at 14.7 psia and 60 F.

17-5 Some states fix motor vehicle taxes on a so called 'taxable horsepower which is taken as  $0.4 n D^2$  where  $n$  is the number of cylinders and  $D$  is the diameter in inches. Derive this formula on the basis of a single-acting four stroke cycle engine having a bmep of 67.5 psi and an average piston speed of 1000 ft per min.

17-6 Calculate the taxable horsepower of the engine in Problem 17-1 and discuss why this differs greatly from the bhp.

17-7 A six cylinder four stroke cycle diesel engine is single acting and has cylinders 5 in  $\times$  7 in. When running at 1200 rpm it has a mechanical efficiency of 60% and a volumetric efficiency of 90% based on air at 14.7 psia and 60 F. If the air fuel ratio is 22 the fuel has a heating value of 19 000 Btu per lb and 7600 Btu are supplied per bhp hr, calculate the bhp and the ihp.

17-8 During a 20 minute dynamometer test of a six cylinder automobile engine  $3\frac{1}{2}$  in  $\times$   $3\frac{1}{2}$  in the following data were recorded

Speed 2260 rpm

Torque 136 lb ft

Weight of gasoline used 10.9 lb

Weight of cooling water 591 lb (supplied from water main and weighed after having passed through the engine cooling system)

Air fuel ratio by weight 15

Temperature of air and fuel supplied 77 F

Temperature of exhaust gases 1600 F

Temperature of cooling water supplied 60 F

Temperature of cooling water from engine 162 F

Barometer 29.55 in Hg

Assuming that the gasoline has a heating value of 20 000 Btu per lb determine as a percentage of the heating value each of the following items

(a) useful work output

(b) energy loss due to jacket water

(c) energy loss due to dry products of combustion ( $c_p = 0.25$ )

(d) energy loss due to combustion of hydrogen (consider that 1.42 lb of  $H_2O$  are formed per pound of fuel and obtain enthalpy from superheated steam table at 1 psia and 1600 F)

(e) unaccounted for (obtained by difference)

17-9 Calculate the volumetric efficiency of the engine in Problem 17-8

17-10 Calculate the bmep of the engine in Problem 17-8

17-11 A diesel engine uses 0.42 lb of fuel per bhp hr. The compression ratio is 14 to 1 and 40 per cent of excess air is used. The heating value of the fuel is 19 000 Btu per lb. The mechanical efficiency is 75%. Calculate (a) the brake relative efficiency and (b) the indicated relative efficiency.

17-12 Calculate the brake fuel rate of an automotive engine having a compression ratio of 6 to 1 and using 90 per cent of theoretical air. The heating value of the fuel is 20 200 Btu per lb. The mechanical efficiency is 72%. Assume an indicated relative efficiency of 85%.

17-13 Calculate the brake fuel rate of an aircraft engine having a compression ratio of 7 $\frac{1}{2}$  to 1 and using 95 per cent of theoretical air. The heating value of the fuel is 20 200 Btu per lb. The mechanical efficiency is 88%. Assume an indicated relative efficiency of 89%.

17-14 A large slow speed diesel engine has a compression ratio of 15 to 1 and uses 175 per cent of theoretical air. The heating value of the fuel is 19 000 Btu per lb. the mechanical efficiency is 92% and the indicated relative efficiency is 85%. Calculate the brake fuel rate.

17-15 It is desired to study the effect of the heating of the incoming charge by the hot clearance gases on volumetric efficiency. Assume that a 10 in  $\times$  12 in engine with a compression ratio of 15 to 1 receives air at 100 F and that the air comes into the

cylinder without picking up heat from either the walls or clearance gas and without mixing with the clearance gas. Consider the clearance gas to have the properties of air and to be at 1000 F.

(a) Calculate the weights of clearance gas and air present, on the basis that the incoming air fills the entire piston displacement.

(b) Now assume that the air is heated by the clearance gas, without mixing and without piston motion, until an equilibrium temperature is reached. Calculate this equilibrium temperature and also the volumes of the air and the clearance gas.

(c) What can be said about the effect of clearance gas on volumetric efficiency?

(d) If the clearance gas loses heat to the walls during suction, will the volumetric efficiency be affected?

17-16. If the cylinder walls in Problem 17-15 give up 0.2 Btu during suction, calculate the suction temperature and the volumetric efficiency. What is the effect of heat from the cylinder walls on the volumetric efficiency?

17-17. At the end of the suction stroke, the pressure in a diesel engine is 2 psi below atmospheric. The compression ratio is 16 to 1, and the suction temperature is 150 F. At the end of exhaust the temperature is 800 F and the pressure is 2 psi above atmospheric. Calculate the volumetric efficiency.

17-18. If the intake valve of the engine in Problem 17-17 is held open until the piston has moved 6 per cent of its return stroke, the pressure becomes 0.5 psi below atmospheric. Calculate the volumetric efficiency.

17-19. The engine in Problem 17-18 uses 40 per cent excess air. The fuel has a heating value of 19,200 Btu per lb and requires theoretically 14.5 lb of air per pound. The indicated relative efficiency is 90%, and the mechanical efficiency is 75%. Calculate the brake thermal efficiency and the bmep.

17-20. A six-cylinder, single-acting, four-stroke-cycle gasoline engine, when supplied with 13 lb of air per lb of fuel, uses 0.6 lb of fuel per bhp-hr. The heating value of the fuel is 20,200 Btu per lb. The volumetric efficiency is 70%. Each cylinder is  $3\frac{1}{2}$  in.  $\times$  4 $\frac{1}{2}$  in. Determine the bhp at 3600 rpm.

17-21. Calculate the bmep of a natural-gas engine receiving 10 cu ft of air per cu ft of gas. The heating value of the fuel is 1100 Btu per cu ft. The volumetric efficiency of the engine is 82%, and the mechanical efficiency is 81%. The engine uses 9 cu ft of fuel per bhp-hr.

17-22. Calculate the indicated thermal efficiency of the engine in Problem 17-21.

17-23. A six-cylinder, four-cycle diesel engine, which is 6 in.  $\times$  8 in., has a 15 to 1 compression ratio. It has a volumetric efficiency of 80% at 900 rpm and uses 0.38 lb of fuel per bhp-hr. The theoretical air-fuel ratio is 14.6 to 1, and 40% excess air is used. Determine: (a) the brake thermal efficiency if the heating value of the oil is 19,200 Btu per lb; (b) the indicated relative efficiency if the mechanical efficiency is 78%; (c) the brake mep.

17-24. For Problem 17-23, determine: (a) the number of cfm of air at 14.7 psia and 60 F; (b) the number of cubic inches of oil per injection per cylinder if the specific gravity of the oil is 0.9.

17-25. Two engines have the following in common: compression ratio, 6 to 1; indicated relative efficiency, 80%; theoretical air-fuel ratio; volumetric efficiency, 82%. One engine uses liquid  $C_8H_{18}$  as a fuel, and the other uses  $CH_4$  (gaseous). Calculate the indicated mep of each engine and compare the results. Assume atmospheric conditions to be 14.7 psia and 60 F.

17-26. Compute the torque of a 5000-hp engine at 200 rpm.

17-27. A six-cylinder, four-cycle gasoline engine is to deliver 120 bhp at 3600 rpm. The bore is to be 80 per cent of the stroke. Assuming a brake mep of 90, determine the bore and the stroke.



17-28 The exhaust gases of a gasoline engine show 4% of  $CO$  and 0.2% of  $H_2$  by weight. There are 12 lb of gases per pound of fuel. Assume that the fuel is  $C_8H_{18}$ . Calculate (a) the heating value of the  $CO$  and  $H_2$  per pound of fuel, (b) the per cent loss of heating value of fuel resulting from incomplete combustion.

17-29 A gasoline engine has a volumetric efficiency of 75%, an indicated relative efficiency of 80%, and a mechanical efficiency of 74%, and it uses the theoretical amount of air. Take the fuel used as  $C_8H_{18}$ . The compression ratio is 7 to 1. Determine the brake mep for standard atmospheric conditions.

17-30 A diesel engine has a volumetric efficiency of 80%, an indicated relative efficiency of 76%, and a mechanical efficiency of 79%, and it uses 160% theoretical air. Take the fuel used as  $C_{12}H_{26}$ . The compression ratio is 16 to 1. Determine the brake mep for standard atmospheric conditions. Compare with Problem 17-29.

17-31 (a) A gasoline engine has a 5 to 1 compression ratio, a mechanical efficiency of 75%, and an indicated relative efficiency of 78%. Take the fuel as  $C_8H_{18}$  and use theoretical air. Calculate the pounds of fuel per bhp-hr.

(b) Work the problem for a 7 to 1 compression ratio.

(c) Work the problem for a 10 to 1 compression ratio.

17-32 An airplane engine is said to use 0.4 lb of fuel per bhp-hr, the fuel having a heating value of 20,000 Btu per lb. The compression ratio is 7.5 to 1, and 95% theoretical air is used. Mechanical efficiency is reported to be 88%. Is the reported performance data reasonable?

17-33 Predict the probable brake mep of a diesel engine which uses 40% excess air and has a compression ratio of 15 to 1. The engine uses a fuel having a heating value of 19,000 Btu per lb and requires, theoretically, 200 cu ft of air per lb of fuel. Mechanical efficiency is 78%. Indicated relative efficiency is 76%. The engine is supercharged, the supercharger delivering a volume of free air equal to 140 per cent of the piston displacement of the engine.

17-34 A two-cycle diesel engine has crankcase compression. The crankcase delivers to the engine cylinder a volume of free air equal to 75 per cent of the piston displacement, and 25 per cent of the air delivered to the cylinder passes directly out the exhaust port and cannot be used for combustion. The engine uses 260 cu ft of air per pound of fuel and 0.44 lb of fuel per bhp-hr. Calculate the brake mep.

17-35 A diesel engine with a 15 to 1 compression ratio uses 50% excess air at full load and has an indicated relative efficiency of 78% and a mechanical efficiency of 84% at the full load. Heating value of fuel is 18,500 Btu per lb. If full load is 1000 bhp, calculate (a) the number of pounds of fuel required per hour, (b) the number of pounds of fuel per bhp-hr, (c) the friction horsepower.

17-36 The fuel supply to the engine in Problem 17-35 is cut in half. Assume that the total air supply to the engine, the indicated relative efficiency, and the friction horsepower remain constant. Calculate (a) the indicated horsepower, (b) the brake horsepower, and (c) the number of pounds of fuel per bhp-hr.

17-37 A gasoline engine uses 0.55 lb of fuel per bhp-hr. Treat the fuel as  $C_8H_{18}$ . Calculate the brake thermal efficiency, based on (a) the higher heating value of the fuel and (b) the lower heating value of the fuel.

17-38 A special carburetor when placed on an automotive engine is said to enable it to get 48 miles per gal at 60 mph. The heating value of the fuel is 145,000 Btu per gal. At the given speed, the power required to overcome wind resistance and mechanical losses in the bearing, differential, and transmission total 35 hp. The compression ratio of the engine is 6 to 1, and 20% excess air is used. Make calculations to show whether the 48 miles per gal is reasonable.

17-39 A 12-cylinder, two-cycle, natural gas engine is to deliver 1500 bhp at 600 rpm. The brake mep is 110. If the stroke is 1.2 times the bore, calculate the bore and stroke.

17-40. An 8-cylinder, four-cycle gasoline engine is to deliver a maximum of 120 bhp. The brake mep is 120. The maximum allowable piston speed is 2500 ft per min. Calculate the bore.

17-41. A diesel engine uses 20 lb of air per pound of fuel and 0.38 lb of fuel per hp-hr. Each pound of fuel produces 1.24 lb of water vapor. Gases leave the engine at 1000 F and pass to a waste-heat boiler where they are cooled to 300 F. The engine horsepower is 600. Take  $c_p$  for the dry gases as 0.25 and  $c_p$  for the vapor as 0.45. Calculate the heat given up per hour to the waste-heat boiler.

17-42. A gasoline has a specific gravity of 0.75. A four-cycle engine using this gasoline produces 45 hp at 3000 rpm. The engine has 6 cylinders and uses 0.6 lb of fuel per hp-hr. Calculate the number of cubic inches of liquid gasoline required per cylinder per cycle.

17-43. A supercharged gasoline engine has a compression ratio of 7 to 1 and uses theoretical air. Assume standard atmospheric conditions. Take the fuel as  $C_8H_{18}$ . The engine is to produce 200 bhp with a brake mep of 190. The indicated relative efficiency is 80% and the mechanical efficiency is 77%. Calculate the number of cubic feet of free air to be supplied by the supercharger per minute.

17-44. The engine in Problem 17-43 has 8 cylinders and operates at 3300 rpm. If the bore is 80 per cent of the stroke, calculate the bore and stroke.

17-45. Compare the indicated mep of a gasoline engine with a 7 to 1 compression ratio when using 120% theoretical air with that obtained when using 80% theoretical air. Assume standard atmospheric conditions, a volumetric efficiency of 80%, and an indicated relative efficiency of 78%. Take the fuel as  $C_8H_{18}$ .

## CHAPTER 18

18-1. Assume an air-standard theoretical gas-turbine cycle, with initial air conditions of 14.7 psia and 60 F. Calculate the net work per pound of air and the thermal efficiency for each of the following temperatures at start of expansion and each of the following pressure ratios: temperatures, 1000 F, 1400 F, and 1800 F; pressure ratios, 2,  $3\frac{1}{2}$ , 5, 8.

18-2. Repeat Problem 18-1, but use an actual cycle with turbine and compressor internal efficiencies of 80% each. Continue to treat the working substance as air conforming to the perfect-gas laws, neglect the change in weight due to the addition of fuel, and neglect the burner and piping pressure drops.

18-3. Repeat Problem 18-2 with internal efficiencies of 60%.

18-4. Repeat Problem 18-3 for a unit that includes a regenerator. Assume that the exhaust gases are cooled to within 100 degrees of the air temperature leaving the compressor.

18-5. Assume an air-standard theoretical gas-turbine cycle in which the pressure ratio is  $3\frac{1}{2}$  to 1 and the temperature at the start of expansion is 1400 F. Calculate the work (net) and the thermal efficiency, per cubic foot of air at start of compression, for the following conditions at start of compression:

- 14.7 psia and 60 F.
- 14.7 psia and 0 F.
- 14.7 psia and 120 F.
- 5 psia and 0 F.

18-6. An actual gas-turbine unit takes in air at 14 psia and 90 F; the compressor discharges at 32.66 psia and 266 F. Calculate the compressor's internal efficiency: (a) by the perfect-gas method; (b) by Table 18-1.

18-7. The compressor of Problem 18-6 discharges into a combustion chamber where fuel having a combined chemical energy and enthalpy of 18,600 Btu per lb is burned. Heat losses and incomplete combustion waste 10 per cent of the fuel. What is the fuel-

air ratio if the products of combustion leave the burner at 31 psia and 1540 F? Solve both by the perfect gas method and by using Table 18-1, and compare the results

18-8 The gases from Problem 18-7 enter the turbine and emerge from it at an exhaust-pipe pressure of 16 19 psia. The turbine is known to have an internal efficiency of 90%. Using the air tables, calculate the enthalpy of the exhaust gases

18-9 Assume that the turbine and the compressor of Problems 18-6, 18-7, and 18-8 have mechanical efficiency values of 98% each. Calculate the thermal efficiency of the unit and the brake horsepower, if 40 lb of air enter the compressor per second. The heating value of the fuel is 19,600 Btu per lb.

18-10 Assume that a regenerator added to the unit of Problem 18-9 brings the temperature of the air entering the combustion chamber up to 1140 F. If other temperatures and pressures are unchanged, what is the thermal efficiency? How does this efficiency compare with that of an air standard Brayton Cycle having the same compressor pressure ratio?

18-11 A gas turbine operates with a pressure ratio of 6 to 1 and with a temperature of 1200 F at the entrance to the turbine element. Assume that atmospheric conditions are 14.7 psia and 60 F. Determine on the air standard basis (a) the net cycle work per pound of air and (b) the thermal efficiency.

18-12 Work Problem 18-11 for a temperature of 1500 F instead of 1200 F.

18-13 Work Problem 18-11 for turbine and compressor efficiencies of 85%.

18-14 Work Problem 18-11 for turbine and compressor efficiencies of 75%.

18-15 Work Problem 18-11 for a temperature of 1500 F and turbine and compressor efficiencies of 85%.

18-16 Work Problem 18-11 for a plant that includes a regenerator which gives perfect heat exchange ( $\epsilon$ , regenerator effectiveness is 100%).

18-17 Work Problem 18-11 for turbine and compressor efficiencies of 85% and a plant that includes a regenerator which has an effectiveness of 75%.

18-18 An actual gas turbine compressor receives air at 14.7 psia and 90 F and discharges at 45 psia. If the compression efficiency is 83%, determine the enthalpy per pound and the temperature at the compressor exit by use of Table 18-1.

18-19 Assume that the fuel for a gas turbine is  $C_{12}H_{24}$ . Air enters the combustion chamber at 440 F. If the allowable temperature at exit is 1540 F, determine the number of pounds of air which must be used per pound of fuel. Neglect the enthalpy of the liquid fuel. Use Table 18-1 for the enthalpies of both the air and the products of combustion.

18-20 Work Problem 18-19 but treat the gases as air and assume that air is a perfect gas.

18-21 Gas turbine engineers often speak of the "work ratio" as being the net work of the cycle divided by the work output of the turbine alone. For a theoretical air-standard gas turbine cycle, compute the net work per pound of air, the work ratio, and the thermal efficiency, if conditions at entrance to the compressor are 14 psia and 90 F, the pressure ratio is 6 to 1, and the maximum temperature is 1200 F.

18-22 Solve Problem 18-21 using compressor entrance temperatures of -60 F, 0 F, 60 F, and 120 F, and other data being unchanged.

18-23 Solve Problem 18-21, using pressure ratios of 2, 4, 8, and 10 to 1.

18-24 Solve Problem 18-21, using maximum temperatures of 800 F, 1000 F, 1400 F, and 1600 F.

18-25 Solve Problem 18-21, using air as a perfect gas but also considering the turbine and the compressor to have internal efficiencies of (a) 90%, (b) 80%, (c) 70%.

18-26 From the results of Problems 18-21 through 18-25, discuss the relative effects of the variables tested. What conclusions do you draw?

18-27. An actual gas-turbine unit takes in air at 14.7 psia and 40 F; the compressor discharges at 70 psia and 390 F. Compute the compressor's internal efficiency: (a) by the perfect-gas method; (b) by Table 18-1.

18-28. Solve Problem 18-27, using a discharge temperature of 415 F.

18-29. The compressor of Problem 18-27 discharges into a combustion chamber where fuel having a combined chemical energy and enthalpy of 18,500 Btu per lb is burned. Heat losses and incomplete combustion cause a waste of 1200 Btu per pound of fuel. What is the fuel-air ratio if the products of combustion leave the burner at 68 psia and 1440 F? Solve by using Table 18-1.

18-30. The gases from Problem 18-29 enter the turbine and emerge from it at exhaust-pipe conditions of 17.9 psia and 940 F. Compute the turbine's internal efficiency: (c) by the perfect-gas method; (b) by Table 18-1. To what do you attribute the difference in results?

18-31. Assume that the compressor of Problem 18-27(b) and the turbine of Problem 18-30(b) each have mechanical efficiencies of 98%. If the fuel of Problem 18-29 has a heating value of 19,500 Btu per lb, estimate the thermal efficiency of the unit.

18-32. Refer to Problem 18-31. Compute the work ratio and the brake horsepower if 50 lb of air enter the compressor per second.

18-33. Refer to Problem 18-31. A regenerator is added to the unit, bringing the temperature of air entering the combustion chamber to 840 F. Using Table 18-1, estimate the thermal efficiency of the unit.

18-34. Refer to Problem 18-33. Compute the effectiveness of the regenerators, based on temperatures.

18-35. Refer to Problem 18-33. How does the thermal efficiency of the actual unit with regenerator compare with that of an air-standard Brayton Cycle having the same compressor pressure ratio?

18-36. Using a regenerator with an effectiveness of 80%, compute the thermal efficiency for the cycle of Problem 18-21. Then, using the same effectiveness of regenerator, compute the thermal efficiencies for the pressure ratios of Problem 18-23. Discuss the influence of pressure ratios on the relative improvement in thermal efficiency that a regenerator can cause.

18-37. Using a regenerator of 80% effectiveness, solve Problem 18-25.

18-38. Using Table 18-1, compute the stagnation enthalpy of air at 140 F moving with a velocity of 448 ft per sec.

18-39. The air of Problem 18-38 expands in a nozzle and emerges at 40 F. Using Table 18-1, compute its exit velocity.

18-40. Considering the expansion of Problems 18-38 and 18-39 to be isentropic, show that the pressure at exit is 52.8 psia if the pressure at 140 F is 100 psia. Use Table 18-1.

18-41. Solve Problems 18-39 and 18-40 by perfect-gas laws.

18-42. Consider a theoretical air-standard gas-turbine cycle, similar to that in Problem 18-21. Sketch the cycle on  $k$ - $S$  coordinates, noting the pressure and temperature at each of the four points. Then consider that the turbine is divided into two units, the high-pressure unit's output being used only to drive the compressor, and the low-pressure turbine furnishing the useful power output. What are the temperature and pressure at the point where the gas leaves the high-pressure turbine?

18-43. Refer to Problem 18-42. The low-pressure turbine is now replaced by a nozzle which exhausts to atmosphere; this is the principle of the jet plane, which obtains its propulsion from the jet reaction. What is the velocity of efflux under ideal conditions?

18-44. Refer to Problem 18-43. For each pound of gas flowing per second, determine the number of pounds of thrust, assuming a properly expanded nozzle.

18-45 Apply the principles of the three preceding problems to the conditions of Problem 18-31, and estimate the thrust for each pound of gas flowing per second. (NOTE For Problem 18-31, expansions are not isentropic, but the data is readily adapted *in toto* by assuming that the nozzle has an efficiency of 89%.)

## CHAPTER 19

19-1 A copper plate 1 in. thick has a temperature of 140 F on one surface and a temperature of 60 F on the other. Calculate the heat transfer rate per square foot.

19-2 Solve Problem 19-1, using mild steel instead of copper.

19-3 Solve Problem 19-1, using 85% magnesia insulation instead of copper.

19-4 A copper plate 1 in. thick has a gas on each surface, each gas has a film coefficient  $\bar{h}$  of 4. One gas is at 140 F and the other is at 60 F. Calculate the heat transfer rate per square foot.

19-5 Solve Problem 19-4, but use liquids instead of gases. Each liquid is to have an  $\bar{h}$  of 400.

19-6 Refer to Problems 19-4 and 19-5. Calculate the heat transfer rate if the high temperature fluid is a gas and the low temperature fluid is a liquid.

19-7 Using the principles of Ohm's Law, estimate (a) the temperature at each surface of the plate in Problem 19-4 and (b) the thermal gradient across the plate,  $\frac{dT}{dx}$ , in degrees per foot.

19-8 Find the quantities in Problem 19-7 for the conditions in Problem 19-5.

19-9 Find the quantities in Problem 19-7 for the conditions in Problem 19-6.

19-10 An alloy steel pipe, having a thermal conductivity of 20, has an outside diameter of 6.625 in. and a wall thickness of 0.562 in. The temperature at the inside surface is 500 F and that at the outside is 400 F. Compute the heat transfer rate per square foot of outside surface and also per linear foot of pipe.

19-11 Refer to Problem 19-10. The pipe is covered with a 2 in. thickness of insulation having a conductivity of 0.04. The temperature at the inside pipe surface is still 500 F but the temperature at the outer surface of the insulation is 100 F. Compute the heat transfer rate per linear foot of pipe.

19-12 Refer to Problem 19-11. Estimate the temperature at the interface.

19-13 Refer to Problem 19-11. The pipe is carrying superheated steam which has a heat transfer film coefficient  $\bar{h}$  of 80 under these conditions. Estimate the temperature of the steam.

19-14 A vertical flat wall has a temperature of 90 F. The air in contact with it has a temperature of 50 F. Compute the convection heat transfer rate.

19-15 An insulated steam line runs horizontally across a space where the air temperature is 50 F. The outer diameter of the insulation is 12.625 in. and its surface temperature is 90 F. Compute the convection heat transfer rate per square foot and also per linear foot.

19-16 The line in Problem 19-15 has an emissivity of 0.90. Compute the radiant heat transfer per square foot and per linear foot.

19-17 Refer to Problems 19-15 and 19-16. Compute the total heat transfer to the air, and the percentage by convection and by radiation.

19-18 Refer to Problem 19-17. Compute the total film coefficient  $\bar{h}$ .

19-19 Two large parallel walls are separated by a narrow space across which radiant energy is transmitted. One wall has a surface temperature of 140 F, and the other has a surface temperature of 40 F. Estimate the radiant heat transfer if each surface is considered a black body.

- 19-20. Solve Problem 19-19 if each surface has an emissivity of 0.92.
- 19-21. Solve Problem 19-19 if each surface is covered with a bright, shiny substance so that the emissivities are each 0.04.
- 19-22. Solve Problem 19-19 if one surface has an emissivity of 0.92 and the other surface has an emissivity of 0.04.
- 19-23. Referring to Problems 19-20, 19-21, and 19-22, compute the percentage reduction in heat transfer by radiation for each of the cases of Problems 19-21 and 19-22, based on Problem 19-20. What conclusions do you draw regarding the use of the shiny foil?
- 19-24. A steam condenser consisting of 1000 tubes arranged in parallel has condensate forming on the outside tube surface while cold water runs through the tubes at a velocity of 8 ft per sec. The tubes have an outside diameter of 0.625 in. and a wall thickness of 0.049 in. If the average temperature of the water is 60 F, compute the water film coefficient  $\bar{h}$ .
- 19-25. The condenser in Problem 19-24 has tubes of a non-ferrous alloy having a conductivity of 64. The steam film conductance  $\bar{h}$  is 2000. Compute the over-all coefficient  $U$  based on inside tube surface area.
- 19-26. Solve Problem 19-24, using velocities of (a) 2 ft per sec, (b) 32 ft per sec, (c) 80 ft per sec.
- 19-27. Separate the variables in equation 19-22a and show that the heat transfer film coefficient  $\bar{h}$  is inversely proportional to the fifth root of the diameter when other quantities are held constant. Then solve Problem 19-24, using an inside diameter of 2.2 in. and a wall thickness of 0.049 in.
- 19-28. Refer to Problems 19-24 and 19-27. If the same quantity of water were to flow in each case, how many tubes would be used for the flow in Problem 19-27?
- 19-29. The tubes in the condenser of Problem 19-24 are 16 ft long. How many square feet of inside tube surface area are there? What is the surface-to-volume ratio (*i.e.*, ratio of heat transfer surface for water to volume of water flowing)?
- 19-30. Consider that for the large-tube conditions of Problem 19-27 the over-all coefficient  $U$  based on inside tube surface area is 668. If the same MTD were used, how long would the tubes need to be in order to get the same heat transfer as in Problems 19-25, 19-28, and 19-29? What conclusions do you draw regarding the reason for using the small tubes?
- 19-31. Steam enters a surface condenser at 2 in. Hg, abs., with 95% quality; the condensate leaves without any subcooling. Cooling water enters the tubes at 70 F and leaves at 82 F. If the weight of condensate is 20,000 lb per hr and  $U$  is 640, determine the cooling surface area and the number of gpm of cooling water.
- 19-32. In a gas-turbine unit, the low-pressure turbine discharges at 860 F into a regenerator, from which the gas emerges at 451 F. The high-pressure compressor discharges air into the regenerator at 287 F; this air leaves the regenerator at 727 F. If the air flow is 72,000 lb per hr and  $U$  is 4, determine (a) the surface area required for heat transfer and (b) the regenerator effectiveness based on temperatures.
- 19-33. An internal-combustion engine consumes 4 gal of gasoline per hour; the heating value is 120,000 Btu per gal. The cooling system is equipped with a radiator whose passages give an equivalent heat transfer surface of 52 sq ft; a fan draws air through the passages at high velocity so that  $U=18$ . The mean temperature of the water is 155 F and that of the air is 62 F. What percentage of the fuel's heating value is dissipated in the radiator?
- 19-34. A convection-type superheater in a power-plant boiler is arranged for parallel flow with the following temperatures: Combustion gases enter at 1812 F and leave at 1172 F; 100,000 lb of steam per hour enter the superheater dry saturated at 1250 psia and leave at 1200 psia and 1000 F. If  $U$  is 9, compute the surface area required.

## MISCELLANEOUS PROBLEMS

1 Taking account of the work of the feed pump calculate the thermal efficiency and the net work per pound of steam for a Rankine Cycle operating between limits of 1000 psia, 1000 F, and 1 in Hg, abs

2 Calculate the net work per pound of air and the thermal efficiency of a theoretical gas turbine cycle which takes in air at 79 F and standard atmospheric pressure, has a pressure ratio of 4 to 1, and has a turbine inlet temperature of 1000 F (Use perfect gas laws)

3 (a) What would be the Carnot efficiency for the temperature extremes of Problem 2?

(b) Some engineers use the term 'work ratio' for the gross turbine work divided by the net work of the cycle. Compute the work ratio for each of the conditions of Problems 1 and 2

4 Solve Problem 1 but consider that instead of a Rankine Cycle, the conditions apply to an actual cycle in which the turbine efficiency is 75% and the pump efficiency is 50% (Use perfect gas laws)

5 Solve Problem 2 but consider that the turbine and the compressor each have an efficiency of 80%.

6 Calculate the work ratio for the conditions of Problems 4 and 5

7 Estimate the mean effective pressure for the conditions of Problem 1

8 Estimate the mean effective pressure for the conditions of Problem 2

9 Discuss briefly the significance of the mep in the above problems considering the type of machinery usually employed for those cycles

10 A certain automobile engine, with a compression ratio of 7 to 1 operates on non premium fuel costing 28 cents per gal. It is suggested that a new cylinder head be installed which will raise the compression ratio to 8.7 to 1. This will necessitate using a premium fuel costing 30 cents per gal. If the thermal efficiencies are in the same ratio as those for theoretical air conditions in Fig. 17-7 estimate whether the new cylinder head will improve the economy of operation

11 Methane gas  $CH_4$  is completely burned with varying excess air conditions. Estimate the percentage of  $CO_2$  by volume in the dry products of combustion for each of the following excess air percentages: 0, 20, 50, 100

12 Refer to Problem 11. If the total pressure is 14.7 psia, estimate the dew point of the products of combustion in each case

13 From Clapeyron's equation show that

$$\left(\frac{dP}{dT}\right)_{\text{saturation}} = \frac{J s_{fg}}{v_{fg}}$$

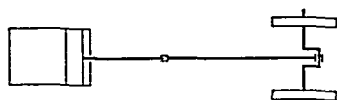
Then calculate the derivative for saturated steam at 32 F, 212 F, and 700 F

14 From the properties of superheated steam at 1600 F under pressures of 1, 2, and 4 psia predict the specific volume, enthalpy, and entropy of steam at 1600 F under a pressure of 0.5 psia

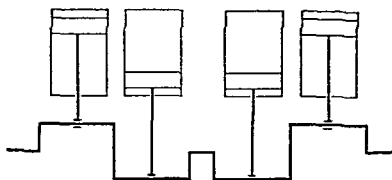
# APPENDIX A

## SOME POSSIBLE CYLINDER ARRANGEMENTS

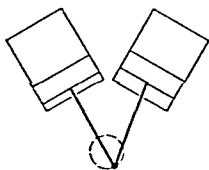
*Reciprocating Machinery*



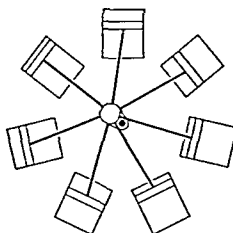
Horizontal, Single Stage, Single Cylinder



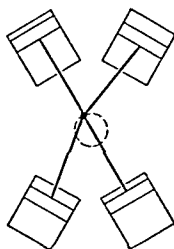
Vertical, Single Stage, Four Cylinder



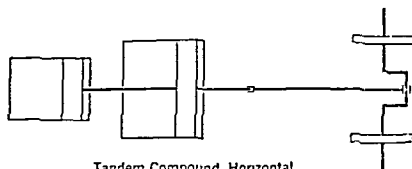
Vee, Single Stage, In Line



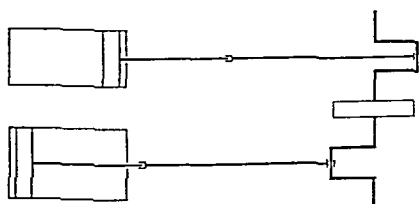
Seven Cylinder, Radial



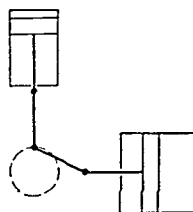
X, Single Stage, In Line



Tandem Compound, Horizontal

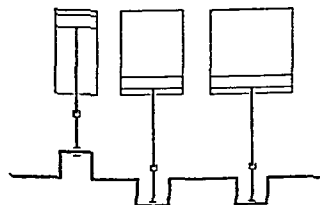


Cross Compound, Horizontal

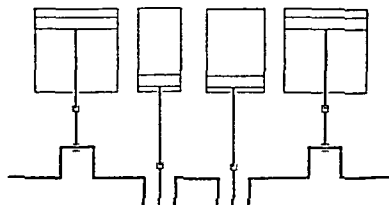


Angle Compound

*Steam Engines*



Triple Expansion



Triple Expansion with Two Low-Pressure Cylinders



# APPENDIX B

## TABLES OF THERMODYNAMIC PROPERTIES OF FREON

The following extract of thermodynamic properties of dichlorodifluoromethane (Freon, F-12) for both the saturated fluid and the superheated vapor has been made with the kind permission of Kinetic Chemicals, Inc., Wilmington, Delaware, and of the American Society of Refrigerating Engineers, data having been obtained through the A.S.R.E. Data Book.

### THERMODYNAMIC PROPERTIES OF SATURATED FREON, F-12

Temp.	Absolute Pressure	Volume		Enthalpy from -40 F			Entropy from -40 F		Temp.
°F <i>t</i>	lb/in. <sup>2</sup> <i>P</i>	Liquid ft <sup>3</sup> /lb <i>v<sub>l</sub></i>	Vapor ft <sup>3</sup> /lb <i>v<sub>g</sub></i>	Liquid Btu/lb <i>h<sub>l</sub></i>	Latent Btu/lb <i>h<sub>fg</sub></i>	Vapor Btu/lb <i>h<sub>g</sub></i>	Liquid Btu/lb/°F <i>s<sub>l</sub></i>	Vapor Btu/lb/°F <i>s<sub>g</sub></i>	°F <i>t</i>
-155	0.1163	0.00954	232.29	-24.61	84.61	60.00	-0.0686	0.2092	-155
-150	.1527	.00957	179.79	-23.50	84.07	60.57	-.0650	.2065	-150
-145	.1985	.00961	140.52	-22.39	83.53	61.14	-.0615	.2040	-145
-140	.2554	.00965	110.92	-21.29	83.01	61.72	-.0580	.2017	-140
-135	.3256	.00969	88.34	-20.19	82.49	62.30	-.0546	.1995	-135
-130	.4116	.00973	70.94	-19.10	81.98	62.88	-.0512	.1975	-130
-125	0.5160	0.00977	57.42	-18.02	81.48	63.46	-0.0480	0.1955	-125
-120	.6417	.00981	46.84	-16.94	80.98	64.04	-.0448	.1937	-120
-115	.7921	.00985	38.49	-15.85	80.48	64.63	-.0416	.1919	-115
-110	.9709	.00989	31.84	-14.78	80.00	65.22	-.0385	.1903	-110
-105	1.182	.00994	26.51	-13.71	79.52	65.81	-.0355	.1888	-105
-100	1.430	.00998	22.20	-12.64	79.04	66.40	-.0325	.1873	-100
-95	1.719	0.01003	18.71	-11.58	78.57	66.99	-0.0295	0.1860	-95
-90	2.054	.01007	15.86	-10.51	78.10	67.59	-.0266	.1847	-90
-85	2.441	.01012	13.51	-9.46	77.64	68.18	-.0238	.1835	-85
-80	2.885	.01016	11.57	-8.40	77.17	68.77	-.0210	.1823	-80
-75	3.393	.01021	9.958	-7.35	76.71	69.36	-.0182	.1813	-75
-70	3.971	.01026	8.608	-6.30	76.25	69.95	-.0155	.1802	-70
-65	4.626	0.01031	7.474	-5.25	75.79	70.54	-0.0128	0.1793	-65
-60	5.365	.01036	6.516	-4.20	75.33	71.13	-.0102	.1783	-60
-55	6.195	.01041	5.704	-3.15	74.87	71.72	-.0076	.1774	-55
-50	7.125	.01047	5.012	-2.11	74.42	72.31	-.0050	.1767	-50
-45	8.163	.01052	4.420	-1.06	73.97	72.91	-.0025	.1759	-45
-40	9.32	0.0106	3.911	0	73.50	73.50	0	0.17517	-40
-38	9.82	.0106	3.727	0.40	73.34	73.74	0.00094	.17490	-38
-36	10.34	.0106	3.553	0.81	73.17	73.98	.00188	.17463	-36
-34	10.87	.0106	3.389	1.21	73.01	74.22	.00282	.17438	-34
-32	11.43	.0107	3.234	1.62	72.84	74.46	.00376	.17412	-32

# THERMODYNAMIC PROPERTIES OF SATURATED FREON, F-12

(Continued)

Temp	Absolute Pressure	Volume		Enthalpy from -40 F			Entropy from -40 F		Temp
$^{\circ}\text{F}$ $t$	lb/in <sup>2</sup> $P$	Liquid ft <sup>3</sup> /lb $v_f$	Vapor ft <sup>3</sup> /lb $v_g$	Liquid Btu/lb $h_f$	Latent Btu/lb $h_{fg}$	Vapor Btu/lb $h_g$	Liquid Btu/lb/ $^{\circ}\text{F}$ $s_f$	Vapor Btu/lb/ $^{\circ}\text{F}$ $s_g$	$^{\circ}\text{F}$ $t$
-30	12.02	0.0107	3.088	2.03	72.67	74.70	0.00471	0.17387	-30
-28	12.62	0.0107	2.950	2.44	72.50	74.94	0.00565	0.17364	-28
-26	13.26	0.0107	2.820	2.85	72.33	75.18	0.00659	0.17340	-26
-24	13.90	0.0108	2.698	3.25	72.16	75.41	0.00753	0.17317	-24
-22	14.58	0.0108	2.583	3.66	71.98	75.64	0.00846	0.17296	-22
-20	15.28	0.0108	2.474	4.07	71.80	75.87	0.00940	0.17275	-20
-18	16.01	0.0108	2.370	4.48	71.63	76.11	0.01033	0.17253	-18
-16	16.77	0.0108	2.271	4.89	71.45	76.34	0.01126	0.17232	-16
-14	17.55	0.0109	2.177	5.30	71.27	76.57	0.01218	0.17212	-14
-12	18.37	0.0109	2.088	5.72	71.09	76.81	0.01310	0.17194	-12
-10	19.20	0.0109	2.003	6.14	70.91	77.05	0.01403	0.17175	-10
-8	20.03	0.0109	1.922	6.57	70.72	77.29	0.01496	0.17158	-8
-6	20.98	0.0110	1.845	6.99	70.53	77.52	0.01589	0.17140	-6
-4	21.91	0.0110	1.772	7.41	70.34	77.75	0.01682	0.17123	-4
-2	22.87	0.0110	1.703	7.83	70.15	77.98	0.01775	0.17107	-2
0	23.87	0.0110	1.637	8.25	69.96	78.21	0.01869	0.17091	0
2	24.89	0.0110	1.574	8.67	69.77	78.44	0.01961	0.17075	2
4	25.96	0.0111	1.514	9.10	69.57	78.67	0.02052	0.17060	4
5*	26.51	0.0111	1.485	9.32	69.47	78.79	0.02097	0.17052	5*
6	27.05	0.0111	1.457	9.53	69.37	78.90	0.02143	0.17045	6
8	28.18	0.0111	1.403	9.96	69.17	79.13	0.02235	0.17030	8
10	29.35	0.0112	1.351	10.39	68.97	79.36	0.02328	0.17015	10
12	30.56	0.0112	1.301	10.82	68.77	79.59	0.02419	0.17001	12
14	31.80	0.0112	1.253	11.26	68.56	79.82	0.02510	0.16987	14
16	33.08	0.0112	1.207	11.70	68.35	80.05	0.02601	0.16974	16
18	34.40	0.0113	1.163	12.12	68.15	80.27	0.02692	0.16961	18
20	35.75	0.0113	1.121	12.55	67.94	80.49	0.02783	0.16949	20
22	37.15	0.0113	1.081	13.00	67.72	80.72	0.02873	0.16938	22
24	38.58	0.0113	1.043	13.44	67.51	80.95	0.02963	0.16926	24
26	40.07	0.0114	1.007	13.88	67.29	81.17	0.03053	0.16913	26
28	41.59	0.0114	0.973	14.32	67.07	81.39	0.03143	0.16900	28
30	43.16	0.0115	0.939	14.76	66.85	81.61	0.03233	0.16887	30
32	44.77	0.0115	0.908	15.21	66.62	81.83	0.03323	0.16876	32
34	46.42	0.0115	0.877	15.65	66.40	82.05	0.03413	0.16865	34
36	48.13	0.0116	0.848	16.10	66.17	82.27	0.03502	0.16854	36
38	49.88	0.0116	0.819	16.55	65.94	82.49	0.03591	0.16843	38

\* Standard ton temperature

# THERMODYNAMIC PROPERTIES OF SATURATED FREON, F-12

(Continued)

Temp.	Absolute Pressure	Volume		Enthalpy from -40 F			Entropy from -40 F		Temp.
°F <i>t</i>	lb/in. <sup>2</sup> <i>P</i>	Liquid ft <sup>3</sup> /lb <i>v<sub>f</sub></i>	Vapor ft <sup>3</sup> /lb <i>v<sub>g</sub></i>	Liquid Btu/lb <i>h<sub>f</sub></i>	Latent Btu/lb <i>h<sub>fg</sub></i>	Vapor Btu/lb <i>h<sub>g</sub></i>	Liquid Btu/lb/°F <i>s<sub>f</sub></i>	Vapor Btu/lb/°F <i>s<sub>g</sub></i>	°F <i>t</i>
40	51.68	0.0116	0.792	17.00	65.71	82.71	0.03680	0.16833	40
42	53.51	.0116	.767	17.46	65.47	82.93	.03770	.16823	42
44	55.40	.0117	.742	17.91	65.24	83.15	.03859	.16813	44
46	57.35	.0117	.718	18.36	65.00	83.36	.03948	.16803	46
48	59.35	.0117	.695	18.82	64.74	83.57	.04037	.16794	48
50	61.39	0.0118	0.673	19.27	64.51	83.78	0.04126	0.16785	50
52	63.49	.0118	.652	19.72	64.27	83.99	.04215	.16776	52
54	65.63	.0118	.632	20.18	64.02	84.20	.04304	.16767	54
56	67.84	.0119	.612	20.64	63.77	84.41	.04392	.16758	56
58	70.10	.0119	.593	21.11	63.51	84.62	.04480	.16749	58
60	72.41	0.0119	0.575	21.57	63.25	84.82	0.04568	0.16741	60
62	74.77	.0120	.557	22.03	62.99	85.02	.04657	.16733	62
64	77.20	.0120	.540	22.49	62.73	85.22	.04745	.16725	64
66	79.67	.0120	.524	22.95	62.47	85.42	.04833	.16717	66
68	82.24	.0121	.508	23.42	62.20	85.62	.04921	.16709	68
70	84.82	0.0121	0.493	23.90	61.92	85.82	0.05009	0.16701	70
72	87.50	.0121	.479	24.37	61.65	86.02	.05097	.16693	72
74	90.20	.0122	.464	24.84	61.38	86.22	.05185	.16685	74
76	93.00	.0122	.451	25.32	61.10	86.42	.05272	.16677	76
78	95.85	.0123	.438	25.80	60.81	86.61	.05359	.16669	78
80	98.76	0.0123	0.425	26.28	60.52	86.80	0.05446	0.16662	80
82	101.7	.0123	.413	26.76	60.23	86.99	.05534	.16655	82
84	104.8	.0124	.401	27.24	59.94	87.18	.05621	.16648	84
86*	107.9	.0124	.389	27.72	59.65	87.37	.05708	.16640	86*
88	111.1	.0124	.378	28.21	59.35	87.56	.05795	.16632	88
90	114.3	0.0125	0.368	28.70	59.04	87.74	0.05882	0.16624	90
92	117.7	.0125	.357	29.19	58.73	87.92	.05969	.16616	92
94	121.0	.0126	.347	29.68	58.42	88.10	.06056	.16608	94
96	124.5	.0126	.338	30.18	58.10	88.28	.06143	.16600	96
98	128.0	.0126	.328	30.67	57.78	88.45	.06230	.16592	98
100	131.6	0.0127	0.319	31.16	57.46	88.62	0.06316	0.16584	100
102	135.3	.0127	.310	31.65	57.14	88.79	.06403	.16576	102
104	139.0	.0128	.302	32.15	56.80	88.95	.06490	.16568	104
106	142.8	.0128	.293	32.65	56.46	89.11	.06577	.16560	106
108	146.8	.0129	.285	33.15	56.12	89.27	.06663	.16551	108
110	150.7	0.0129	0.277	33.65	55.78	89.43	0.06749	0.16542	110
120	171.8	.0132	.240	36.16	53.99	90.15	.07180	.16495	120
130	194.9	.0134	.208	38.69	52.07	90.76	.07607	.16438	130
140	220.2	.0138	.180	41.24	50.00	91.24	.08024	.16363	140

\* Standard ton temperature

## THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12

Temp °F	Abs Pressure 8 lb/in <sup>2</sup> (Sat n Temp -43.8 F)		
	<i>v</i>	<i>h</i>	<i>s</i>
(at sat n)	(4.502)	(72.80)	(0.17596)
-40	4.569	73.56	0.17777
-30	4.684	74.87	18085
-20	4.799	76.20	18390
-10	4.914	77.54	18691
0	5.028	78.89	0.18991
10	5.142	80.26	19284
20	5.257	81.64	19574
30	5.370	83.02	19860
40	5.484	84.43	20143
50	5.598	85.85	0.20425
60	5.711	87.27	20703
70	5.824	88.72	20977
80	5.938	90.18	21250
90	6.051	91.64	21519
100	6.165	93.13	0.21786
110	6.278	94.63	22051
120	6.391	96.13	22314
130	6.504	97.64	22573
140	6.617	99.18	22831
150	6.730	100.73	0.23087
160	6.843	102.29	23340
170	6.955	103.87	23591
180	7.068	105.44	23842
190	7.181	107.05	24090
200	7.294	108.67	0.24337
210	7.407	110.28	24581
220	7.520	111.93	24825
230	7.633	113.57	25066

Temp °F	Abs Pressure 12 lb/in <sup>2</sup> (Sat n Temp -30.1 F)		
	<i>v</i>	<i>h</i>	<i>s</i>
(at sat n)	(3.093)	(74.69)	(0.17389)
-30	3.094	74.71	0.17392
-20	3.172	76.02	17695
-10	3.250	77.37	17998
0	3.328	78.73	0.18299
10	3.405	80.10	18594
20	3.483	81.48	18884
30	3.560	82.87	19173
40	3.637	84.28	19458
50	3.714	85.71	0.19739
60	3.790	87.14	20018
70	3.867	88.59	20293
80	3.943	90.05	20566
90	4.019	91.52	20836
100	4.095	93.00	0.21104
110	4.170	94.50	21367
120	4.246	96.01	21631
130	4.323	97.53	21891
140	4.400	99.07	22151
150	4.474	100.62	0.22406
160	4.549	102.18	22659
170	4.624	103.75	22911
180	4.700	105.34	23162
190	4.774	106.94	23409
200	4.850	108.55	0.23656
210	4.926	110.18	23901
220	5.000	111.82	24144
230	5.076	113.47	24385
240	5.152	115.15	24626

# THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12

(Continued)

Temp. °F	Abs. Pressure 16 lb/in. <sup>2</sup> (Sat'n Temp. -13.0 F)			Temp. °F	Abs. Pressure 20 lb/in. <sup>2</sup> (Sat'n Temp. -8.2 F)			
	<i>t</i>	<i>v</i>	<i>h</i>		<i>s</i>	<i>t</i>	<i>v</i>	<i>h</i>
(at sat'n)	(2.370)	(76.11)	(0.17254)	(at sat'n)	(1.925)	(77.27)	(0.17160)	
-10	2.417	77.20	0.17498	0	1.965	78.39	0.17407	
0	2.476	78.56	0.17800	10	2.013	79.76	.17704	
10	2.535	79.94	.18095	20	2.060	81.14	.17996	
20	2.594	81.33	.18387	30	2.107	82.55	.18286	
30	2.652	82.73	.18676	40	2.155	83.97	.18573	
40	2.710	84.13	.18962	50	2.203	85.40	0.18858	
50	2.768	85.55	0.19244	60	2.250	86.85	.19138	
60	2.827	86.99	.19524	70	2.297	88.31	.19415	
70	2.886	88.45	.19800	80	2.343	89.78	.19688	
80	2.944	89.91	.20075	90	2.390	91.26	.19959	
90	3.002	91.39	.20345	100	2.437	92.75	0.20229	
100	3.059	92.88	0.20613	110	2.483	94.26	.20494	
110	3.117	94.38	.20879	120	2.530	95.78	.20759	
120	3.175	95.89	.21143	130	2.577	97.31	.21020	
130	3.232	97.42	.21403	140	2.623	98.85	.21280	
140	3.290	98.95	.21662	150	2.669	100.40	0.21537	
150	3.347	100.50	0.21917	160	2.716	101.97	.21792	
160	3.404	102.07	.22172	170	2.762	103.56	.22045	
170	3.461	103.65	.22425	180	2.808	105.15	.22297	
180	3.519	105.24	.22677	190	2.854	106.76	.22545	
190	3.576	106.85	.22925	200	2.901	108.38	0.22794	
200	3.633	108.47	0.23173	210	2.947	110.01	.23039	
210	3.690	110.10	.23418	220	2.992	111.65	.23283	
220	3.747	111.74	.23662	230	3.038	113.31	.23524	
230	3.803	113.40	.23903	240	3.084	114.98	.23766	
240	3.860	115.07	.24143	250	3.130	116.67	0.24005	
250	3.917	116.76	0.24382	260	3.177	118.36	.24242	
				270	3.223	120.07	.24477	

## THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12

(Continued)

Temp °F	Abs. Pressure 28 lb./in. <sup>2</sup> (Sat'n Temp 77°F)		
<i>t</i>	<i>v</i>	<i>h</i>	<i>s</i>
(at sat'n)	(1.409)	(79.10)	(0.17032)
10	1.415	79.41	0.17099
20	1.450	80.81	17393
30	1.485	82.23	17685
40	1.520	83.66	17975
50	1.555	85.11	0.18261
60	1.590	86.56	18544
70	1.625	88.03	18823
80	1.659	89.51	19097
90	1.693	90.99	19371
100	1.727	92.49	0.19642
110	1.761	94.01	19909
120	1.795	95.53	20174
130	1.828	97.07	20436
140	1.862	98.62	20698
150	1.896	100.18	0.20956
160	1.930	101.75	21212
170	1.963	103.33	21466
180	1.997	104.93	21719
190	2.030	106.55	21967
200	2.063	108.17	0.22216
210	2.096	109.81	22462
220	2.129	111.46	22706
230	2.163	113.12	22949
240	2.196	114.80	23191
250	2.229	116.49	0.23430
260	2.262	118.19	23669
270	2.295	119.91	23905
280	2.329	121.65	24141

Temp °F	Abs. Pressure 36 lb./in. <sup>2</sup> (Sat'n Temp 20.4°F)		
<i>t</i>	<i>v</i>	<i>h</i>	<i>s</i>
(at sat'n)	(1.113)	(80.54)	(0.16947)
30	1.140	81.90	0.17227
40	1.168	83.35	17518
50	1.196	84.81	0.17806
60	1.223	86.27	18089
70	1.250	87.74	18369
80	1.278	89.22	18647
90	1.305	90.71	18921
100	1.332	92.22	0.19193
110	1.359	93.75	19462
120	1.386	95.28	19729
130	1.412	96.82	19991
140	1.439	98.37	20254
150	1.465	99.93	0.20512
160	1.492	101.51	20770
170	1.518	103.11	21024
180	1.545	104.72	21278
190	1.571	106.34	21528
200	1.597	107.97	0.21778
210	1.623	109.61	22024
220	1.650	111.27	22270
230	1.676	112.94	22513
240	1.702	114.62	22756
250	1.728	116.31	0.22996
260	1.754	118.02	23235
270	1.780	119.74	23472
280	1.807	121.47	23708
290	1.833	123.22	23942

# THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12

(Continued)

Temp. °F	Abs. Pressure 44 lb/in. <sup>2</sup> (Sat'n Temp. 31.0 F)			Temp. °F	Abs. Pressure 52 lb/in. <sup>2</sup> (Sat'n Temp. 40.4 F)		
	<i>v</i>	<i>h</i>	<i>s</i>		<i>v</i>	<i>h</i>	<i>s</i>
(at sat'n)	(0.922)	(81.72)	(0.16882)	(at sat'n)	(0.788)	(82.75)	(0.16831)
40	0.943	83.03	0.17142	50	0.808	84.17	0.17114
				60	0.827	85.65	.17400
50	0.966	84.48	0.17432	70	0.847	87.14	.17684
60	0.989	85.96	.17717	80	0.867	88.64	.17966
70	1.012	87.45	.18000	90	0.886	90.15	.18244
80	1.035	88.94	.18279				
90	1.058	90.44	.18556	100	0.906	91.68	0.18518
				110	0.925	93.22	.18789
100	1.080	91.95	0.18828	120	0.945	94.77	.19059
110	1.103	93.48	.19099	130	0.964	96.33	.19325
120	1.125	95.02	.19367	140	0.983	97.90	.19590
130	1.147	96.57	.19630				
140	1.170	98.14	.19895	150	1.002	99.48	0.19850
				160	1.021	101.07	.20109
150	1.192	99.72	0.20154	170	1.040	102.68	.20365
160	1.214	101.31	.20412	180	1.059	104.30	.20621
170	1.236	102.91	.20667	190	1.078	105.93	.20873
180	1.258	104.52	.20922				
190	1.280	106.14	.21173	200	1.097	107.58	0.21125
				210	1.116	109.23	.21374
200	1.302	107.78	0.21424	220	1.134	110.89	.21620
210	1.324	109.42	.21672	230	1.153	112.56	.21865
220	1.346	111.08	.21918	240	1.172	114.26	.22110
230	1.367	112.75	.22161				
240	1.389	114.44	.22405	250	1.190	115.96	0.22352
				260	1.208	117.67	.22591
250	1.411	116.14	0.22646	270	1.227	119.40	.22829
260	1.432	117.85	.22885	280	1.245	121.14	.23065
270	1.454	119.57	.23123	290	1.263	122.90	.23299
280	1.475	121.31	.23359				
290	1.496	123.06	.23592	300	1.281	124.66	0.23532
				310	1.298	126.42	.23763
300	1.518	124.82	0.23826				
310	1.539	126.59	.24058				

# THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12

(Continued)

Temp °F	Abs Pressure 60 lb/in. <sup>2</sup> (Sat n Temp 48.7 F)		
	<i>v</i>	<i>h</i>	<i>s</i>
(at sat n)	(0.688)	(83.65)	(0.16791)
50	0.690	83.83	0.16829
60	0.708	85.33	17120
70	0.726	86.84	17407
80	0.743	88.35	17689
90	0.760	89.87	17968
100	0.778	91.41	0.18246
110	0.795	92.96	18519
120	0.812	94.51	18789
130	0.829	96.07	19056
140	0.846	97.65	19323
150	0.863	99.24	0.19585
160	0.880	100.84	19846
170	0.897	102.45	20104
180	0.913	104.07	20360
190	0.930	105.71	20613
200	0.946	107.36	0.20865
210	0.962	109.02	21113
220	0.979	110.69	21361
230	0.995	112.37	21607
240	1.012	114.06	21853
250	1.028	115.77	0.22094
260	1.044	117.49	22334
270	1.060	119.23	22573
280	1.076	120.97	22810
290	1.092	122.73	23045
300	1.108	124.50	0.23280
310	1.124	126.28	23513
320	1.140	128.07	23745

Temp °F	Abs Pressure 100 lb/in. <sup>2</sup> (Sat n Temp 80.9 F)		
	<i>v</i>	<i>h</i>	<i>s</i>
(at sat n)	(0.419)	(86.89)	(0.16659)
90	0.430	88.32	0.16926
100	0.442	89.93	0.17210
110	0.454	91.54	17493
120	0.465	93.15	17773
130	0.477	94.76	18049
140	0.488	96.37	18321
150	0.499	97.99	0.18590
160	0.510	99.63	18856
170	0.521	101.28	19120
180	0.531	102.94	19381
190	0.542	104.61	19638
200	0.553	106.29	0.19894
210	0.563	107.98	20148
220	0.574	109.68	20401
230	0.585	111.39	20650
240	0.595	113.11	20899
250	0.606	114.84	0.21145
260	0.616	116.58	21389
270	0.626	118.33	21631
280	0.636	120.10	21870
290	0.646	121.88	22108
300	0.657	123.67	0.22347
310	0.667	125.47	22583
320	0.677	127.28	22817
330	0.687	129.10	23050
340	0.697	130.94	23281
350	0.707	132.80	0.23510
360	0.718	134.68	23738



**THERMODYNAMIC PROPERTIES OF SUPERHEATED FREON, F-12**  
(Continued)

Temp. °F	Abs. Pressure 140 lb/in. <sup>2</sup> (Sat'n Temp. 104.5 F)			Temp. °F	Abs. Pressure 207 lb/in. <sup>2</sup> (Sat'n Temp. 132.1 F)			
	<i>t</i>	<i>v</i>	<i>h</i>		<i>s</i>	<i>t</i>	<i>v</i>	<i>h</i>
(at sat'n)	(0.298)	(88.99)	(0.16566)	(at sat'n)	(0.202)	(90.86)	(0.16424)	
110	0.304	89.92	0.16725	140	0.208	92.30	0.16661	
120	0.314	91.60	.17021					
130	0.323	93.28	.17306	150	0.216	94.10	0.16966	
140	0.332	94.96	.17590	160	0.224	95.90	.17262	
				170	0.231	97.70	.17551	
150	0.341	96.65	0.17868	180	0.238	99.51	.17838	
160	0.350	98.34	.18142	190	0.245	101.32	.18115	
170	0.358	100.03	.18412					
180	0.366	101.72	.18678	200	0.252	103.13	0.18388	
190	0.374	103.42	.18941	210	0.258	104.94	.18659	
				220	0.265	106.76	.18927	
200	0.383	105.14	0.19205	230	0.272	108.58	.19192	
210	0.391	106.86	.19466	240	0.278	110.40	.19455	
220	0.399	108.59	.19724					
230	0.407	110.33	.19976	250	0.284	112.23	0.19713	
240	0.415	112.09	.20229	260	0.290	114.06	.19967	
				270	0.296	115.89	.20217	
250	0.423	113.85	0.20479	280	0.302	117.73	.20467	
260	0.431	115.63	.20728	290	0.308	119.58	.20715	
270	0.439	117.42	.20974					
280	0.447	119.22	.21219	300	0.314	121.44	0.20961	
290	0.455	121.03	.21461	310	0.320	123.31	.21204	
				320	0.326	125.19	.21445	
300	0.462	122.85	0.21701					
310	0.470	124.67	.21939					
320	0.477	126.50	.22174					

# APPENDIX C STEAM TABLES\*

TABLE 1. SATURATION: TEMPERATURES

Temp. Fahr. t	Abs. Press. Lb Sq in. p	Specific Volume			Enthalpy			Entropy			Temp. Fahr. t
		Sat. Liquid v <sub>f</sub>	Evap. v <sub>fg</sub>	Sat. Vapor v <sub>g</sub>	Sat. Liquid h <sub>f</sub>	Evap. h <sub>fg</sub>	Sat. Vapor h <sub>g</sub>	Sat. Liquid s <sub>f</sub>	Evap. s <sub>fg</sub>	Sat. Vapor s <sub>g</sub>	
32°	0.08854	0.01602	3300	3.006	0.00	1074.8	1075.8	0.0000	2.1877	2.1877	32°
35	0.09005	0.01602	2917	2917	3.02	1074.1	1077.1	0.0001	2.1799	2.1770	35
40	0.12170	0.01602	2444	2444	8.05	1071.3	1079.3	0.0102	2.1135	2.1507	40
45	0.14752	0.01602	2030.1	2030.1	13.00	1068.1	1081.5	0.0262	2.1107	2.1429	45
50	0.17811	0.01603	1703.2	1703.2	18.07	1065.0	1083.7	0.0361	2.0903	2.1261	50
60°	0.2563	0.01604	1200.0	1200.7	28.00	1059.0	1088.0	0.0555	2.0303	2.0648	60°
70	0.3631	0.01606	807.8	807.9	38.04	1054.3	1082.3	0.0715	1.9902	2.0647	70
80	0.5060	0.01608	633.1	633.1	48.02	1048.6	1080.6	0.0932	1.9128	2.0360	80
90	0.6982	0.01610	468.0	468.0	57.90	1042.0	1100.0	0.1115	1.8072	2.0087	90
100	0.9492	0.01613	350.3	350.1	67.97	1037.2	1105.2	0.1295	1.8531	1.9826	100
110°	1.2748	0.01617	265.3	265.1	77.04	1031.6	1109.5	0.1471	1.8106	1.9577	110°
120	1.6924	0.01620	203.25	203.27	87.02	1025.8	1113.7	0.1645	1.7691	1.9349	120
130	2.2225	0.01625	157.32	157.32	97.80	1020.0	1117.9	0.1816	1.7296	1.9112	130
140	2.8886	0.01629	122.90	122.90	107.80	1014.1	1122.0	0.1984	1.6910	1.8804	140
150	3.718	0.01631	97.06	97.07	117.80	1008.2	1126.1	0.2140	1.6537	1.8685	150
160°	4.711	0.01639	77.27	77.20	127.80	1002.3	1130.2	0.2311	1.6171	1.8485	160°
170	5.992	0.01645	62.04	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.8263	170
180	7.510	0.01651	50.21	50.23	147.92	990.2	1138.1	0.2630	1.5480	1.8100	180
190	9.349	0.01657	40.91	40.96	157.95	984.1	1142.0	0.2785	1.5147	1.7932	190
200	11.529	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4821	1.7762	200
210°	14.123	0.01670	27.80	27.82	178.05	971.0	1149.7	0.3090	1.4508	1.7598	210°
212	14.696	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4410	1.7506	212
220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.7440	220
230	20.780	0.01684	19.365	19.382	198.23	958.8	1157.0	0.3387	1.3901	1.7288	230
240	24.969	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.7140	240

250°	29 825	0.01700	13 894	13 821	218 48	945 5	1164 0	0.3375	1 3323	1 6698	250°
260	35 429	0.01707	11 746	11 763	228 64	938 7	1167 3	0.3317	1 3043	1 6890	260
270	41 853	0.01717	10 044	10 061	238 06	931 8	1170 6	0.3258	1 2769	1 6727	270
280	49 203	0.01726	8 628	8 645	249 06	924 7	1173 8	0.4096	1 2501	1 6597	280
290	57 556	0.01735	7 444	7 461	259 31	917 5	1176 8	0.4234	1 2238	1 6472	290
300°	67 013	0.01745	6 449	6 466	269 59	910 1	1179 7	0.4369	1 1980	1 6350	300°
310	77 08	0.01755	5 609	5 626	279 92	902 6	1182 5	0.4504	1 1727	1 6231	310
320	89 06	0.01765	4 896	4 914	290 28	894 9	1185 2	0.4637	1 1478	1 6115	320
330	103 06	0.01776	4 289	4 307	300 68	887 0	1187 7	0.4769	1 1233	1 6002	330
340	118 01	0.01787	3 770	3 788	311 13	879 0	1190 1	0.4900	1 0992	1 5891	340
350°	134 63	0.01799	3 334	3 352	321 63	870 7	1192 3	0.5029	1 0754	1 5783	350°
360	153 04	0.01811	2 939	2 957	332 18	862 2	1194 4	0.5158	1 0519	1 5677	360
370	173 37	0.01823	2 606	2 625	342 79	853 5	1196 3	0.5286	1 0287	1 5573	370
380	195 77	0.01835	2 317	2 335	353 45	844 6	1198 1	0.5413	1 0059	1 5471	380
390	220 37	0.01850	2 0651	2 0836	364 17	835 4	1199 6	0.5539	0 9832	1 5371	390
400°	247 31	0.01864	1 8447	1 8623	374 97	826 0	1201 0	0.5664	0 9608	1 5272	400°
410	276 75	0.01878	1 6512	1 6700	385 83	816 3	1202 1	0.5788	0 9388	1 5174	410
420	308 83	0.01894	1 4811	1 5000	396 77	806 3	1203 1	0.5912	0 9166	1 5078	420
430	343 72	0.01910	1 3308	1 3499	407 79	796 0	1203 8	0.6035	0 8947	1 4982	430
440	381 59	0.01926	1 1979	1 2171	418 90	785 4	1204 3	0.6158	0 8730	1 4887	440
450°	422 6	0.0194	1 0799	1 0993	430 1	774 5	1204 6	0.6280	0 8513	1 4793	450°
460	466 9	0.0196	0 9748	0 9944	441 4	763 2	1204 6	0.6402	0 8298	1 4700	460
470	514 7	0.0198	0 8811	0 9009	452 8	751 5	1204 3	0.6523	0 8083	1 4606	470
480	566 1	0.0200	0 7972	0 8172	464 4	739 4	1203 7	0.6645	0 7868	1 4513	480
490	621 4	0.0202	0 7221	0 7423	476 0	726 8	1202 8	0.6766	0 7653	1 4419	490
500°	680 8	0.0204	0 6545	0 6749	487 8	713 9	1201 7	0.6887	0 7438	1 4325	500°
510	742 4	0.0206	0 5944	0 6149	499 9	698 4	1199 2	0.7008	0 7223	1 4236	510
520	808 5	0.0215	0 5434	0 5640	512 6	686 6	1193 2	0.7130	0 7008	1 4136	520
530	880 1	0.0221	0 4947	0 5163	526 2	674 2	1189 4	0.7251	0 6793	1 3942	530
540	956 8	0.0228	0 4499	0 4723	540 9	661 9	1187 3	0.7372	0 6578	1 3742	540
550	1039 8	0.0235	0 4089	0 4321	556 9	649 4	1185 5	0.7493	0 6363	1 3542	550
560	1129 9	0.0242	0 3712	0 3953	574 0	636 6	1183 5	0.7614	0 6148	1 3342	560
570	1218 6	0.0249	0 3367	0 3617	592 6	623 2	1181 5	0.7735	0 5933	1 3142	570
580	1306 8	0.0256	0 3054	0 3313	611 7	610 1	1179 5	0.7856	0 5718	1 2942	580
590	1394 9	0.0263	0 2772	0 3040	631 9	596 6	1177 5	0.7977	0 5503	1 2742	590
600°	1483 6	0.0270	0 2519	0 2796	653 4	583 4	1175 5	0.8098	0 5288	1 2542	600°
610	1573 6	0.0277	0 2294	0 2580	676 6	569 9	1173 5	0.8219	0 5073	1 2342	610
620	1665 7	0.0284	0 2097	0 2392	701 3	556 6	1171 5	0.8340	0 4858	1 2142	620
630	1760 4	0.0291	0 1926	0 2231	727 3	543 4	1169 5	0.8461	0 4643	1 1942	630
640	1858 6	0.0298	0 1779	0 2094	754 3	530 2	1167 5	0.8582	0 4428	1 1742	640
650	1960 1	0.0305	0 1654	0 1989	782 3	517 0	1165 5	0.8703	0 4213	1 1542	650
660	2065 7	0.0312	0 1549	0 1894	811 3	503 6	1163 5	0.8824	0 4000	1 1342	660
670	2175 1	0.0319	0 1464	0 1820	841 4	490 1	1161 5	0.8945	0 3785	1 1142	670
680	2289 1	0.0326	0 1397	0 1765	872 3	476 0	1159 5	0.9066	0 3570	1 0942	680
690	2407 7	0.0333	0 1347	0 1729	904 3	462 6	1157 5	0.9187	0 3355	1 0742	690
700°	2530 3	0.0340	0 1303	0 1699	937 3	449 1	1155 5	0.9308	0 3140	1 0542	700°
705 4	2566 2	0.0343	0 1289	0 1685	952 7	440 0	1153 5	0.9429	0 2925	1 0342	705 4

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TABLE 2. SATURATION: PRESSURES

Abs. Press. Lb Sq In. p	Temp. Fahr. t	Specific Volume		Enthalpy			Entropy			Internal Energy		Abs. Press. Lb Sq In. p
		Sat. Liquid v <sub>f</sub>	Sat. Vapor v <sub>g</sub>	Sat. Liquid h <sub>f</sub>	Evap. h <sub>fg</sub>	Sat. Vapor h <sub>g</sub>	Sat. Liquid s <sub>f</sub>	Evap. s <sub>fg</sub>	Sat. Vapor s <sub>g</sub>	Sat. Liquid u <sub>f</sub>	Sat. Vapor u <sub>g</sub>	
1.0	101.74	0.01014	333.6	69.70	1036.3	1106.0	0.1326	1.8456	1.9782	69.70	1044.3	1.0
2.0	126.08	0.01023	173.73	93.90	1022.2	1110.2	0.1749	1.7451	1.9200	93.98	1051.9	2.0
3.0	141.48	0.01030	118.71	109.37	1013.2	1122.6	0.2008	1.6855	1.8863	109.36	1056.7	3.0
4.0	152.97	0.01036	90.63	120.86	1006.4	1127.3	0.2198	1.6427	1.8925	120.85	1060.2	4.0
5.0	162.24	0.01040	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	130.12	1063.1	5.0
6.0	170.06	0.01045	61.98	137.00	996.2	1134.2	0.2472	1.5820	1.8292	137.04	1065.4	6.0
7.0	176.85	0.01049	53.64	144.76	992.1	1136.9	0.2581	1.5586	1.8107	144.74	1067.4	7.0
8.0	182.80	0.01053	47.34	150.70	988.5	1139.3	0.2674	1.5383	1.8037	150.77	1069.2	8.0
9.0	188.28	0.01056	42.40	156.22	985.2	1141.4	0.2759	1.5203	1.7962	156.19	1070.8	9.0
10	193.21	0.01059	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	161.14	1072.2	10
14.096	212.00	0.01072	20.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	1077.5	14.096
15	213.03	0.01072	20.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	181.06	1077.8	15
20	227.96	0.01083	20.080	196.10	960.1	1156.3	0.3356	1.3962	1.7319	196.10	1081.9	20
25	240.07	0.01092	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7139	208.34	1085.1	25
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6993	218.73	1087.8	30
35	259.28	0.01708	11.898	227.91	939.2	1167.1	0.3807	1.3063	1.6870	227.80	1090.1	35
40	267.25	0.01715	10.408	236.03	933.7	1169.7	0.3919	1.2844	1.6763	235.90	1092.0	40
45	274.44	0.01721	9.401	243.36	928.6	1172.0	0.4019	1.2650	1.6660	243.22	1093.7	45
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249.43	1095.3	50
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	256.12	1096.7	55
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	261.90	1097.9	60
65	297.07	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	267.23	1099.1	65
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	272.38	1100.2	70
75	307.60	0.01753	5.816	277.43	904.5	1181.9	0.4472	1.1787	1.6260	277.10	1101.2	75
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	281.76	1102.1	80
85	316.25	0.01761	5.168	286.30	897.8	1184.2	0.4587	1.1571	1.6155	286.11	1102.9	85
90	320.27	0.01766	4.896	290.50	894.7	1185.3	0.4641	1.1471	1.6112	290.27	1103.7	90
95	324.12	0.01770	4.652	294.56	891.7	1186.2	0.4692	1.1376	1.6068	294.25	1104.5	95
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	298.08	1105.2	100
110	334.77	0.01782	4.040	305.66	883.2	1188.0	0.4832	1.1117	1.5948	305.30	1106.5	110

120	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0862	1.5878	312.05	1107.6	120
130	347.32	0.01708	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	318.38	1108.6	130
140	353.02	0.01627	3.220	324.82	868.2	1193.0	0.5069	1.0852	1.5751	324.35	1109.6	140
150	358.42	0.01546	3.015	330.51	863.6	1194.1	0.5138	1.0556	1.5694	330.01	1110.5	150
160	363.53	0.01465	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	335.39	1111.2	160
170	368.41	0.01382	2.675	341.09	854.9	1196.0	0.5266	1.0324	1.5590	340.52	1111.9	170
180	373.06	0.01302	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	345.42	1112.5	180
190	377.51	0.01223	2.404	350.79	846.8	1197.6	0.5381	1.0116	1.5497	350.15	1113.1	190
200	381.79	0.01143	2.288	355.36	843.0	1198.4	0.5435	1.0018	1.5453	354.68	1113.7	200
250	400.95	0.01065	1.8438	376.00	825.1	1201.1	0.5675	0.9588	1.5263	375.14	1115.6	250
300	417.33	0.01000	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	393.79	1117.1	300
350	431.72	0.01913	1.3260	409.69	794.2	1203.9	0.6056	0.8910	1.4965	408.45	1118.0	350
400	444.59	0.0193	1.1613	424.0	780.5	1204.5	0.6214	0.8620	1.4844	422.6	1118.5	400
450	456.28	0.0195	1.0320	437.2	767.4	1204.6	0.6356	0.8378	1.4734	435.5	1118.7	450
500	467.01	0.0197	0.9278	449.4	755.0	1204.4	0.6487	0.8147	1.4634	447.6	1118.6	500
550	476.94	0.0199	0.8424	460.8	743.1	1203.9	0.6608	0.7934	1.4542	458.8	1118.2	550
600	486.21	0.0201	0.7668	471.6	731.6	1203.2	0.6720	0.7734	1.4454	469.4	1117.7	600
650	494.90	0.0203	0.7083	481.8	720.5	1202.3	0.6826	0.7548	1.4374	479.4	1117.1	650
700	503.10	0.0205	0.6554	491.5	709.7	1201.2	0.6925	0.7371	1.4296	488.8	1116.3	700
750	510.86	0.0207	0.6092	500.8	699.2	1200.0	0.7019	0.7204	1.4223	498.0	1115.4	750
800	518.23	0.0209	0.5687	509.7	688.9	1198.6	0.7108	0.7045	1.4153	506.6	1114.4	800
850	525.26	0.0210	0.5327	518.2	678.8	1197.1	0.7194	0.6891	1.4085	515.0	1113.3	850
900	531.98	0.0212	0.5008	526.6	668.8	1195.4	0.7275	0.6744	1.4020	523.1	1112.1	900
950	538.43	0.0214	0.4717	534.6	659.1	1193.7	0.7355	0.6602	1.3957	530.9	1110.8	950
1000	544.61	0.0216	0.4456	542.4	649.4	1191.8	0.7430	0.6467	1.3897	538.4	1109.4	1000
1100	556.31	0.0220	0.4001	557.4	630.4	1187.8	0.7575	0.6205	1.3780	552.9	1106.4	1100
1200	567.22	0.0223	0.3619	571.7	611.7	1183.4	0.7711	0.5956	1.3667	566.7	1103.0	1200
1300	577.46	0.0227	0.3293	585.4	593.2	1178.6	0.7840	0.5719	1.3559	580.0	1099.4	1300
1400	587.10	0.0231	0.3012	598.7	574.7	1173.4	0.7963	0.5491	1.3454	592.7	1095.1	1400
1500	596.23	0.0235	0.2765	611.6	556.3	1167.9	0.8082	0.5269	1.3351	605.1	1091.2	1500
2000	635.82	0.0257	0.1878	671.7	463.4	1135.1	0.8619	0.4230	1.2849	662.2	1065.6	2000
2500	668.13	0.0287	0.1307	730.6	360.5	1091.1	0.9125	0.3197	1.2322	717.3	1030.6	2500
3000	695.35	0.0346	0.0858	802.5	217.8	1020.3	0.9731	0.1885	1.1615	783.4	973.7	3000
3206.2	705.40	0.0503	0.0503	902.7	0	902.7	1.0580	0	1.0580	872.9	872.9	3206.2

TABLE 3. SUPERHEATED VAPOR

Temperature—Degrees Fahrenheit														
	200°	300°	400°	500°	600°	700°	800°	900°	1000°	1100°	1200°	1400°	1600°	1800°
Abs. Press. Lb./Sq. in. (Sat. Temp.)														
1 (101.74) s	392.6 1150.4 2.0512	452.3 1195.8 2.1153	512.0 1241.7 2.1720	571.0 1288.3 2.2233	631.2 1335.7 2.2702	690.8 1383.8 2.3137	750.4 1432.8 2.3642	809.0 1482.7 2.4023	869.5 1533.5 2.4283	929.1 1585.2 2.4625	988.7 1637.7 2.4952	1107.8 1745.7 2.5500	1227.0 1857.5 2.6137	1346.2 1969.7 2.6685
5 (102.24) s	78.16 1148.8 1.8718	90.25 1195.0 1.9370	102.26 1241.2 1.9942	114.22 1288.0 2.0466	126.10 1335.4 2.0927	138.10 1383.6 2.1361	150.03 1432.7 2.1767	161.05 1482.6 2.2148	173.87 1533.4 2.2500	185.70 1585.1 2.2851	197.71 1637.7 2.3178	221.0 1745.7 2.3702	245.4 1857.4 2.4363	269.8 1969.7 2.4911
10 (103.21) s	38.85 1146.0 1.7927	45.00 1193.0 1.8505	51.04 1240.6 1.9172	57.05 1287.5 1.9680	63.03 1335.1 2.0160	69.01 1383.4 2.0590	74.98 1432.5 2.1002	80.95 1482.4 2.1383	86.92 1533.2 2.1744	92.88 1585.0 2.2080	98.84 1637.0 2.2413	110.77 1745.6 2.3028	122.69 1857.3 2.3598	1346.2 1969.7 2.4146
14.696 (212.00) s	30.53 1192.8 1.8160	34.08 1230.9 1.8743	38.78 1287.1 1.9261	42.86 1334.8 1.9734	46.94 1383.2 2.0170	51.00 1432.3 2.0576	55.07 1482.3 2.0958	59.13 1533.1 2.1319	63.19 1584.8 2.1662	67.25 1637.5 2.1980	71.31 1687.0 2.2263	75.37 1745.5 2.2603	83.48 1857.3 2.3174	95.42 1969.7 2.3722
20 (227.96) s	22.36 1191.0 1.7808	25.43 1230.2 1.8306	28.46 1280.6 1.8818	31.47 1334.4 1.9392	34.47 1382.9 1.9920	37.46 1432.1 2.0235	40.45 1482.1 2.0618	43.44 1533.0 2.0978	46.42 1584.7 2.1321	49.41 1637.4 2.1648	52.40 1687.0 2.1983	55.37 1745.4 2.2303	61.34 1857.2 2.2834	73.28 1969.7 2.3382
40 (207.25) s	11.040 1186.8 1.6994	12.628 1230.5 1.7508	14.168 1284.8 1.8010	15.688 1331.3 1.8510	17.198 1381.9 1.9058	18.702 1431.3 1.9467	20.20 1481.4 1.9850	21.70 1532.4 2.0212	23.20 1584.3 2.0555	24.69 1637.0 2.0883	26.18 1687.0 2.1148	27.68 1743.1 2.1408	30.00 1857.0 2.2069	41.94 1969.7 2.2617
60 (202.71) s	7.259 1181.6 1.6492	8.357 1233.6 1.6996	9.403 1283.0 1.7508	10.427 1331.8 1.8012	11.441 1380.9 1.8505	12.440 1430.5 1.9015	13.452 1480.1 1.9400	14.454 1531.0 1.9762	15.453 1583.8 2.0100	16.451 1636.6 2.0434	17.448 1686.7 2.0719	18.446 1744.8 2.1049	20.44 1856.5 2.1621	31.88 1969.7 2.2170
80 (312.03) s	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791	6.220 1230.7 1.6791
100 (327.81) s	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518	4.937 1227.6 1.6518

120 (341 25)	4 081 1224 4 1 6287	4 636 1277 2 1 6869	5 165 1327 7 1 7370	5 683 1377 8 1 7822	6 195 1428 1 1 8237	6 702 1478 8 1 8625	7 247 1530 2 1 8990	7 710 1582 4 1 9385	8 212 1635 3 1 9664	8 714 1743 9 2 0281	10 213 1856 0 2 0854
140 (353 02)	3 408 1221 1 1 6087	3 954 1275 2 1 6683	4 413 1326 4 1 7190	4 861 1376 8 1 7645	5 301 1427 3 1 8063	5 738 1478 2 1 8451	6 172 1529 7 1 8817	6 604 1581 9 1 9163	7 035 1634 9 1 9493	7 895 1743 5 2 0110	8 762 1855 7 2 0683
160 (363 53)	3 008 1217 6 1 5908	3 443 1273 1 1 6519	3 849 1325 0 1 7033	4 244 1375 7 1 7491	4 631 1426 4 1 7911	5 015 1477 5 1 8301	5 396 1529 1 1 8667	5 775 1581 4 1 9014	6 152 1634 5 1 9344	6 906 1743 2 1 9982	7 656 1855 5 2 0535
180 (373 06)	2 649 1214 0 1 5745	3 044 1271 0 1 6373	3 411 1323 5 1 6894	3 764 1374 7 1 7355	4 110 1425 6 1 7776	4 432 1476 8 1 8167	4 792 1528 6 1 8534	5 129 1581 0 1 8882	5 466 1634 1 1 9212	6 186 1742 9 1 9831	6 904 1855 2 2 0404
200 (381 79)	2 361 1210 3 1 5594	2 726 1268 9 1 6240	3 090 1322 1 1 6767	3 380 1373 6 1 7232	3 693 1424 8 1 7655	4 002 1476 2 1 8048	4 309 1528 0 1 8415	4 613 1580 5 1 8763	4 917 1633 7 1 9094	5 521 1742 6 1 9713	6 123 1855 0 2 0287
220 (389 86)	2 125 1206 5 1 5453	2 465 1266 7 1 6117	2 772 1320 7 1 6652	3 066 1372 6 1 7130	3 352 1424 0 1 7545	3 634 1475 5 1 7930	3 913 1527 5 1 8308	4 101 1580 0 1 8656	4 467 1633 3 1 8987	5 017 1742 3 1 9607	5 565 1854 7 2 0181
240 (397 37)	1 9276 1202 5 1 5319	2 247 1264 5 1 6003	2 533 1319 2 1 6546	2 804 1371 5 1 7017	3 068 1423 2 1 7444	3 327 1474 8 1 7839	3 584 1526 9 1 8209	3 839 1579 6 1 8558	4 093 1632 9 1 8889	4 597 1742 0 1 9510	5 100 1854 5 2 0084
260 (404 42)	1 7675 1257 9 1 5701	2 063 1262 3 1 5897	2 330 1317 7 1 6447	2 582 1370 4 1 6922	2 827 1422 3 1 7352	3 067 1474 2 1 7748	3 305 1526 3 1 8118	3 541 1579 1 1 8407	3 776 1632 5 1 8799	4 242 1741 7 1 9420	4 707 1854 2 1 9993
280 (411 05)	1 6047 1260 0 1 5796	1 9047 1260 0 1 5796	2 156 1316 2 1 6354	2 392 1369 4 1 6834	2 621 1421 5 1 7265	2 845 1473 5 1 7662	3 066 1525 8 1 8033	3 286 1578 6 1 8383	3 504 1632 1 1 8716	3 938 1741 4 1 9337	4 370 1854 0 1 9912
300 (417 33)	1 4623 1251 5 1 5481	1 7675 1257 9 1 5701	2 005 1314 7 1 6258	2 227 1368 3 1 6751	2 442 1420 6 1 7184	2 652 1472 8 1 7582	2 859 1525 2 1 7954	3 065 1578 1 1 8305	3 269 1631 7 1 8638	3 674 1741 0 1 9260	4 078 1853 7 1 9835
320 (431 72)	1 2851 1245 1 1 5281	1 4023 1251 5 1 5481	1 7036 1310 9 1 6070	1 8980 1363 5 1 6563	2 084 1418 5 1 7002	2 266 1471 1 1 7403	2 445 1523 8 1 7777	2 622 1577 0 1 8130	2 798 1630 7 1 8463	3 147 1740 3 1 9086	3 493 1853 1 1 9663
400 (444 59)	1 2851 1245 1 1 5281	1 2851 1245 1 1 5281	1 4770 1306 9 1 5894	1 6508 1363 7 1 6398	1 8161 1416 4 1 6842	1 9767 1469 4 1 7247	2 134 1522 7 1 7623	2 290 1575 5 1 7977	2 445 1629 6 1 8311	2 751 1739 5 1 8636	3 055 1852 5 1 9513

TABLE 3. SUPERHEATED VAPOR (Continued)

Temperature—Degrees Fahrenheit													
		500°	550°	600°	650°	700°	800°	900°	1000°	1200°	1400°	1600°	
Abs. Press. Lb./Sq In. (Sat. Temp.)		1.1231	1.2155	1.3005	1.3332	1.3652	1.3967	1.4278	1.4581	1.4874	1.5110	1.5310	2.714
	v	1238.4	1272.0	1302.8	1314.0	1320.2	1327.5	1348.8	1359.9	1414.3	1467.7	1521.0	1851.9
	s	1.5095	1.5437	1.5735	1.5845	1.5951	1.6054	1.6153	1.6250	1.6600	1.7108	1.7510	1.9381
500	v	0.9027	1.0800	1.1501	1.1893	1.2188	1.2478	1.2763	1.3044	1.4405	1.5715	1.6900	2.442
	h	1231.3	1266.8	1298.6	1310.7	1322.6	1334.2	1345.7	1357.0	1412.1	1466.0	1519.9	1851.3
	s	1.4019	1.5280	1.5588	1.5701	1.5810	1.5915	1.6010	1.6115	1.6571	1.6982	1.7303	1.9292
550	v	0.8852	0.9686	1.0431	1.0714	1.0980	1.1259	1.1523	1.1783	1.3038	1.4241	1.5414	2.210
	h	1223.7	1261.2	1294.3	1306.8	1318.0	1330.8	1342.5	1354.0	1409.0	1464.3	1518.2	1850.6
	s	1.4751	1.5131	1.5451	1.5508	1.5580	1.5787	1.5890	1.5991	1.6452	1.6868	1.7250	1.9155
600	v	0.7947	0.8753	0.9463	0.9720	0.9988	1.0241	1.0480	1.0732	1.1890	1.3013	1.4000	2.033
	h	1215.7	1255.5	1280.0	1302.7	1315.2	1327.4	1339.4	1351.1	1407.7	1462.5	1516.7	1850.0
	s	1.4586	1.4900	1.5223	1.5443	1.5558	1.5667	1.5773	1.5875	1.6343	1.6792	1.7147	1.9050
700	v	0.7277	0.7934	0.8177	0.8411	0.8630	0.8830	0.9007	0.9182	1.0204	1.1253	1.2221	1.7405
	h	1243.2	1280.6	1294.3	1307.5	1319.7	1332.8	1345.0	1356.5	1403.2	1450.0	1513.9	1848.8
	s	1.4722	1.5084	1.5212	1.5333	1.5440	1.5540	1.5645	1.5753	1.6147	1.6573	1.6963	1.8881
800	v	0.6154	0.6779	0.7006	0.7223	0.7433	0.7635	0.7833	0.8033	0.8703	0.9833	1.0470	1.5214
	h	1250.8	1270.7	1285.4	1299.4	1312.9	1325.9	1338.6	1351.0	1398.6	1455.4	1511.0	1847.5
	s	1.4407	1.4803	1.5000	1.5120	1.5250	1.5360	1.5470	1.5572	1.5972	1.6407	1.6801	1.8720
900	v	0.5294	0.5873	0.6080	0.6294	0.6491	0.6680	0.6863	0.7040	0.7710	0.8506	0.9262	1.3509
	h	1215.0	1260.1	1275.0	1290.9	1305.1	1318.8	1332.1	1345.9	1393.9	1451.8	1508.1	1846.3
	s	1.4210	1.4653	1.4800	1.4938	1.5060	1.5187	1.5303	1.5427	1.5814	1.6257	1.6650	1.8505



1000 (544 61)	0 4533 1198 3 1 3961	0 5140 1268 8 1 4450	0 5550 1265 9 1 4610	0 5546 1291 9 1 4757	0 5733 1297 0 1 4803	0 5912 1311 4 1 5021	0 6084 1325 3 1 5141	0 6878 1359 2 1 5670	0 7604 1448 2 1 6121	0 8294 1505 1 1 6525	0 9615 1617 3 1 7245	1 0893 1723 9 1 7856	1 2146 1845 0 1 8474
1100 (558 31)	0 4532 1256 7 1 4251	0 4738 1255 3 1 4425	0 4929 1272 4 1 4653	0 5110 1288 5 1 4883	0 5291 1303 7 1 4728	0 5485 1318 3 1 4862	0 5678 1334 3 1 4989	0 6191 1384 3 1 5535	0 6366 1444 5 1 5995	0 7503 1502 2 1 6405	0 8716 1615 2 1 7130	0 9885 1729 4 1 7775	1 1031 1843 8 1 8363
1200 (567 22)	0 4016 1223 5 1 4052	0 4222 1243 9 1 4243	0 4410 1262 4 1 4413	0 4585 1279 0 1 4568	0 4752 1295 7 1 4710	0 4900 1311 0 1 4843	0 5017 1329 3 1 4909	0 5617 1379 3 1 5409	0 6260 1440 7 1 5970	0 6843 1499 2 1 6293	0 7967 1613 1 1 7025	0 9040 1729 9 1 7672	1 0101 1842 8 1 8263
1400 (587 10)	0 3174 1193 0 1 3639	0 3390 1218 4 1 3877	0 3580 1240 4 1 4079	0 3753 1260 3 1 4258	0 3912 1278 5 1 4419	0 4062 1295 5 1 4567	0 4214 1309 1 1 4677	0 4714 1433 1 1 5060	0 5281 1483 1 1 5560	0 5805 1493 2 1 6093	0 6780 1603 9 1 6836	0 7727 1723 7 1 7489	0 8640 1840 0 1 8083
1600 (604 90)	0 2733 1167 8 1 3489	0 2939 1215 2 1 3741	0 3112 1238 7 1 3952	0 3271 1259 6 1 4137	0 3417 1278 7 1 4303	0 3553 1295 6 1 4444	0 3684 1311 7 1 4576	0 4034 1359 4 1 4804	0 4553 1425 3 1 5147	0 5027 1487 0 1 5314	0 5606 1604 6 1 6069	0 6738 1720 5 1 7398	0 7545 1837 5 1 7920
1800 (621 03)	0 2407 1185 1 1 3377	0 2597 1214 0 1 3538	0 2760 1238 5 1 3655	0 2937 1259 6 1 3855	0 3097 1278 7 1 4044	0 3252 1295 6 1 4232	0 3402 1311 7 1 4421	0 3502 1329 4 1 4610	0 3688 1347 2 1 4800	0 4421 1480 8 1 5752	0 5218 1600 4 1 6520	0 5968 1717 3 1 7185	0 6693 1833 0 1 7786
2000 (635 82)	0 1936 1145 6 1 2945	0 2161 1184 9 1 3300	0 2337 1214 8 1 3564	0 2489 1238 5 1 3833	0 2607 1259 6 1 4044	0 2760 1278 7 1 4232	0 2907 1295 6 1 4421	0 3074 1311 7 1 4610	0 3252 1329 4 1 4800	0 3935 1474 5 1 5603	0 4668 1596 1 1 6384	0 5352 1714 1 1 7055	0 6011 1832 5 1 7660
2500 (668 13)	0 1484 1132 3 1 2687	0 1686 1176 8 1 3073	0 1848 1214 8 1 3564	0 2007 1238 5 1 3855	0 2161 1259 6 1 4044	0 2337 1278 7 1 4232	0 2489 1295 6 1 4421	0 2607 1311 7 1 4610	0 2760 1329 4 1 4800	0 3061 1458 4 1 5273	0 3678 1585 3 1 6088	0 4244 1706 1 1 6775	0 4784 1828 2 1 7389
3000 (695 36)	0 0984 1060 7 1 1966	0 1161 1100 7 1 2687	0 1348 1132 3 1 2945	0 1535 1167 8 1 3203	0 1722 1203 7 1 3461	0 1909 1238 5 1 3719	0 2096 1278 7 1 3977	0 2283 1311 7 1 4232	0 2470 1347 2 1 4480	0 2476 1441 5 1 4984	0 3018 1574 3 1 5837	0 3505 1698 0 1 6540	0 3966 1819 9 1 7163
3200 (705 40)	0 0983 1059 6 1 1965	0 1160 1100 7 1 2687	0 1347 1132 3 1 2945	0 1534 1167 8 1 3203	0 1721 1203 7 1 3461	0 1908 1238 5 1 3719	0 2095 1278 7 1 3977	0 2282 1311 7 1 4232	0 2469 1347 2 1 4480	0 2475 1441 5 1 4984	0 2917 1569 8 1 5837	0 3367 1691 6 1 6540	0 3703 1817 2 1 7080

## APPENDIX D

### SOLUTION OF VAN DER WAALS' EQUATION FOR $V$

Let it be assumed that the pressure  $P$  and the temperature  $T$  of a gas are known, and that the volume  $V$  is to be found by Van der Waals' equation, which is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Inspection will show that a cubic equation will result. The solution of such an equation by synthetic division will be given here to illustrate the procedure. For example, we will consider the following equation for carbon dioxide:

$$\left(61.2 + \frac{925}{V^2}\right)(V - 0.686) = 481$$

This equation reduces to

$$61.2V^3 + 925V - 635 = 523V^2$$

or

$$V^3 - 8.54V^2 + 15.1V - 10.38 = 0$$

The first step in solving this equation by synthetic division is to set down the coefficients and to try various integers as roots until the remainder from the constant term is found to be a minimum. Trying 7 for  $V$ , we obtain:

$$\begin{array}{r|l} 1 & -8.54 + 15.1 & -10.38 \\ +7 & -10.78 + 30.24 \\ \hline 1 & -1.54 + 4.32 + 19.86 \end{array}$$

The fact that the remainder is positive indicates that 7 is too large. So we try 6 and get the following result:

$$\begin{array}{r|l} 1 & -8.54 + 15.1 & -10.38 \\ +6 & -15.24 & 0.84 \\ \hline 1 & -2.54 & -0.14 - 11.22 \\ +6 & +20.76 \\ \hline 1 & +3.46 + 20.62 \\ +6 & \\ \hline 1 & +9.46 \end{array}$$

It is seen that 6 is the proper digit to the left of the decimal point. The digit in the first decimal place may now be found by making up an equation from the remainders in the previous attempt. This new equation is

$$1+9.46+20.62-11.22$$

For the first decimal place, we will try 0.5. The division follows

$$\begin{array}{r|l} 1+9.46+20.62-11.22 & 5 \\ +0.5 + 4.98+12.80 & \\ \hline 1+9.96+25.60+ & 1.58 \end{array}$$

Since 0.5 is too large, we will try 0.4. We then get

$$\begin{array}{r|l} 1+9.46+20.62-11.22 & 4 \\ +0.4 + 3.94+9.82 & \\ \hline 1+9.86+24.56- & 1.40 \\ +0.4 + 4.10 & \\ \hline 1+10.26+28.66 & \\ +0.4 & \\ \hline 1+10.66 & \end{array}$$

If still another decimal place is wanted, a new set of coefficients is

$$1+10.66+28.66-1.40$$

The value in the second decimal place is found to be between .04 and .05, being very close to the latter. Hence, the desired result is  $V=6.45$  cu ft per mol.

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